1. Introduction

We have discussed dynamics of western boundary layers in the subtropical and subpolar basin. In particular, inertial boundary layer plays an important role in the western boundary region. Now we move towards the equator, where a most unique situation is the vanishing of the Coriolis parameter. The dynamics of the equatorial current system is much more complicated than that at middle latitudes; in this lecture we will discuss only one of the most outstanding components of the equatorial currents -- the equatorial undercurrent. The fundament structure of the equatorial current system and associated structure of temperature and salinity is shown in Fig. 1.

![Fig. 1. Structure of the equatorial thermocline and circulation (Wyrtki and Kilonsky, 1984).](image)

The most outstanding feature is the existence of the Equatorial Undercurrent (EUC), which is located within a latitudinal band of 2° around the equator. Vertically, EUC is at the depth of 100 m to 250 m below the surface, and there is strong westward surface current above the core of the eastward undercurrent. In addition, there is another westward current below the eastward undercurrent. There are many other complicated currents near the equator; however, we will confine our attention to EUC in this section, including the equatorial thermocline associated with
EUC. Another major feature is the equatorial thermocline and the thermocline ridge near 10°N, which is associated with the special feature of wind stress curl near the Inter-Tropical Converge Zone and will be explained in the next lecture.

As discussed in previous lectures on ventilated thermocline, warm water is subducted in the subtropical gyre interior. These water masses move within the subtropical gyre interior; some of the subducted water moves back poleward via the western boundary currents, but part of its moves toward the equator in two ways, i.e., through the low latitude western boundary or through the interior communication window. These links have been identified through many observational studies. For example, Wyrtki and Kilonsky (1984) identified the source of the water mass in the equatorial current system using the Hawaii-to-Tahiti Shuttle Experiment; Gouriou and Toole (1993) studied the mean circulation of the upper layers of the western equatorial Pacific Ocean. These studies showed that the water mass in the equatorial current system comes from the subtropics. In particular, Gouriou and Toole (1993) pointed out that the water mass at the beginning of the Undercurrent comes primarily from the Southern Hemisphere.

The interior communication window was also identified through tracer studies in 1970 and 1980. Fine and her colleagues (Fine et al., 1987; McPhaden and Fine, 1988) have analyzed the tritium data and found a local tritium maximum around 140°W along the equator, which they rightly attributed to the ventilation of the subtropical water via subduction. Liu (1994) studied the interior communication associated with the circulation in the subtropical-tropical regime, using a ventilated thermocline model driven by zonally mean wind stress, and concluded that communication in the lower layer between the subtropics and tropics is possible. On the other hand, McCreary & Lu (1994) and Lu & McCreary (1995) used a layered numerical model to study the influence of the ITCZ on flow between the subtropics and tropics and estimated an interior communication rate of 3 Sv, very close to an early estimate of 3 Sv across 10°N (Wijffels, 1993).

Recent analysis of tracer and hydrographic data has led to a communication rate that is substantially higher. For example, Johnson and McPhaden (1999) estimated that this communication rate is about 5 Sv for the North Pacific and 16 Sv for the South Pacific. The communication rate in the Atlantic was estimated by Fratantoni et al. (2000) as 1.8 Sv for the North Atlantic and 2.1 Sv for the South Atlantic. The mass flux through the interior communication window was discussed by Huang and Wang (2001), using a simple index that is based on the wind stress data only. This index can be used to illustrate the asymmetric nature of the interior communication between the tropics and the subtropics. In addition, this index can be used to infer the decadal variability of the interior communication. However, this index cannot be used to infer the dynamical details of the equatorial currents.

Since the Coriolis parameter vanishes near the equator, geostrophy is invalid; thus, higher order dynamics, such as the inertial terms and friction, must be included. In a series of studies, Pedlosky (1987, 1996) has developed a theory for the dynamic connection between the subtropical thermocline and the Equatorial Undercurrent. The essence of the theory is that to the lower order the Equatorial Undercurrent can be treated in terms of the ideal-fluid model. In order to conserve potential vorticity the inertial term is retained near the equator.

First, we argue that there is a small latitudinal band near the equator where the geostrophy breaks down. In replacement of the geostrophy, we search a new balance between the inertial terms and other terms. The outer edge of this special zone can be defined as the latitude where the Coriolis term is equal to the inertial term. The commonly used Rossby number is defined as

\[ Ro = \frac{U}{fL} \]  

Near the equator, the Coriolis parameter is approximately
\( f = \beta y \).  

Approaching the equator, \( f \) declines, thus, the Rossby number \( Ro = \frac{U}{\beta y^2} \) increases. Thus, within a distance of \( d_f \):

\[
d_f = \sqrt{\frac{U}{\beta}}
\]

the advection term, or the relative vorticity, will be dynamically important. From observation, \( U \approx 1 m/s \), \( \beta = 2.28 \times 10^{-11} / s/m \); thus, \( d_f \approx 208 km \). Accordingly, within 2 degrees off the equator, we expect that the inertial terms in the momentum equation is non-negligible.

Since the Equatorial Undercurrent is linked to the ventilated thermocline at middle latitude, we will first describe the ventilated thermocline at extra-equatorial regime.

2. The extra-equatorial solution

Solution of equator is described by the LPS model

\[
h^2 = \frac{D_0^2}{1 + G \left(1 - \frac{f}{f_1}\right)^2}
\]

\[
h_2 = \frac{f}{f_1} h; h_i = \left(1 - \frac{f}{f_1}\right) h
\]

where \( G = \gamma_1 / \gamma_2 \) and \( f_1 \) is the Coriolis parameter at \( y = y_1 \), function \( D_0^2 \) is the forcing, and for the case with zonal wind

\[
D_0^2 = -\frac{2(x_e - x)}{\gamma_2} \left(-\tau + \frac{f}{\beta} \frac{\partial \tau}{y}\right)
\]

In an equatorial \( \beta \)-plane we have \( f = \beta y \). Since the wind stress varies on the planetary scale, the ratio of the last two terms in (6) is \( O(1/L) \), so the last term can be neglected near the equator. We will match the interior solution with an equatorial solution at \( y = 1 \), with the assumption that \( l \ll L \).

Note that the semi-geostrophy holds near the equator, so the zonal velocity is balanced by the meridional pressure gradient, or \( \gamma_2 H / l \approx \beta l U \). Thus, the characteristic velocity is linked to the layer thickness through

\[
H = U \beta l^2 / \gamma_2
\]

Introducing the non-dimensional variables

\[
h^* = h / H, (x^*, y^*) = (x / L, y / L)
\]

Equation (4) is reduced to

\[
h^2 = \frac{\gamma_2 \tau_0 L}{\beta^2 l^4 U^2} \frac{(x_e - x)(-\tau)}{1 + G(1 - y / y_1)^2}
\]

This equation should have an \( O(1) \) balance, so we have the scales

\[
l = \left( \frac{\gamma_2 \tau_0 L}{\rho_1 \beta^4} \right)^{1/8}, H = \left( \frac{\tau_0 L}{\gamma_2 \rho_0} \right)^{1/2}, U = \left( \frac{\gamma_2 \tau_0 L}{\rho_0} \right)^{1/4}
\]

The corresponding scale for the streamfunction is \( \Psi = HUI \). For the Atlantic Ocean, \( \tau_0 = 1 \text{ dyn/cm}^2, \gamma_2 = 1 \text{ cm/s}^2, \beta = 2 \times 10^{-13} / \text{ cm}, L = 3000 \text{ km} \), so that \( l = 280 \text{ km}, H = 125 \text{ m}, U = 157 \text{ cm/s} \), and \( \Psi = 58 \times 10^6 \text{ m}^3 / \text{s} \). For the Pacific Ocean, \( L = 14000 \text{ km} \), so that \( l = 339 \text{ km}, H = 265 \text{ m}, U = 230 \text{ cm/s} \), and \( \Psi = 225 \times 10^6 \text{ m}^3 / \text{s} \). These scales are quite close to the observed
Equatorial Undercurrent in the oceans. Note that the undercurrent has fractional thickness, on the order of 0.1H, so the volume flux contribution to the undercurrent from each hemisphere is on the order of 5 Sv for the Atlantic and 20 Sv for the Pacific.

Assuming $x_e=L$, the matching condition for the equatorial solution is that at the outer edge of the equatorial boundary layer, the solution should approach

$$h = \sqrt{\frac{-2(1-x)\tau}{1+G(1-y/y_1)^2}} \quad (11)$$

In order to find the solution we can use the potential vorticity and Bernoulli conservation laws. Far from the equator, potential vorticity and Bernoulli conservations lead to

$$\frac{y}{h_2} = Q_2(B_2) = Q_2(h) \quad (12)$$

because the $h=const.$ or $B_2=const.$ lines are streamlines in the second layer. Tracing back to the latitude of outcropping, we have

$$\frac{y}{h_2} = \frac{y_1}{h} \quad (13)$$

Thus,

$$Q_2(h) = \frac{y_1}{h} \quad (14)$$

so

$$Q_2(B_2) = \frac{y_1}{B_2} \quad (15)$$

Note that $h$ remains finite approaching the equator; however, geostrophy must breaks down near the equator because the Coriolis parameter vanishes at the equator. Thus, the interior solution is not valid near the equator, and the equatorial region must be treated as a boundary layer.

3. The equatorial undercurrent as an inertial boundary current

Near the equator, the boundary current is in semi-geostrophy

$$u_2 \frac{\partial u_2}{\partial x} + v_2 \frac{\partial u_2}{\partial y} - yv_2 = -\frac{\partial h}{\partial x} \quad (16)$$

$$yu_2 = -\frac{\partial h}{\partial y} \quad (17)$$

From these two equations we can derive the conservation laws, i.e., potential vorticity and Bernoulli function:

$$Q_2 = \frac{y - \partial u_2 / \partial y}{h_2} \quad , \quad B_2 = h \frac{u_2^2}{2} \quad (18)$$

are conserved along the streamlines. Note that the contribution due to the zonal gradient of the meridional velocity is neglected in the relative vorticity. Thus, the solution of the equatorial boundary layer can be obtained by solving two equations

$$\frac{\partial u_2}{\partial y} = y - y_1 \frac{h - h_i}{h + u_2^2 / 2} \quad (19)$$
\[ \frac{\partial h}{\partial y} = -yu_2 \]  

These two equations are solved over the domain of \( y=\{0, y_1\} \). There are three important points: first, the solution should match the interior solution at the outer edge of the equatorial boundary layer.

Note that the layer thickness along the matching latitude is

\[ H_0^2 = \frac{2(1-x)}{1 + (1 - y_n / y_1)} \]  

(21)

The corresponding thicknesses for the upper and lower layers are

\[ h_1 = H_0(1 - y_n / y_1), h_2 = H_0 y_n / y_1 \]  

(22)

In particular, the thickness of the upper layer must match with the interior solution at the matching latitude, so

\[ h \rightarrow h_{\text{interior}}(y_1), \text{as } y \rightarrow y_1. \]

Second, there is an additional boundary condition at the equator. We assume that the equator is a streamline, so that \( B_2 = B_{20} \) is a constant, which is set to be the Bernoulli function at the outer edge of the western boundary layer at some specified latitude.

Third, the upper layer thickness \( h_1 \) is unknown. For simplicity, one can chose to have \( h_1 = h_{y_n} \), where \( y_n \) is the latitude where the boundary layer solution matches with the interior solution. The solution can be found by a shooting method, as described by Pedlosky (1996). This model can be improved in several aspects; however, this is beyond of the focus of this lecture.

4. The asymmetric nature of the Equatorial Undercurrent in the Pacific

In the oceans there are western boundary currents in both hemispheres, and they all feed the Equatorial undercurrent; thus, there is a competition between them. For the case of a symmetric forcing, the solution is reduced to the case discussed above. For the case of asymmetric forcing, the western boundary current from the hemisphere with stronger forcing overshoots the equator where the two western boundary currents merge and form an Undercurrent that is asymmetric with respect to the equator. Layer thicknesses are continuous across the matching streamline, but zonal velocity can be discontinuous.

Flow across the equator has been examined by many investigators, e.g., Anderson and Moore (1979), Killworth (1991), Edwards and Pedlosky (1998). Since the Coriolis parameter changes sign across the equator, it works as a potential vorticity barrier for cross-equator flow. Killworth (1991) showed that cross-equator flow is confined within a few deformation radii of the equator. In order to move beyond such a range, a frictional force is required to alternate the potential vorticity of the water parcels. The problem associated with frictional flow across the equator has been studied by many investigators, such as Edwards and Pedlosky (1998).

A further complication arises with the bifurcation of the western boundary currents. At low latitudes, the westward-flowing current bifurcates when it meets the western boundary, thus forming the poleward and equatorward western boundary currents. The best example is the northward Kuroshio and the equatorward Mindanao Current in the North Pacific. The bifurcation of the western boundary current involves complicated dynamics in three-dimensional space, including eddies and flow over complicated topography. However, a barotropic model may provide a simple solution that can be used as a crude estimate.

A. Model formulation
Throughout, we assume that the circulation is steady and can be treated in terms of an ideal-fluid model; thus both potential vorticity and the Bernoulli function are conserved along streamlines
\[ \mathbf{u}_n \cdot \nabla q = 0, \quad \mathbf{u}_n \cdot \nabla B = 0, \]
where the subscript \( n \) labels the layer. We make such assumptions for both the thermocline and the EUC (Equatorial Undercurrent) in the nature of a null hypothesis with regard to vertical mixing. That is, we attempt to see how much of the structure of the mid-latitude and equatorial thermocline can be explained on the basis of the ideal-fluid theory. This is of course not the only legitimate point of view. Numerical experiments (e.g., Blanke and Raynaud, 1997 and Lu et al., 1998) require mixing for numerical reasons and suggest the possible importance of mixing for the dynamics.

Note that the upper layer thickness is another unknown. In order to carry out the boundary layer calculation, we also need to specify an additional constraint on the upper layer thickness. Pedlosky (1987) first assumed that \( h_1 \) is independent of latitude within the equatorial boundary layer and found some interesting solutions. Another choice is to assume that the upper layer thickness is compensated within the equatorial region (Pedlosky, 1996). This choice is based on the following idea: the geostrophic velocity in the upper layer is much smaller than that in the second layer; thus, to the lowest-order approximation the meridional pressure gradient in the upper layer is negligible compared with that in the second layer. This approximation implies that near the equator the current in the upper layer is dominated by the local wind stress.

Of course this specification of the depth of the upper layer is entirely arbitrary. In the original theory as described by Pedlosky (1996) the two extremes of specification of the upper layer thickness, i.e., either independent of \( y \) with no vertical shear across the interface, or completely compensated, had little effect on the undercurrent or the depth of the equatorial thermocline. In the interest of keeping our model as simple and comprehensible as possible, we retain this arbitrary, and admittedly deficient element of the theory. Thus, we will use the following additional constraint on the upper layer thickness
\[ h_1(x, y) = h_1(x, y_m) + h(x, y_m) - h(x, y) \] (23)
where \( y_m \) is the latitude where the equatorial thermocline solution is matched with the interior thermocline solution. This is the compensated solution. Similar to the solution discussed in previous section, as long as \( B_0 \) is specified, the solution can be found by a shooting method, as described by Huang and Pedlosky (2000).

A major assumption implicitly made in the model discussed above is that the solution is symmetric with respect to the equator. This implicit assumption does not hold exactly in the real oceans. Due to the asymmetric nature of the atmospheric general circulation, wind stress near the equator is not symmetric. It is readily seen that a straightforward application of this model will lead to discontinuity of the solution for the equatorial thermocline.

A natural requirement is that the solution should be continuous across the equator. Thus, the minimum requirement is that the thickness of both the upper and lower layers should be continuous across the equator. Since the Bernoulli function carried by the corresponding western boundary currents in the two hemispheres is not the same, these two boundary currents should not match exactly along the equator. In other words, the matching latitude at each longitudinal section should be a free boundary determined from the interior dynamics.

Thus, the suitable boundary value problem for the branch originating from the Northern hemisphere branch is
\( h^n = H^n_0 \text{ at } y = y_m \) \hspace{1cm} (24)

Second, the Bernoulli function should be a constant along the matching line near the equator
\[ h^n + u^2 / 2 = B^n_0 \text{ at } y = y_{sep} \] \hspace{1cm} (25)

where \( B^n_0 \) is the Bernoulli function of the western boundary current at the bifurcation latitude in the Northern hemisphere, determined by the barotropic circulation in the subtropical basin interior. In addition, we will use the following additional constraint on the upper layer thickness
\[ h^n_1(x, y) = h^n(x, y_m) + \Delta h^n \frac{y_m - y}{2y_m} - h^n(x, y) \] \hspace{1cm} (26)

where
\[ \Delta h^n = h^n_1(x, -y_m) + h^n(x, -y_m) - \left[ h^n_1(x, y_m) + h^n(x, y_m) \right] \]

The suitable boundary value problem for the branch originating from the Southern hemisphere branch is
\[ h^s = H^s_0 \text{ at } y = -y_m \] \hspace{1cm} (27)

Second, the Bernoulli function should be a constant along the matching line near the equator
\[ h^s + u^2 / 2 = B^n_0 \text{ at } y = y_{sep} \text{ and} \]
\[ h^n_1(x, y) = h^n_1(x, -y_m) + h^s(x, -y_m) + \Delta h^s \frac{y + y_m}{2y_m} - h^s(x, y) \] \hspace{1cm} (29)

Note that for the case with symmetric forcing, our formulation is reduced to the solution discussed by Pedlosky (1996), with the equator as the separating line of these two branches, i.e., \( y_{sep} = 0 \).

Since the Bernoulli function plays a role similar to the pressure head, it is expected that the western boundary current that has the larger Bernoulli function should overshoot the equator and invade the other hemisphere, as shown in the sketch in Fig. 2. In addition, there may be a small difference between the interior solutions at the matching latitude in the two hemispheres, thus, there is a small pressure gradient in the upper layer, resulting from \( \Delta h^n \neq 0 \); however, this may be a small contribution to the solution, so we will assume that \( \Delta h^n = 0 \) for the following analysis.
The boundary value problem consists of two sets of ordinary differential equation systems. Each set is subjected to the matching boundary conditions at the matching latitude $y_m$; these two solutions match along a free boundary $y_{sep}$, which is part of the solution. Across this matching boundary, the upper and lower layer thicknesses are continuous, as required. However, the zonal velocity may be discontinuous because the Bernoulli function from the two hemispheres can be different. A solution with $B^* = 1.25B^0$ is shown in Figs. 3 and 4.

The solution is very similar to the case with a symmetric forcing, and it is slightly asymmetric with respect to the equator, Fig. 3. The thermocline depth clearly has a dumbbell shape in the equatorial regime, resembling the gross structure of the equatorial thermocline observed in the oceans. The most outstanding feature of this solution is the discontinuity of the zonal velocity across the separating streamline, and this discontinuity can be seen clearly in the meridional sections shown in Fig. 4.

Fig. 2. Sketch of the Equatorial Undercurrent in a two-hemisphere basin.
The western boundary currents from the two hemispheres meet near the equator and form the eastward-moving Equatorial Undercurrent. Due to the difference in the Bernoulli head, the zonal velocity is discontinuous across the separating streamline, as shown in Fig. 4a. As the undercurrent moves eastward, the zonal velocity core becomes stronger, and the separating streamline moves slightly toward the equator, as shown by the line labeled as \( y_{sep} \) in lower part of Fig. 4a.

The meridional structure of the undercurrent can be represented by the meridional section taken along the western boundary, as shown in Figs. 4b, 4c, 4d. As required by the matching boundary condition, the thermocline depth is continuous, Fig. 4d. However, both the zonal velocity and the Bernoulli function are discontinuous across the separation line, Fig. 4b and 4c.

It is important to note that this ideal-fluid solution may not be very stable. In fact, the solution consists of a rather narrow band north of the equator, where a patch of negative potential vorticity originating from the Southern Hemisphere is adjacent to the positive potential vorticity in the Northern hemisphere, Fig. 5a. The coexistence of potential vorticity with opposite signs indicates that symmetric instability may occur and thus modify the solution.
Fig. 4: The zonal and meridional structure of the undercurrent. a) The zonal variation of the zonal velocity along the matching streamline, SH (NH) indicates the water particle originating from the Southern (Northern) Hemisphere, $y_{sep}$ indicates the zonal variation of the separation point; b) the zonal velocity at the western boundary; c) the Bernoulli function along the western boundary; d) the thermocline depth along the western boundary.

Fig. 5. The meridional profile of (a) the total vorticity and (b) the streamfunction at the western boundary.
Second, it is readily shown that the zonal velocity profile satisfies the sufficient condition for barotropic instability. The barotropic instability is likely to smooth out the velocity cusp near the equator. Even for the hemispherical symmetric solution first obtained by Pedlosky (1987), it is readily shown that the zonal profile of the solution satisfies the sufficient condition for barotropic instability. The fact that symmetric instability may arise from the hemispheric asymmetric solution may also not be a significant deficiency. To some extent, it may be a realistic property. It was suggested recently by Hua et al. (1997) that the observed equatorial mean circulation may marginally satisfy the condition for symmetric instability. Using a numerical model, they further demonstrated that for a basic state flow with this kind of instability, the nonlinear equilibrated state of the equatorial ocean circulation exhibits some secondary flows which resemble the observed multiple equatorial jets underneath the Equatorial Undercurrent.

B. Application to the Pacific Ocean

1) Evidence of off-equator shift of the Equatorial Undercurrent core

Direct field measurements also indicated the asymmetrical nature of the zonal velocity profile in the Equatorial Undercurrent. Hayes (1982) studied the zonal geostrophic velocity profile at two sections in the eastern equatorial Pacific. Geostrophic velocity calculated from hydrographic data is consistent with that obtained from the free-fall acoustically tracked velocimeter (TOPS). The geostrophic velocity south of the equator (at 0.5oS and 1oS) was clearly stronger than that north of the equator. Wyrtki and Kilonsky (1984) calculated the geostrophic zonal velocity from the data collected during the Hawaii-Tahiti Shuttle Experiment. Their results clearly showed that the center of the Equatorial Undercurrent is located at 0.5oS.

However, in the western part of the Pacific equatorial ocean, the core of the Undercurrent is located north of the equator. Tsuchiya et al. (1989) made a detailed analysis of the water mass properties collected during the Western Equatorial Pacific Ocean Circulation Study (WEPOCS). Their analysis clearly showed that the major portion of the water in the Equatorial Undercurrent at its beginning north of Papua New Guinea is supplied from the south by a narrow western boundary undercurrent (New Guinea Coastal Undercurrent). In fact, the water mass north of the equator can be traced back to its source in the south.

Gouriou and Toole (1993) analyzed the mean circulation in the western equatorial Pacific Ocean. Their Fig. 8B clearly showed there is a patch of negative potential vorticity north of the equator, adjacent to the positive potential vorticity in the environment. Such a patch of negative potential vorticity must come from the Southern hemisphere.

Joyce (1988) argued that the wind stress on the equatorial ocean can force a cross-equatorial flow in the upper ocean. By applying a generalized Sverdrup relation to the equatorial oceans, he inferred that there is a southward cross-equatorial flow in the eastern equatorial Pacific Ocean, and a northward cross-equatorial flow in the western equatorial Pacific Ocean. His calculation, however, did not include the contribution due to the relative vorticity term, nor the connection with the western boundary currents.

The total mass flux going through the western boundary at the beginning of the Undercurrent can be estimated from the streamfunction profile shown in Fig. 4b. For example, the mass flux from the Southern and Northern hemispheres is about 0.1 and 0.05 in non-dimensional units. For the Pacific Ocean, the scale of the streamfunction is about 206 Sv; thus, the mass flux contribution from the western boundary currents is about 20 Sv and 10 Sv. The cross-equatorial
flux, \( \psi_0 \), is about 0.03 non-dimensional unit, which corresponds to about 6 Sv in dimensional units. Tsuchiya et al. (1989) estimated that the mass flux of 5 Sv is fed to the eastward interior circulation between \( 3^\circ S \) and the equator. If there were no Indonesian Throughflow, this would be the mass flux fed to the Undercurrent. This flux rate is rather close to the 6 Sv estimated in the discussion above.

2) Determination of the cross-equatorial flow in the source regime of the undercurrent

The dynamic factor that controls the position of the separating streamline is the strength of the Bernoulli function at the latitude of the western boundary bifurcation. In reality, the bifurcation of the western boundary is a complicated three-dimensional phenomenon, which is determined by many dynamic factors, such as the large-scale forcing fields and the resulting pressure field, the shape of the coastline, bottom topography, and eddies. However, a barotropic bifurcation latitude may provide a simple and useful first step to this complicated problem.

The total poleward Ekman flux across a latitudinal section is

\[
M_E = -\int_0^L \frac{\tau^x}{f \rho_0} \, dx
\]  

and the equatorward Sverdrup flux is

\[
M_G = \int_0^L \frac{\bar{w}}{\beta} \, dx
\]

where \( w_0 \) is the Ekman pumping velocity calculated from wind stress. The total meridional mass flux involved in the wind-driven gyre across a latitudinal section of the basin interior is the sum of these two fluxes, i.e., \( M_{SV} = M_E + M_G \). The return flow into the western boundary is \( M_{sv} \). The bifurcation latitude of the western boundary current is the latitude where \( M_{sv} \) vanishes; and it can be found out from the wind stress distribution in the basin. For the climatological mean wind stress, this gives the separation latitude as 14.5°N for the North Pacific and 15°S for the South Pacific, values which are close to the bifurcation points estimated from a numerical model, e.g., Huang and Liu (1999).

For a given amount of wind stress, the depth of the main thermocline can be calculated, using a simple reduced-gravity model. The corresponding Bernoulli function can be calculated according to the definition: \( B = g' \eta \), where \( g' \) is the reduced gravity, chosen as \( g' = 2 \text{cm/s}^2 \) for the Pacific, \( H \) is the depth of the main thermocline.

This simple approach is, however, unsatisfactory because wind stress is slightly stronger in the Northern hemisphere and the corresponding Bernoulli head in the Northern Hemisphere is larger than that in the Southern hemisphere. As a result, this simple approach would lead to a conclusion that water from the Northern hemisphere western boundary should invade the Southern hemisphere. However, observations indicate that most water forming the Equatorial Undercurrent does come from the south, as discussed above. Thus, in order to have the water from the Southern hemisphere overshoot the equator, another mechanism is needed. A strong contender is the Indonesian Throughflow. With the Throughflow, the western boundary current system changes and the source of the undercurrent also change accordingly, as shown in Fig. 2b.

Australia can be treated as a big island, so the simple island rule can apply and yield a circulation around the island that is a simple function of wind stress. The island rule has been discussed in many papers, and the most important physical mechanisms associated with this rule have been discussed thoroughly by Godfrey (1989, 1996).
Although mass flux associated with the Throughflow predicted from the island rule can be large, in the order of 17 Sv, other physical processes reduce the flux to about 10Sv. In our simple purely inertial model, water from the Southern Hemisphere could not penetrate into the Northern Hemisphere or feed the Throughflow without changing sign of the potential vorticity. The western boundary current from the Southern Hemisphere has to go through the undercurrent first. Over the eastward trajectory this mass flux upwells into the Ekman layer and then moves poleward in forms of Ekman transport. Thus, mass flux from the Southern Hemisphere becomes part of the Sverdrup mass flux in the ocean interior of the Northern Hemisphere. Within the simple ideal-fluid formulation of our model the only option is to assume that the Throughflow is fed from the western boundary from the Northern Hemisphere; hence, we will simply assume that, indeed, about 10Sv of water leaves the basin and forms the Indonesian Throughflow.

In the Northern Hemisphere, thus, the separation latitude is no affected by the Throughflow; mass flux in the western boundary current is zero at the separation latitude. However, the effective western boundary current that feeds the undercurrent actually comes from the ocean interior. To find the Bernoulli head of this western boundary layer, one has to search eastward, starting from the western boundary at the separation latitude, for a place where the equatorward flux, integrated from the eastern boundary, satisfies

\[ \psi_{\text{effective wbc}} = \psi_{\text{wbc}} + \Delta \psi \]

where \( \psi_{\text{wbc}} = -M_{Sv} \) is the Sverdrupian flux at the outer edge of the western boundary; \( \Delta \psi = 10Sv \) is the volumetric contribution to the Throughflow from the wind-driven circulation.

In the Southern Hemisphere, the separation latitude is now determined by the constraint that the total mass flux in the western boundary layer

\[ \psi_{\text{effective wbc}} = -M_{Sv} + \Delta \psi \]

vanishes. Without the Throughflow, the separation latitude in the Southern Hemisphere is near 15oS; with the Throughflow, however, the separation latitude is pushed southward to 17oS.

Appendix. The separation of one-layer inertial boundary layer

1) Why does the boundary layer separate?

Using semi-geostrophy, it is readily seen that the maximum streamfunction satisfies

\[ \psi_{\text{c}} = g \int_{1}^{y_n} \frac{h_y}{f} dy \]

(A1)

where \( y = 1 \) is set as the beginning of the inertial western boundary current, \( y_n \) indicates the northern limit of the inertial western boundary where it separates from the coast. The layer thickness at the western wall is

\[ h_w = \left( h_n^2 - \frac{2 f_n^2}{g^2} \psi_{\text{c}} \right)^{1/2} = \left( h_n^2 - 2 f_n^2 \int_{1}^{y_n} \frac{h_y}{f} dy \right)^{1/2} \]

(A2)

Therefore,

\[ \frac{dh_w}{dy} = -\frac{\beta}{h_w} \int_{1}^{y_n} \frac{h_y}{f} dy < 0 \]

(A3)

Since inertial western boundary is valid only when there is inflow, so \( h_y > 0 \). Consequently, \( h_w \) is a monotonically declining function of \( y \). At a critical point \( y_c \), the boundary layer separates from the coast, i.e.,
\[ h_w(y_c) = 0, \text{ and } \frac{dh_w}{dy} = -\infty \]

Therefore, solution breaks down near the separation latitude.

2) Equatorward inertial western boundary layers

Although poleward inertial western boundary layers have the tendency of separating from the coast, inertial western boundary currents moving toward equator do not have such problems. In the subpolar gyre and the tropical gyre, there are equatorward inertial western boundary currents. The dynamical structure of these currents has not received much attention in the past. It is straightforward to show that inertial model works just fine for these currents, and there is no crisis associated with the boundary layer separation.

Similar to our analysis in subsection (1), we have a relation

\[ \frac{dh_w}{dy} = -\frac{\beta}{h_w} \int_{y_s}^{y_w} \frac{hh_y}{f} \, dy = \frac{\beta}{h_w} \int_{y_s}^{y_w} \frac{hh_y}{f} > 0 \]

where \( y_s < y_w \) is the south of the starting latitude. Therefore, the layer thickness along the wall actually increases toward the equator.

Another interesting point is that: since current is moving toward the place with smaller planetary vorticity \( f \), the current needs to generated positive relative vorticity shear; thus, the current width should be increased equatorward, and its velocity should also decline. All these properties of the equatorward inertial western boundary currents are opposite to that of the poleward moving inertial western boundary current.

The solution of the inertial western boundary current can be easily described, using the streamfunction coordinates.

\[ h^2 = h_w^2 + \frac{2f}{g'} \psi \]

\[ \frac{v^2}{2} + g' h = G(\psi) \]

where \( G(\psi) \) is the Bernoulli function given at upstream locations. Finally, the physical coordinate can be recovered from the transformation

\[ x = \int_0^\nu \frac{d\psi}{h\psi} \]

It is speculated that this approach may be useful tool for the description of the Labrador Current in the North Atlantic and the Mindanao Current in the North Pacific.

References


