The three-dimensional steady circulation in a homogenous ocean induced by a stationary hurricane

Zhu Min Lu
Key Laboratory of Tropical Marine Environmental Dynamics,
South China Sea Institute of Oceanology, Guangzhou, China

Rui Xin Huang
Woods Hole Oceanographic Institution, Woods Hole, MA 02543, USA

January 28, 2010


Corresponding author address: Dr. Zhu Min Lu, South China Sea Institute of Oceanology, Chinese Academy of Science, Guangzhou 510301, China
E-mail: luzhumin@scsio.ac.cn
Abstract

Based on the classical Ekman layer theory, a simple analytical solution of the steady flow induced by a stationary hurricane in a homogenous ocean is discussed. The model consists of flow converging in an inward spiral in the deeper layer and diverging in the upper layer.

The simple analytical model indicates that both the upwelling flux and the horizontal transport increase linearly with increasing radius of maximum winds. Furthermore, they both have a parabolic relationship with the maximum wind speed. The Coriolis parameter also affects the upwelling flux: the response to a hurricane is stronger at low latitudes than that at middle latitudes. Numerical solutions based on a regional version of an ocean general circulation model are similar to the primary results obtained through the analytical solution. Thus, the simplifications made in formulating the analytical solution are reasonable.

Although the analytical solution in this paper is sought for a rather idealized ocean, it can help us to understanding results from the more complicated numerical model. These conceptual models provide a theoretical limit structure of the oceanic response to a moving hurricane over a stratified ocean.
1. Introduction

Hurricanes (typhoons) constitute an important component of the atmosphere-ocean system. Recent studies suggest that they contribute a noticeable share of the mechanical energy input into the ocean, acting like a blender in the ocean, and thus significantly influencing the ocean heat transport (Emanuel 2003, Sriver and Huber 2007, Liu et al. 2008, Jansen and Ferrari 2009). However, it is clear that our understanding of these strong nonlinear events remains rather rudimentary at best.

Oceanic response to a hurricane has been studied extensively over the past half century. The early studies were based on the fundamental issues of hurricane-induced motions in the ocean by means of rather idealized models. For example, Longuet-Higgins (1965) studied the response of a stratified ocean to a stationary or moving wind system. Many aspects of hurricane dynamics have been explored by previous investigators such as Geisler (1970), Price (1981, 1983), Greatbatch (1983, 1984), Shay et al. (1989), Price et al. (1994). Most of these studies have focused on the response of the upper ocean to hurricane forcing, in particular cooling of the upper ocean, and mechanical stirring induced by the strong wind stress associated with a hurricane.

There have been a few studies focused on the deep ocean response to hurricanes (Chang 1985, Shay et al. 1989, Ginis and Sutyrin 1995). Recently, there are more observations highlighting the importance of deep ocean response. For example, satellite observations suggest that the hurricane-induced phytoplankton blooms may be mainly attributed to the injection of nutrients from depths as deep as 100m (Babin
et al 2004, Gierach and Subrahmanyam 2008). Since the low level of nutrient concentration in the Sargasso Sea as shown by ocean color analysis, the chlorophyll-a (chl-a) enhancement induced by a hurricane must be associated with the upwelling of cold water from the deep ocean (Babin et al 2004).

Hence, it is of great interest to explore the impact of hurricanes on the circulation of the middle and bottom part of water column. The prominent hurricane-induced upwelling feature within the core of a hurricane is associated with the outward Ekman transport. In fact, the strong upwelling in the center of a hurricane requires a continuous supply of water from the deep ocean.

Although it is rather difficult to observe the response in the deeper layer, a few observations have shown that the deep ocean responses to moving hurricanes in the form of near-inertial oscillation (Shay and Elsberry 1987, Morozov and Velarde 2008). Using current meter measurements, Shay and Elsberry (1987) have reported that bottom currents increased significantly as the features of rapid response. The theoretical study of Greatbatch (1984) has shown that the response to a “large” enough or “fast” enough hurricane will not feel the effect of the ocean stratification and extend through the depth of the ocean.

Over the past ten to twenty years, the study of oceanic response to hurricanes has been extended and refined in many different directions. Based on these new research results, our understanding of hurricane induced circulation has been greatly extended. With such a wealth of new information there is, however, a need to put it into a simple and clear framework.
In order to build up such a framework, we must start from the basic foundation. For the case of a hurricane moving over a continuously stratified ocean, the analytical solution for the oceanic response is extremely complicated; so far, no analytical solution has been found. Thus, we begin by examining the case of a steady hurricane blowing over a homogenous ocean. Under such assumptions a simple analytical solution can be found; such an analytical solution is a great tool helping us to understand the fundamental dynamic aspects of hurricane induced motions in the ocean. Therefore, although such a case does not exist in the real world, this model provides us with a fundamental framework to study the variation of oceanic response due to the gradual increase of moving velocity and stratification.

As the second step, we will move to cases with a moving hurricane over a continuously stratified ocean. With the knowledge building up from the first step, it will be much easier to understand how the circulation changes in response to the gradually increasing moving speed of the hurricane, and how the upwelling and near-inertial oscillations change in response to the gradual build up of stratification.

Hurricanes are one of powerful drivers of oceanic circulation. Although whether hurricanes can contribute to the global oceanic circulation is under debate, it is clear that their contribution to local circulation is vitally important, and that is the focus of this study. In the upper ocean, strong wind stress drives radial outward Ekman transport. This radially outward flow is supported by the balance between frictional force and the Coriolis force.

Due to the continuity of mass, there must be a strong upwelling underneath the
center of the hurricane to supply this mass flux. In the subsurface layer immediately below the surface layer, frictional force is negligible and the flow is primarily cyclo-geostrophic. Thus, any inward cyclo-geostrophic flow must be balanced by the corresponding radial pressure gradient force. Since we assume that the hurricane is axisymmetric, there is no pressure gradient in the azimuthal direction. Therefore, under the assumption of a steady state circulation, the upwelling beneath the center of the hurricane cannot be supplied with mass from the middle depth of the ocean. The only way to supply this upwelling in the center of the hurricane is, thus, from the radial inflow in the bottom boundary layer. This argument applies to a stationary hurricane over a non-stratified ocean only. In the case of a moving hurricane and a stratified ocean, the supply of water feeding the wind-stress driven upwelling in the center of a hurricane can come from rather shallow depth.

Due to the existence of bottom friction, an inward mass flux is possible under the balance between the Coriolis force, the inward pressure gradient and the bottom friction. Because friction in the middle of the water column is assumed to be negligible, the inward radial pressure gradient required for the maintenance of this inflow bottom boundary layer is linked to the deformation of the sea surface elevation. At the middle level, this radial pressure gradient is balanced by the Coriolis force associated with an azimuthal cyclo-geostrophic velocity.

Therefore, the circulation driven by a stationary hurricane consists of four components: the outward Ekman transport in the upper ocean, the inward Ekman transport in the bottom boundary layer, the upwelling in the center of the hurricane,
and the azimuthal cyclo-geostrophic current between the top and bottom boundary layers. These four components combine and are manifested as a beautiful eddy over the whole depth of the ocean, Fig. 1.

It is worthwhile to emphasize that according to Newton's Third Law, the mass transport in the atmospheric boundary layer is exactly the same as the mass transport in the oceanic boundary layer in the upper ocean. i.e.,

\[ M_{\text{atmo}} = M_{\text{ocean,top}} \]

Note that in this formula we have omitted the azimuthal component of the Ekman transport driven by the radial wind stress, which should also obey the same constraint. Similarly, the corresponding azimuthal component of the ageostrophic flow in the bottom boundary layer will be omitted in the discussion below. Since flow in the middle depth of the ocean is assumed to be nearly inviscid, with very little ageostrophic component, the total amount of upwelling and the total amount of inward mass transport in the bottom boundary layer should be nearly the same as the outward mass transport in the atmospheric and oceanic boundary layer in the upper ocean, i.e.,

\[ M_{\text{atmo}} = M_{\text{ocean,top}} \equiv M_{\text{ocean,up}} \equiv M_{\text{ocean,bot}} . \]

In this study, we will discuss the vertical structure of circulation induced by a stationary hurricane using an analytical model. Most importantly, an analytical solution can provide a simplified and succinct picture of the circulation. This paper is outlined as follows. An analytical solution for the circulation induced by a stationary hurricane is presented in Section 2. Section 3 discusses the relationships between
accumulated transport and the physical parameters of a hurricane. A regional version of the Massachusetts Institute of Technology ocean general circulation model (MIT OGCM) described in Marshall et al (1997) is utilized in Section 4 to validate the structure of the circulation induced by a stationary hurricane. Finally, conclusions are summarized in Section 5.

2. Analytical solutions for the circulation driven by a stationary hurricane

As the first step, we examine the oceanic response to a stationary hurricane. The wind stress pattern of a hurricane can be separated into the azimuthal and radial components. The azimuthal component of the wind drives an outward radial Ekman transport in the upper ocean; the inward radial wind component drives a cyclonic Ekman transport in the azimuthal direction; however, this component of Ekman transport does not contribute to the upward motion below the Ekman layer. Thus, it is not linked to the circulation pattern discussed below, and it will be omitted in the following discussion.

Both the hurricane wind field and the induced ocean circulation are axisymmetric. This problem can be analyzed, using the cylinder coordinate system \((r, \theta, z)\), Fig. 2. The origin of the coordinates is at the center of the hurricane and \(z\) increases upward with \(z=0\) on the mean sea surface. For simplicity, the model is formulated for a homogeneous ocean on an f-plane, with a flat bottom. The ocean depth \(H\) is assigned 1200m in this study.

In order to obtain concise analytical solutions, we also assume that the curvature
term (the centrifugal force term) is negligible, compared with other terms in the horizontal momentum balance. The circulation including the curvature term will be examined in connection with the numerical solutions discussed in Section 4. Neglecting the time-dependent terms and the advection terms, the horizontal momentum equations are

\[-f v_\theta = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \frac{\partial}{\partial z} \left( A_z \frac{\partial v_r}{\partial z} \right), \quad (1a)\]

\[f v_r = \frac{\partial}{\partial z} \left( A_z \frac{\partial v_\theta}{\partial z} \right), \quad (1b)\]

where \(v_r, v_\theta\) are the radial and azimuthal velocity, \(p\) is pressure, \(\rho\) is density of sea water, \(f\) is the Coriolis parameter and \(A_z\) is the vertical eddy viscosity.

The parameterization of vertical viscosity is one of the major unsettled problems in dynamical oceanography. Since it is related to the availability of mechanical energy sustaining turbulence in the ocean, it must vary with time and space. In particular, the physical processes that regulate the supply of turbulent kinetic energy in the bottom boundary layer underneath a hurricane remain largely unknown. Due to this great uncertainty, we will make use of a bold assumption that \(A_z\) is constant throughout the water column. This assumption should certainly affect the structure of the solution, and the results should be interpreted with caution.

Equations (1a, 1b) can be simplified by decomposing the velocity into two parts, the geostrophic and ageostrophic components:

\[v_r = v_{r,geo} + v_{r,ageo} = v_{r,geo}, \quad (2a)\]
\[ v_\theta = v_{\theta,\text{geo}} + v_{\theta,\text{ageo}} = \frac{1}{f} \frac{\partial p}{\partial r} + v_{\theta,\text{ageo}}, \quad (2b) \]

where \( v_{r,\text{geo}}, v_{\theta,\text{geo}} \) are the radial and azimuthal geostrophic velocities, which are in balance with the pressure gradient. Since the solution is assumed axisymmetric, there is no azimuthal pressure gradient, and the corresponding geostrophic velocity component vanishes, \( v_{r,\text{geo}} = 0 \). We also assume that the vertical shear of the geostrophic velocity can be neglected, so that the ageostrophic velocity \( \begin{pmatrix} v_{r,\text{ageo}} \\ v_{\theta,\text{ageo}} \end{pmatrix} \) satisfies the following equations

\[ -f v_{\theta,\text{ageo}} = \frac{\partial}{\partial z} \left( A_z \frac{\partial v_{r,\text{ageo}}}{\partial z} \right), \quad (3a) \]

\[ f v_{r,\text{ageo}} = \frac{\partial}{\partial z} \left( A_z \frac{\partial v_{\theta,\text{ageo}}}{\partial z} \right). \quad (3b) \]

Introducing a complex velocity

\[ W_{\text{ageo}} = v_{r,\text{ageo}} + i v_{\theta,\text{ageo}}, \quad (4) \]

Eqs. (3a, 3b) are reduced to

\[ \frac{d^2 W_{\text{ageo}}}{dz^2} - i \frac{f}{A_z} W_{\text{ageo}} = 0. \quad (5) \]

The basic solution of equation (5) is

\[ W_{\text{ageo}} = (c_1 + i d_1) e^{i \lambda z} + (c_2 + i d_2) e^{-i \lambda z}, \quad \lambda = \left( 1 + i \right) / \delta, \quad (6) \]

where \( \delta \) is the Ekman depth defined as

\[ \delta = \sqrt{2 A_z / f}. \quad (7) \]

Four constraints are needed to determine solution (6). At the surface, wind stress in the azimuthal direction \( \tau_\theta \) gives rise to a boundary condition
\[ \rho A_z \frac{dW_{\text{geo}}}{dz} \bigg|_{z=0} = \rho A_z \lambda \left[ (c_1 + id_1) - (c_2 + id_2) \right] = i \tau^0, \quad \text{(8)} \]

i.e.,

\[ (c_1 - c_2) + i(d_1 - d_2) = \frac{1 + i \delta \tau^0}{2 \rho A_z}. \quad \text{(9)} \]

The real/imaginary components of Eq. (9) provide two constraints. On the sea floor \( z = -H \), there is no geostrophic velocity in the radial direction, and the no-slip condition applies to the radial ageostrophic velocity and gives rise to

\[ e^{-H/\delta}[c_1 \cos(H/\delta) + d_1 \sin(H/\delta)] + e^{H/\delta}[c_2 \cos(H/\delta) - d_2 \sin(H/\delta)] = 0. \quad \text{(10)} \]

In the azimuthal direction, the sum of geostrophic and ageostrophic velocity should vanish on the sea floor. However, the geostrophic velocity in the azimuthal direction in the subsurface ocean is one of the unknowns. To overcome this problem, we can use the following constraint: the total volume transport in the radial direction integrated over the whole depth of the ocean should be zero, as required by the mass continuity in a steady state. This constraint can be written in the following form

\[ \text{Re} \left\{ \int_{-H}^{0} [c_1 + id_1]e^{i\tau z} + [c_2 + id_2]e^{-i\tau z}] \, dz \right\} = 0. \quad \text{(11)} \]

Eqs. (9-11) consist of four equations for four unknowns \( c_1, d_1, c_2, d_2 \), and they can be written in the following matrix form

\[ A \mathbf{X} = \mathbf{B}, \quad \text{(12)} \]

\[ A = \begin{bmatrix}
0 & 1 & 0 & -1 \\
e^{-\frac{H}{\delta}} \cos \frac{H}{\delta} & e^{-\frac{H}{\delta}} \sin \frac{H}{\delta} & e^{\frac{H}{\delta}} \cos \frac{H}{\delta} & -e^{\frac{H}{\delta}} \sin \frac{H}{\delta} \\
e^{-\frac{H}{\delta}} \left( 2 \cos \frac{H}{\delta} - \sin \frac{H}{\delta} \right) & e^{\frac{H}{\delta}} \left( \cos \frac{H}{\delta} + 2 \sin \frac{H}{\delta} \right) & -e^{-\frac{H}{\delta}} \sin \frac{H}{\delta} & -e^{-\frac{H}{\delta}} \cos \frac{H}{\delta}
\end{bmatrix}, \quad \text{(13)} \]

\[ \mathbf{B} = \begin{bmatrix}
\delta \tau^0 / 2 \rho A_z & \delta \tau^0 / 2 \rho A_z & \delta \tau^0 / \rho A_z
\end{bmatrix}^T, \quad \text{(14)} \]
\[
X = \begin{bmatrix}
   c_1 & d_1 & c_2 & d_2 \\
\end{bmatrix}^T
\]  

(15)

After solving Eq. (12), the corresponding ageostrophic flow is

\[
v_{r,ageo} = e^\frac{z}{\delta} \left( c_1 \cos \frac{z}{\delta} - d_1 \sin \frac{z}{\delta} \right) + e^{-\frac{z}{\delta}} \left( c_2 \cos \frac{z}{\delta} + d_2 \sin \frac{z}{\delta} \right),
\]  

(16a)

\[
v_{\theta,ageo} = e^\frac{z}{\delta} \left( c_1 \sin \frac{z}{\delta} + d_1 \cos \frac{z}{\delta} \right) + e^{-\frac{z}{\delta}} \left( -c_2 \sin \frac{z}{\delta} + d_2 \cos \frac{z}{\delta} \right).
\]  

(16b)

The geostrophic velocity in the ocean interior can be derived from the constraint that the total azimuthal velocity on the sea floor be zero, i.e.,

\[
v_{\theta,geo} = -v_{\theta,ageo} \bigg|_{z=-H} = e^\frac{H}{\delta} \left( c_1 \sin \frac{H}{\delta} - d_1 \cos \frac{H}{\delta} \right) - e^{-\frac{H}{\delta}} \left( c_2 \sin \frac{H}{\delta} + d_2 \cos \frac{H}{\delta} \right). \quad (17)
\]

Within the framework of our model, the geostrophic velocity in the subsurface ocean has no vertical shear. If \( H > 2\delta \), \( e^{-H/\delta} \) is much smaller than \( e^{H/\delta} \) and thus negligible.

After some manipulation, we obtain the following approximate formula

\[
v_{\theta,geo} \approx \delta \tau^\theta / \rho A_z.
\]

According to Eq. (12), the circulation induced by a hurricane is controlled by three parameters: the ocean depth, the Ekman depth, and the wind stress. In particular, the parameterization of Ekman depth is a crucial part of the analytical solution.

Although the structure of the Ekman layer in the ocean was proposed one hundred years ago, it had not been confirmed by in-situ observations. It was not until the middle of 1980s that a clear picture of the Ekman layer was observed in the upper ocean through instrumentation (Price et al., 1987). Furthermore, in-situ observations indicate that the structure of the surface circulation is different from the Ekman spiral
predicted by the simple theory based on constant viscosity (Chereskin and Roemmich, 1991; Price and Sundermeyer, 1999).

Turbulent motions in the Ekman layer crucially depend on the availability of mechanical energy to sustain the strong dissipation. In most cases, energy supporting mixing in the upper ocean comes primarily from surface wind. Thus, vertical viscosity in the Ekman layer is not constant; instead, it may depend on the wind stress and the latitude. Instead of using Eq. (7), one can also use the following empirical formula

$$\delta = \frac{f}{\rho \frac{\tau}{g}^{\gamma}}$$

where \( \tau \) is the magnitude of wind stress and \( \gamma = 0.25 - 0.40 \), determined from observations (Coleman et al. 1990, Price and Sundermeyer 1999). Based on six datasets available at that time, Wang and Huang (2004) found that the best-fit value of \( \gamma \) is 0.5.

Furthermore, the Ekman layer involved in the hurricane study has to deal with cases of extremely strong winds, beyond the normal range of bulk formula. Thus, there are no adequate in-situ observations to calibrate the parameterization of vertical momentum mixing and Ekman layer depth. As a compromise, both Eqs. (7) and (18) will be utilized in this study to investigate the sensitivity to the parameterization of vertical viscosity. In particular, we will examine the circulation induced by hurricanes under different parameterizations.

\textit{a. Wind stress profile of a hurricane.}

A hurricane rotates similar to a rigid body from its center to the radius of the
maximum winds, and outside of this radius winds decay rapidly with radius (Emanuel 2003). There are some simple models of the wind stress describing the near-surface azimuthal wind of a hurricane, such as a modified Rankine vortex (Ginis and Sutyrin 1995) and the Holland wind stress model (Holland 1980). For simplicity, we adopt the following model for the angular velocity distribution:

\[
\omega = \begin{cases} 
  ar & r < r_0 \\
  a r_0^{b+1} / r^b & r \geq r_0 
\end{cases}, \quad (19)
\]

where \( a, b \) is a constant, \( r_0 \) is the radius of maximum wind speed. In common practice, the maximum wind speed \( V_{\text{max}} \) is more frequently specified as a basic parameter for a hurricane, and constant \( a \) can be inferred from

\[
a = V_{\text{max}} / r_0^2. \quad (20)
\]

For simulating hurricane Katrina in 2005 (Shen et al 2006), the maximum wind speed \( V_{\text{max}} \) is approximately 60 m s\(^{-1}\), and other parameters are \( b = 2.95 \) and \( r_0 = 50 \text{ km} \).

Based on Eq. (19), the corresponding azimuthal velocity is

\[
V = \begin{cases} 
  V_{\text{max}} r^2 / r_0^2 & r < r_0 \\
  V_{\text{max}} r_0^{b-1} / r^{b-1} & r \geq r_0 
\end{cases}. \quad (21)
\]

The azimuthal wind stress is calculated from the bulk formula

\[
t^\theta = \rho_a C_D \rho V^2 = \rho_a C_D V_{\text{max}}^2 F(r), \quad (22)
\]

where \( \rho_a = 1.26 \text{ kg m}^{-3} \) is air density, \( C_D = 2.5 \times 10^{-3} \) is the drag coefficient, and \( F(r) \) is a piecewise function

\[
F(r) = \begin{cases} 
  (r / r_0)^4 & r < r_0 \\
  (r_0 / r)^{2(b-1)} & r \geq r_0 
\end{cases}. \quad (23)
\]

The bulk formula (22) is commonly used, and it is based on observations of low or
moderate wind speed. Recent studies indicate that drag coefficients at high wind speeds are much more complicated (Powell et al. 2003; Donelan et al. 2004; Black et al. 2007). The more accurate parameterization of wind stress at high speed wind typically for hurricanes is a hotly-debated issue and a research frontier. A thorough review and discussion is beyond the scope of our study. For simplicity, we set the drag coefficient as a constant in this study. The peak of wind speed appears clearly in radius 50 km and the corresponding maximum wind stress is approximately 9 N m\(^{-2}\), Fig. 3. In order to explore the sensitivity of the analytical model to parameterization of turbulent mixing in the Ekman layer, we present solutions obtained from the following two parameterizations.

\textit{b. Solutions for a fixed viscosity}\quad A_z = 0.5 m^2 s^{-1}

Due to the strong wind associated with the subtropical cyclones, mixing in the ocean is greatly enhanced (Huang et al. 2007; Sriver and Huber 2007); hence, for a model based on constant vertical viscosity, a large value should be used. In this section, we set \( A_z = 0.5 \) m\(^2\) s\(^{-1}\) and \( \delta \) is calculated from (7). From Eq. (12), it is readily seen that solution (15) depends on the wind stress only. Since wind stress is a function of \( r \) only, the ageostrophic flow (16) can be rewritten as the product of a function of \( z \) and a function of \( r \):

\begin{align*}
\nonumber
v_{r,ageo} &= v_{r,ageo}^{\text{max}}(z)F(r), \quad (24a) \\
\nonumber
v_{\theta,ageo} &= v_{\theta,ageo}^{\text{max}}(z)F(r), \quad (24b)
\end{align*}

where \( v_{r,ageo}^{\text{max}} \) and \( v_{\theta,ageo}^{\text{max}} \) are the radial and azimuthal components of the ageostrophic flow forced by the maximum wind stress. Since the flow is
axisymmetric, the continuity equation is
\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \psi_{r,ageo} \right) + \frac{\partial w}{\partial z} = 0, \]  
(25)
where \( w \) is the vertical velocity. Integrating (25) leads to
\[ w = -\int_{H}^{z} \frac{1}{r} \frac{\partial}{\partial r} \left( r \psi_{r,ageo} \right) \, dz = G(z) P(r), \]  
(26)
where
\[ G(z) = -\int_{H}^{z} \nu_{r,ageo}^{\max}(z) \, dz, \]  
(27)
\[ P(r) = \begin{cases} 5 r^3 / r_0^4 & r < r_0, \\ (3-2b) r_0^{2b-2} / r^{2b-1} & r \geq r_0. \end{cases} \]  
(28)
The flow in the \( r-z \) plane can be conveniently described in terms of a streamfunction
\[ \psi(z, r) = \int_{0}^{r} w \cdot 2 \pi r dr = G(z) Q(r), \]  
(29)
where
\[ Q(r) = \begin{cases} 2 \pi r^5 / r_0^4 & r < r_0, \\ 2 \pi r_0^{2b-2} / r^{2b-3} & r \geq r_0. \end{cases} \]  
(30)
The ageostrophic velocity profiles induced by the maximum wind stress (9N m\(^{-2}\)) at 18\(^\circ\)N and 36\(^\circ\)N are plotted in Fig. 4. According to Eq. (7), boundary layer thickness is inversely proportional to the square root of the Coriolis parameter, and this can be seen from Fig. 4. As a result, the geostrophic flow regime between the upper and lower boundary layers at middle latitudes is more pronounced than that at low latitudes. The strength of the geostrophic flow in the middle depth range can be inferred from the lower boundary condition (17) that the total azimuthal velocity vanishes at the sea floor. The flow in the upper (bottom) Ekman layer appears in the form of an outward (inward) spiral. The geostrophic and ageostrophic flow induced by the same wind stress are stronger at low latitudes than those at middle
latitudes. This has a very important implication that hurricane at low latitudes may contribute more mechanical mixing.

According to formulae (24)-(30), the ageostrophic flow can be concisely described in terms of modal functions $F(r), P(r), Q(r), G(z)$, shown in Figs. 5 and 6. Function $F(r)$ represents the non-dimensional ageostrophic flow profile, so it is similar to the wind stress profile in Fig. 2b. The amplitude of the flow increases with $r$ to the maximum value at $r = r_0$, then decays to zero at about 200 km from the center (Fig. 5b). In the radial direction, both the vertical velocity and upwelling flux attain their maximum at $r = r_0$; however, vertical velocity becomes negative outside the core of the hurricane, i.e., for $r \geq r_0$, and this is consistent with the observation that downwelling appears around the periphery of the maximum wind band of a hurricane (Figs. 5b, 5c).

The vertical velocity and upwelling flux vary almost linearly in the Ekman layers, Fig. 6; however, they are nearly constant within the core of the geostrophic regime at the middle depth. It is clear that using this parameterization, the Coriolis parameter (or the latitudinal location of the hurricane) has no impact on the radial variations of vertical velocity and upwelling flux (Figs. 5b, 5c); however, in the geostrophic layer the vertical and upwelling flux at 18 °N are about twice those at 36 °N, Fig. 6. Thus, the vertical response for the same hurricane is stronger at low latitudes than at middle latitudes.

c. Solutions for $\delta = \sqrt{\frac{\zeta^0}{f^2\rho}}$

Under this assumption, vertical viscosity can be written as
\[ A_z = \gamma^2 \tau^0 / 2 f \rho; \]  

Thus, vertical viscosity is proportional to wind stress and inertial frequency. Since wind stress varies with the radial coordinate, vertical viscosity is now a function of radial coordinate as well.

We also assume that Eq. (31) applies to the whole water column, including the bottom boundary layer. Although Eq. (18) is based on observations, its application to strong wind with large wind shear remains to be reexamined through further study. In particular, parameterization of turbulent motions in the bottom boundary layer is a great challenge and results based on such an assumption should be interpreted with caution.

In the present case, the ageostrophic velocity cannot be expressed in a form similar to Eq. (24). The ageostrophic velocities for viscosity (31) are shown in Fig. 7. Compared with Fig. 4, it is readily seen that the ageostrophic flows are much weaker than those of a constant viscosity, especially in the regime of high wind speed, and this is due to the fact that under assumption (31), viscosity in high wind regime is very strong. A noteworthy fact in Fig. 7 is that although the geostrophic flow \( \nu_{\theta, \text{geo}} \bigg|_{z=-H} \) occurs at the radius of maximum wind speed, the maximum radial velocity does not; instead, double peaks appear on both sides of \( r = r_0 \).

Assuming that the boundary layer at the sea floor is well-separated from that on the sea surface, flow in bottom Ekman layer is directly linked to the geostrophic flow at the middle depth. Thus, when the geostrophic flow at middle level is enhanced, flux in the bottom boundary layer increases. However, under extremely
strong wind condition, the Ekman depth defined by (31) is so thick that the Ekman layers in the upper ocean and on the sea floor merge. As a result, flow in the bottom Ekman layer is not only affected by the geostrophic flow, but also the surface wind stress. Under the same conditions, the double-peak structure of radial velocity at middle latitudes (Fig. 7c) is less pronounced than that at low latitudes (Fig. 7a) because the Ekman depth is shallower at middle latitudes and these two boundary layers are well separated.

d. The common feature of solutions for $A_z = 0.5 m^2 s^{-1}$ and $\delta = \frac{\sqrt{f^2 \rho}}{\gamma}$

Notably, due to

$$\int_{-H}^0 W_{ageo} dz = \left[ \left( \tau^\theta - \tau_b^\theta \right) + i \tau_b^r \right] \sqrt{f^2 \rho},$$

where $\tau_b^r$ and $\tau_b^\theta$ are radial and azimuthal bottom stress, respectively, Eq. (11) implies that the azimuthal bottom stress is equal to the azimuthal surface stress. On the other hand, $\tau_b^r$ can be written as

$$\tau_b^r = \frac{\rho \delta^\theta}{2} \left[ c_1 e^{\frac{H}{\delta}} \left( \sin \frac{H}{\delta} + \cos \frac{H}{\delta} \right) + d_1 e^{\frac{H}{\delta}} \left( \sin \frac{H}{\delta} - \cos \frac{H}{\delta} \right) \right] + c_2 e^{\frac{H}{\delta}} \left( \sin \frac{H}{\delta} - \cos \frac{H}{\delta} \right) + d_2 e^{\frac{H}{\delta}} \left( \sin \frac{H}{\delta} + \cos \frac{H}{\delta} \right). \quad (33)$$

Assuming $H > 2\delta$, we have $e^{H/\delta} \gg e^{-H/\delta}$. After some manipulations, we obtain the approximate expression

$$\tau_b^r \approx -\tau^\theta. \quad (34)$$

Therefore, under the assumption $H > 2\delta$, the choice of vertical viscosity does not affect the bottom stress, and the magnitude of bottom stress is always larger than
that of surface stress and varies linearly with the magnitude of surface stress.

A very important feature of the analytical model is that azimuthal ageostrophic velocity in the bottom boundary layer is much stronger than that in the surface boundary layer, as can be seen from Figs. 4 and 6. Strong bottom boundary velocity and shear imply strong erosion of seabed. The combination of the azimuthal and radial velocity gives rise to the inward spiral velocity field, which may induce strong movement of sediment on the sea floor.

The vertical structure can be illustrated by a streamfunction in the r-z plan which represents the upwelling/downwelling, Fig. 8. For a stationary hurricane, the induced upwelling in the core brings water from the bottom Ekman layer on the sea floor to the upper ocean, as sketched in Fig. 1. In this sense, the oceanic circulation induced by a stationary hurricane is a giant eddy spanning the entire water depth very much like a hurricane in the atmosphere. The induced upwelling with a constant viscosity $A_z = 0.5 \text{ m}^2 \text{ s}^{-1}$ is stronger than that with viscosity $A_z = \gamma^2 \tau^0 / 2 \rho f$. The Coriolis parameter also affects the upwelling flux. As discussed above, under the assumption of constant viscosity, the response to a hurricane is stronger at low latitudes than at middle latitudes. However, due to the merging of the surface and bottom boundary layers the overturning streamfunction for the case with $A_z = \gamma^2 \tau^0 / 2 \rho f$ may appear in the form of two cells, Figs. 8c and 8d, similar to the structure shown in Fig. 7. As a result, the maximum upwelling flux occurs at a different radius with the different choice of vertical viscosity.
3. Relationship between oceanic response and hurricane physical parameters

Through examining the chl-a response to 13 hurricanes in 1998-2001, Babin et al. (2004) found that there is a linear relationship between percent chl-a enhancement and some hurricane physical parameters, such as mean wind speed, tropical storm force wind radius, and hurricane-force wind radius. To assess the physical effects of hurricanes we will establish some quantitative relations between the oceanic response and hurricane physical parameters. The corresponding relations for these two parameterizations are discussed separately.

a. \( A_z = 0.5 \text{m}^2 \text{s}^{-1} \)

Recall the upwelling flux (29). At \( r = r_0 \), it is reduced to

\[
\psi(z, r_0) = 2\pi r_0 G(z).
\]

(35)

Since the maximum of \( G(z) \) is in the geostrophic layer and depends on \( V_{\text{max}}^2 \) only, we conclude that under the assumption of a constant viscosity, the maximum upwelling flux induced by a stationary hurricane is proportional to the square of its maximum wind speed and the radius of the maximum wind speed.

Under the assumption of constant viscosity, the Ekman depth \( \delta \) is constant. If \( H > 2\delta \), the geostrophic velocity (17) is approximately

\[
v_{\theta, \text{geo}} = \frac{\delta \tau^\theta}{\rho A_z} = \frac{\tau^\theta}{\rho} \sqrt{\frac{2}{A_z f}} = \frac{2 \tau^\theta}{\rho \delta}. \]

(36)

The accumulated geostrophic transport can be calculated by

\[
Q_{\text{geo}}(r) = \int_0^r H \nu_{\theta, \text{geo}}dr = \frac{\tau_{\theta, \text{max}}^\theta}{\rho} \frac{H}{A_z f} \int_0^r F(r)dr,
\]

(37)
and the final result is

\[
Q_{\text{geo}}(r) = \begin{cases} 
\frac{\tau_{0,\text{max}} H}{5 \rho} \sqrt{\frac{2}{A_z} \frac{r^5}{r_0^5}} , & r < r_0 \\
\frac{\tau_{0,\text{max}} H}{\rho} \left[ \frac{2}{5} \frac{r_0}{r} \right] - \frac{r_0}{3 - 2b} + \frac{r^{3-2b}}{(3-2b)r_0^{2-2b}} \right] , & r \geq r_0
\end{cases}
\quad , \quad (38)
\]

where \( \tau_{0,\text{max}} \) is the maximum wind stress.

From formula (32), we obtain the accumulated azimuthal ageostrophic transport

\[
Q_{\text{ageo}}(r) = \int_{-H}^{0} \int_{r}^{0} d\rho \gamma \tau_{\theta,\text{geo}}(z) dz dr = \frac{\tau_{b,\text{max}}}{f} \int_{0}^{r} f(r) dr , \quad (39)
\]

where \( \tau_{b,\text{max}} \) is the maximum radial bottom stress. According to (34),

\[
\tau_{b,\text{max}} = -\tau_{0,\text{max}} ; \quad \text{hence,}
\]

\[
\frac{Q_{\text{geo}}(r)}{Q_{\text{ageo}}(r)} = -\frac{2H}{\delta} , \quad (40)
\]

which indicates that the geostrophic motion dominates the horizontal circulation induced by a hurricane.

For \( A_z = 0.5 \text{ m}^2\text{s}^{-1} \), the Ekman depth at 18°N and 36°N is 149m and 108m, respectively, so condition \( H > 2\delta \) is satisfied.

\[ b. \quad \delta = \gamma \sqrt{\tau_0 / f^2 \rho} \]

If \( H > 2\delta \), the geostrophic velocity can be represented as

\[
\hat{v}_{\theta,\text{geo}} \approx \frac{\delta \tau_0}{\rho A_z} = 2 \sqrt{\tau_0 / \gamma^2 \rho} ; \quad (41)
\]

furthermore, the accumulated geostrophic transport satisfies

\[
Q_{\text{geo}}^f(r) = \begin{cases} 
\frac{2H}{3 \gamma} \sqrt{\frac{\tau_{0,\text{max}} H}{\rho} \frac{r^3}{r_0^3}} , & r < r_0 \\
\frac{2H}{\gamma} \sqrt{\frac{\tau_{0,\text{max}} H}{\rho} \left[ \frac{2}{5} \frac{r_0}{r} \right] - \frac{r_0}{3 - 2b} + \frac{r^{3-2b}}{(3-2b)r_0^{2-2b}} \right] , & r \geq r_0
\end{cases}
\quad , \quad (42)
\]
Note that it is independent of latitude. In this case, the Ekman depth increases as wind stress is increased and as a result, condition $H > 2\delta$ may not always be satisfied.

Fig. 9 shows the accumulated geostrophic transport for a typical case with $H = 1200$ m, $r_0 = 50$ km, $V_{\text{max}} = 60$ m s$^{-1}$. Since the Ekman depth $\delta = \gamma \sqrt{\frac{\tau_0}{f^2 \rho}}$ at $18^\circ$N is 1040m corresponding to the maximum wind stress, condition $H > 2\delta$ cannot be satisfied, Eq. (42) is not suitable for cases with strong wind stress.

However, as seen in Fig.9b, the discrepancy between the analytical geostrophic transport obtained by numerically integrating $Q_{\text{geo}}(r) = \int_0^r H \nu \rho(r) dr$ and Eq. (42) is so small that Eq. (42) can also be used to approximately estimate the geostrophic transport for $\delta = \gamma \sqrt{\frac{\tau_0}{f^2 \rho}}$ without satisfying the condition $H > 2\delta$.

When $r \leq 2r_0$, the accumulated geostrophic transport increases quickly and levels off at $r = 4r_0$ (Figs. 9a, 9b). As seen from formula (38), $Q_{\text{geo}}(r) \propto 1/\sqrt{f}$, so that under the assumption of a constant viscosity the horizontal current induced by a hurricane at low latitudes is stronger than at middle latitudes (Fig. 9a). This is consistent with the numerical results by Greatbatch’s (1984). In both cases the geostrophic transport is quite large, suggesting that the current induced by a hurricane may play an important role in stirring up the whole depth of the ocean.

In Eqs. (38) and (42), the accumulated geostrophic transports are both a linear function of the ocean depth. To illustrate their variation quantitatively with the ocean depth, the accumulated geostrophic transport at $r = 4r_0$ for $r_0 = 50$ km, $V_{\text{max}} = 60$ m s$^{-1}$ is displayed in Fig. 10. To compare the error of Eq. (42) arising from $H < 2\delta$ under
the assumption of \( \delta = \gamma \sqrt{\tau^0/f^2 \rho} \), the results obtained by integrating \( Q'_{\text{geo}}(r) = \int_0^r H \theta_{\text{geo}} dr \) are also illustrated in Fig.10. As seen from Fig. 10, Eq. (42) can be utilized to delineate approximately the change of geostrophic transport with depth. In a 600m deep ocean, the total horizontal transport induced by a hurricane is about 10, 30, 42Sv for three specified case, respectively.

The accumulated geostrophic transport (at \( r = 4r_0 \)) and maximum upwelling flux vary with the maximum wind speed and the radius of maximum wind speed, Figs. 11 and 12. According to Eqs.(35) and (38), under the assumption of constant viscosity they both change linearly with the radius of maximum wind speed and quadratically with the maximum wind speed.

The similar parameter dependence for these two transports is due to that fact that the bottom Ekman flow is dominated by geostrophic flow if the upper and bottom Ekman layer are well separated. Since \( Q'_{\text{geo}}(r) \propto \sqrt{\tau^0_{\text{max}}} \), there is a linear relationship between the accumulated geostrophic transport and maximum wind speed for the case \( \delta = \gamma \sqrt{\tau^0/f^2 \rho} \). For a typical radius of maximum wind speed \( r_0 = 50 \text{ km} \) and a typical maximum wind speed \( V_{\text{max}} = 60 \text{ m s}^{-1} \), the total geostrophic transports are 25, 61 and 84Sv separately (Figs. 11a and 12a) while the maximum upwelling flux are 31 and 63Sv (Figs. 11b and 12b).

4. Numerical results based on an Oceanic General Circulation Model

The analytical solutions discussed above are based on the classical theory of the Ekman layer; thus, they are highly idealized. Many important processes omitted
in the analytical solution can be included in numerical simulations. In this study, we employ a regional version of MIT OGCM for an inviscid, incompressible fluid governed by hydrostatic, Boussinesq primitive equations. The model has a nonlinear implicit free surface and is forced by surface wind stress only, without thermohaline forcing, Eq. (22). The model ocean is configured as an f-plane ocean for a stationary hurricane in local Cartesian coordinates, with x axis eastward, y axis northward, z axis upward. The coordinate origin is located at the undisturbed sea surface. The corresponding velocity components are (u, v, w). The model domain is 1000km×1000 km, with horizontal resolution 10km×10km. The model ocean is 1200 m deep with vertically uniform resolution of 40 m. The no-slip boundary condition is imposed on the bottom boundary. The vertical viscosity is set to 0.5m$^2$s$^{-1}$, and the horizontal viscosity 4000m$^2$s$^{-1}$. The Coriolis parameter $f$ is set to $0.4506 \times 10^{-4}$s$^{-1}$, corresponding to 18°N. The wind stress profile is the same as defined by Eqs. (19) - (23) and shown in Fig. 3 and the hurricane is located at the center of the model domain.

The model was integrated over 7 days from an initial state of rest to a quasi-steady state. The accumulated geostrophic transport obtained from the analytical model and MIT OGCM model is displayed in Fig. 13. For the case with the maximum wind speed equal to 20m s$^{-1}$, the accumulated geostrophic transport obtained from the numerical model is quite close to that obtained from the analytical model. As the maximum wind speed increases, however, the discrepancy between the model and the analytical solution with vertical viscosity set to 0.5m$^2$s$^{-1}$
is enlarged. The primary reason is that the 10 km horizontal resolution is not fine enough to resolve the spike-like stress profile shown in Fig. 3b. In fact, the maximum wind stress in the numerical model for the case with the maximum wind speed set to 60 m s\(^{-1}\) is approximately 8 N m\(^{-2}\), which is slightly smaller than the analytical solution shown in Fig. 3b. The accumulated geostrophic transport in MIT OGCM model has a quadratic relation with the maximum wind speed.

Fig. 14 shows the flows of a homogenous ocean model, in which the parameters are the same as discussed in Section 3a. The horizontal velocity \( u \) in section \( y=500 \) km corresponding to the radial velocity in the analytical solution is plotted in Fig. 10a and vectors \( (u, 1000*w) \) are superimposed. Similar to Fig. 4, the upper and bottom Ekman layer are distinct and their thicknesses are comparable with the analytical solution. Velocity vectors in Fig. 9a show the circulation in the x-z plane, with upwelling inside the maximum wind band and downwelling outside. The horizontal flow and vertical velocity at depth 80 m, 600 m and 1120 m are plotted in Figs. 9b, c, d. Obviously, the horizontal flow is an outward (inward) spiral in bottom (upper) Ekman layer; however, it appears as circular motion at the middle depth, which is a strong indicator of the nearly azimuthal cyclo-geostrophic flow at this level. The contours of vertical velocity in Figs. 9b, c, d confirm the axisymmetric character and the vertical movement in Fig. 9a.

There are indeed some differences between the analytical and numerical solutions. First, the upwelling extends to the outside of the maximum wind speed radius. It is clear that the step-function-like discontinuity in the analytical solution
is smoothed out by horizontal diffusion in the numerical solution. Second, flow in the bottom Ekman layer displayed in Fig. 14a is weaker than that obtained from the analytical solution. Therefore, the volume flux compensating the outward Ekman transport in the upper ocean is not entirely from the bottom boundary layer.

Overall, results from the analytical model are quite similar to those obtained from the numerical model. Apparently, omitting the centrifugal force terms and other high order terms in the analytical model is a good approximation, and the analytical model can be used as a tool to understand the more complicated results from the numerical model. Therefore, if there were stationary hurricanes, the analytical solutions can provide a framework. No hurricane is truly stationary and no ocean is completely homogenized; thus, the solutions discussed above are only an idealization, which can serve as the theoretical limit for hurricane moving extremely slowly over an ocean with extremely weak stratification.

5. Conclusions.

The analytical solution discussed in this study provides an idealized picture of the zero-order structure of the oceanic circulation induced by a stationary hurricane. Most importantly, our study reveals the hurricane-induced upwelling water associated with outward surface Ekman transport comes from the bottom Ekman layer and appears in the form of an upward spiral eddy. In some sense, the circulation structure induced by a stationary hurricane is quite similar to that of a hurricane in the atmosphere. As required by Newton’s Third law, the mass
transport in the atmospheric boundary layer and the oceanic boundary layer should be equal in magnitude, but with opposite signs.

Despite the crude assumptions made in the analytical solution, it gives a succinct description to the circulation induced by a stationary hurricane. The assumption of a purely geostrophic flow in the middle level gives rise to constraint (11) in the analytical model. Such an assumption is clearly an idealization, and it implies the existence of an intense bottom Ekman flow. Nevertheless, our numerical experiments indicate that the analytical solution is quite close to the numerical solutions which include some of the ageostrophic processes. In fact, results of the MIT OGCM model confirm the overall structure of the circulation generated by a stationary hurricane. At the same time, it is found that the horizontal diffusion, nonlinear terms, centrifugal force terms have little effect on the overall structure of the circulation.

Both the horizontal transport and upwelling flux induced by a hurricane are huge and have a strong link with the hurricane physical parameters. With a choice of constant viscosity, they both vary linearly with the radius of maximum wind speed and quadratically with maximum wind speed; however, for a variational viscosity associated linearly with wind stress, the geostrophic transport is linearly dependent on the maximum wind stress. Therefore, it is not strange that some statistic relations between chl-a enhancement and hurricane physical parameters can be revealed in Babin et al (2004).

Finally, we stress again that the analytical solution for a stationary hurricane
discussed in this paper is highly idealized and provides only a conceptual picture to
the circulation induced by a hurricane. In reality, hurricanes often move with a
typical speed of 5 to 20 km per hours. The lateral translation of a hurricane brings
about many complicated dynamical processes, including the exciting of
near-inertial oscillations in the ocean, which play a vitally important role in the
hurricane induced perturbation. The study of related topics is currently under way,
and will be reported in a follow-up paper.

Acknowledgments. ZML was supported by CAS (kzcx2-yw-226, SQ200813), NSFC
under Grants No.40906009, No.40776008, and Dr. Xiaodong Shang through “100
Talents Program” of CAS. Comments and suggestions from the two anonymous
reviewers helped us to improve the manuscript greatly.
REFERENCES


List of Figures

FIG. 1. Sketch of the oceanic circulation induced by an axisymmetric and stationary hurricane.

FIG. 2. The surface and bottom Ekman layers and the geostrophic current in the cylinder coordinate system used in formulating the analytical solution.

FIG. 3. Radial profile of a) the wind speed and b) wind stress.

FIG. 4. The ageostrophic velocity profiles induced by the maximum wind stress (9N m\(^{-2}\)) at latitude 18°N (a) and 36°N (b).

FIG. 5. The modal functions: a) F(r) for ageostrophic velocities; b) P(r) for the upwelling velocity; c) Q(r) for the upwelling flux.

FIG. 6. The modal function G(z) for vertical velocity and upwelling flux.

FIG. 7. The ageostrophic velocity of the analytical solution under the assumption of \(A_z = \gamma^2 \tau''/2f\rho\).

FIG. 8. The streamfunction (in Sv) in the r-z plane. FIG. 9. The accumulated geostrophic transport vs. \(r\) for \(H=1200m\), \(r_0=50km\), \(V_{\text{max}}=60m\ \text{s}^{-1}\). a) \(A_z=0.5\ \text{m}^2\ \text{s}^{-1}\); b) \(A_z=\gamma^2 \tau''/2f\rho\). In the label, “numerical integration” means that the accumulated geostrophic transport is calculated by numerical integrating \(Q'_{\text{geo}}(r) = \int_0^r H\nu_{\theta,\text{geo}}dr\).

FIG. 10. The accumulated geostrophic transport at \(r = 4r_0\) vs. \(H\) for \(r_0 =50 \ \text{km}\), \(V_{\text{max}}=60m\ \text{s}^{-1}\). In the label, “numerical integration” means that the accumulated geostrophic transport is calculated by numerically integrating \(Q'_{\text{geo}}(r) = \int_0^r H\nu_{\theta,\text{geo}}dr\).

FIG. 11. The accumulated geostrophic transport at \(r = 4r_0\) a) and maximum upwelling flux b) vs. \(r_0\) for \(H=1200m\), \(V_{\text{max}}=60m\ \text{s}^{-1}\).
Fig. 12. The accumulated geostrophic transport at $r = 4r_0$ a) and maximum upwelling flux b) vs. $V_{\text{max}}$ for $H = 1200\, m, r_0 = 50\, \text{km}$.

Fig. 13. The accumulated geostrophic transport obtained from the analytical model (marked by circles and diamonds) and MIT OGCM model (marked by squares). The radius of maximum wind speed is $r_0 = 50\, \text{km}$.

Fig. 14. Flow fields of a homogenous ocean model driven by a stationary hurricane. a) $u$ in $y=500\, \text{km}$ section (gray scale, in m s$^{-1}$) superimposed by vectors $(u, 1000\, w)$; b) $w$ (gray scale, in 10$^{-3}$ m s$^{-1}$) superimposed by vectors $(u, v)$, at 80 m depth; c) and d) are the same as b), but at depth of 600 m and 1120 m, respectively.
FIG. 1. Sketch of the oceanic circulation induced by an axisymmetric and stationary hurricane.
FIG. 2. The surface and bottom Ekman layers and the geostrophic current in the cylinder coordinate system used in formulating the analytical solution.
Fig. 3. Radial profile of: a) the wind speed and b) wind stress.
Fig. 4. The ageostrophic velocity profiles induced by the maximum wind stress (9N m$^{-2}$) at latitude 18°N (a) and 36°N (b).
FIG. 5. The modal functions: a) $F(r)$ for ageostrophic velocities; b) $P(r)$ for the upwelling velocity; and c) $Q(r)$ for the upwelling flux.
Fig. 6. The modal function $G(z)$ for vertical velocity and upwelling flux.
Fig. 7. The ageostrophic velocity of the analytical solution under the assumption of $A_z = \gamma^z \mathcal{C}/2f\rho$. 

---

a) $v_{r,geo} (\text{ms}^{-1})$ at latitude 18°N

b) $v_{\theta,geo} (\text{ms}^{-1})$ at latitude 18°N

c) $v_{r,geo} (\text{ms}^{-1})$ at latitude 36°N

d) $v_{\theta,geo} (\text{ms}^{-1})$ at latitude 36°N

---

FIG. 7. The ageostrophic velocity of the analytical solution under the assumption of

$A_z = \gamma^z \mathcal{C}/2f\rho$. 

---
Fig. 8. The streamfunction (in Sv) in the r-z plane.
Fig. 9. The accumulated geostrophic transport vs. $r$ for $H=1200\,m$, $r_0=50\,km$, $V_{\max}=60\,m\,s^{-1}$. a) $A_z=0.5\,m^2\,s^{-1}$; b) $A_z=\gamma^2 \tau^\theta/2\rho f$. In the label, “numerical integration” means that the accumulated geostrophic transport is calculated by numerical integrating $Q_{\text{geo}}(r) = \int_0^r H\nu_{\text{geo}}\,dr$. 
Fig. 10. The accumulated geostrophic transport at $r = 4r_0$ vs. $H$ for $r_0 = 50$ km, $V_{\text{max}} = 60$ m s$^{-1}$. In the label, “numerical integration” means that the accumulated geostrophic transport is calculated by numerically integrating $Q'_{\text{geo}}(r) = \int_0^r H_{\theta_{\text{geo}}}(r)\, dr$. 
FIG. 11. The accumulated geostrophic transport at $r = 4r_0$, a) and maximum upwelling flux b) vs. $r_0$ for $H = 1200\text{m}$, $V_{\text{max}} = 60\text{m s}^{-1}$.
FIG. 12. The accumulated geostrophic transport at $r = 4r_0$, a) and maximum upwelling flux b) vs. $V_{\text{max}}$ for $H = 1200m$, $r_0 = 50$km.
Accumulated geostrophic transport (Sv)

Maximum wind speed (m s$^{-1}$)

Az = 0.5 m$^2$ s$^{-1}$

Az = $\gamma^2 v / 2 \rho f$

MITgcm, Az = 0.5 m$^2$ s$^{-1}$

Fig. 13. The accumulated geostrophic transport obtained from the analytical model (marked by circles and diamonds) and MIT OGCM model (marked by squares). The radius of maximum wind speed is $r_0 = 50$ km.
FIG. 14. Flow fields of a homogenous ocean model driven by a stationary hurricane. a) u in y=500km section (gray scale, in m s$^{-1}$) superimposed by vectors (u, 1000*w); b) w (gray scale, in 10$^{-3}$ m s$^{-1}$) superimposed by vectors (u, v), at 80 m depth; c) and d) are the same as b), but at depth of 600 m and 1120 m, respectively.