Lecture 3. Sandstrom Theory

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1. The "Sandstorm theorem":

A. Is the ocean a heat engine?

The atmosphere and oceans work together as a heat engine, if we neglect the small contribution of tidal energy to the circulation in the oceans. The atmosphere can be considered as a heat engine, which is driven by differential heating, with an efficiency of 0.8% (the corresponding Carnot efficiency is about 33%). In this sense although the oceans is subject to differential heating similar to the atmosphere, it is not a heat engine at all. In fact, the driving force for the oceanic circulation is the mechanical energy in the forms of wind stress and tides. In comparison, differential heating is only a precondition for the thermohaline circulation, and not the driving force of the oceanic general circulation. Thus, the ocean is not a heat engine; instead, it is a machine driven by external mechanical energy that transports thermal energy, fresh water, CO_2, and other tracers.

B. Sandstrom Theorem:

Sandstrom's original papers (1908, 1916) are in German, not easily accessible, but the citation from Dafant's book is very concise and accurate:

A closed steady circulation can only be maintained in the ocean if the heat sources are situated at a lower level than the cold sources (Defant, 1961, page 491).

Sandstrom (1916) considered the mechanical energy balance of the steady circulation in the oceans. In order to overcome friction, there should be a net input of mechanical energy over each closed streamline

$$w = -\oint_s v dp \qquad (1)$$

where v and p are the specific volume and pressure respectively, and the integration is taken along closed streamlines s. He simplified the oceanic circulation in terms of a heat engine by assuming four idealized stages within each cycle of the engine:

i) Heating-induced expansion under a constant pressure ($1 \to 2$);

ii) Adiabatic transition from the heating source to the cooling source ($2 \to 3$);

iii) Cooling-induced contraction under a constant pressure ($3 \to 4$); and

iv) Adiabatic transition from the cooling source to the heating source ($4 \rightarrow 1$).

According to this idealized cycle, the net amount of work is negative, if the system is heated under low pressure, but cooled under high pressure, Fig. 1a. Positive work is possible only for the counter-clockwise cyclic process, in which heating takes place at a higher pressure and cooling takes place at a lower pressure, as shown in Fig. 1b. Thus, he came to a conclusion that a closed steady circulation can be maintained in the ocean only if the heating source is situated at a level lower than the cooling source.

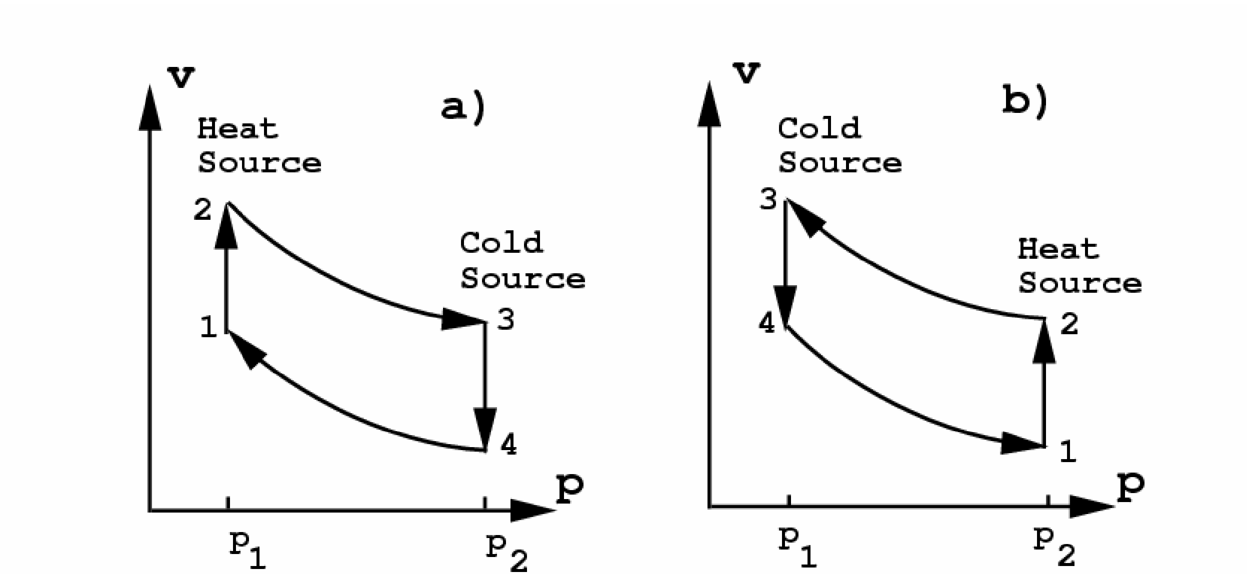


Fig. 1. An idealized Carnot cycle.

In order to illustrate his theorem, Sandstrom carried out laboratory experiments. In the first experiment, the heating source was put at a level lower than the cooling source. Strong circulation was observed between the levels of heating and cooling. In the second experiment, the heating source was put at a level higher than the cooling source. He reported that no circulation was observed, and a stable stratification was observed between the heating and cooling levels.

Sandstrom's theorem was questioned by Jeffreys (1925), who pointed out that any horizontal density (temperature) gradients must induce a circulation. By including the diffusion terms in the density balance equation, Jeffreys concluded that a circulation should be induced even if the heating source were put at a level higher than the cooling source.

The debate over Sandstrom's theorem continues to the present time. Further reference to Sandstrom's theorem may be found in many books and review articles, e.g. Hodske et al. (1957), Defant (1961), Dutton (1986), and Colin de Verdiere (1993).

Many authors cite Sandstrom's theorem because they believe the theorem is based on sound thermodynamic principles, and nobody wants to take the risk of violating the second law of thermodynamics. However, the application of Sandstrom's theorem to the oceanic circulation does pose a serious puzzle. The ocean is mostly heated and cooled from the upper surface. (Compared with other sources of energy, energy due to the geothermal heating is much smaller; however, its contribution to the oceanic general circulation may not be totally negligible, as will be discussed shortly.) Due to thermal expansion, the sea surface level at low latitudes where heating takes place is about one meter higher than the sea level at high latitudes where cooling takes place. Therefore, according to Sandstrom's theorem, there should be no convectively driven circulation. Thus, the existence of the strong meridional overturning circulation in the oceans, poses a serious challenge for Sandstrom's theorem.

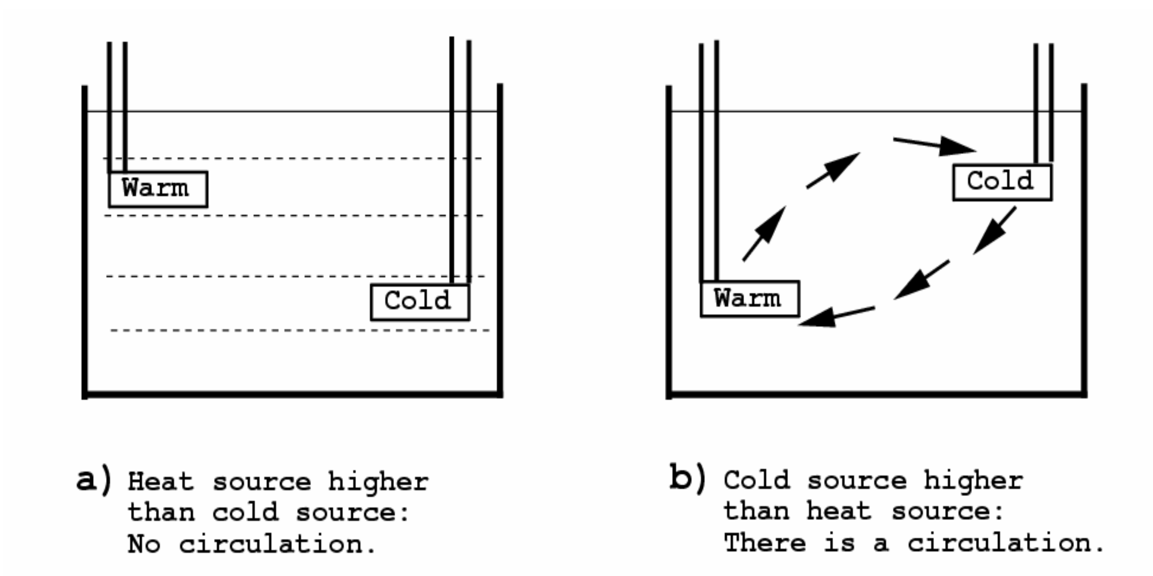


Fig. 2. Laboratory experiments demonstrating the Sandstrom's Theorem.

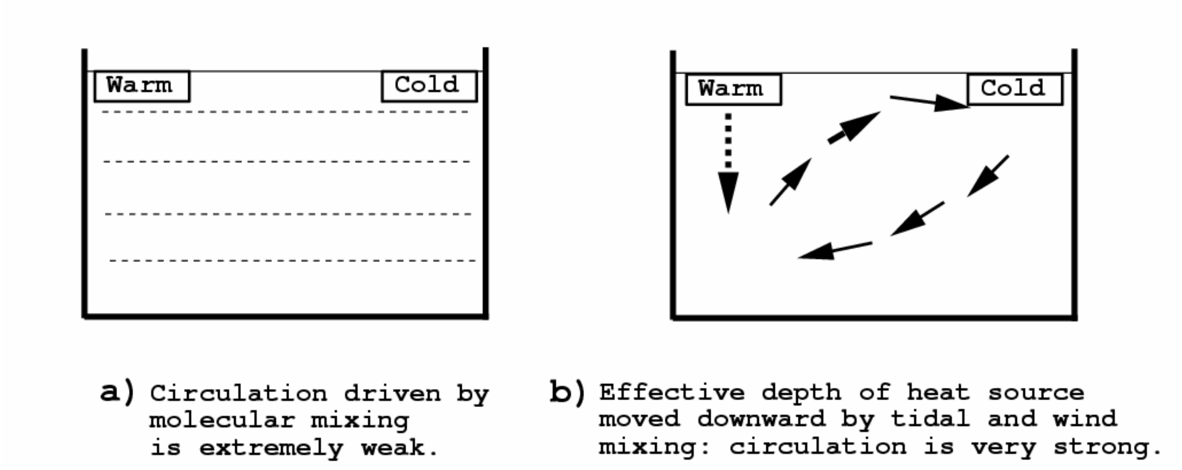
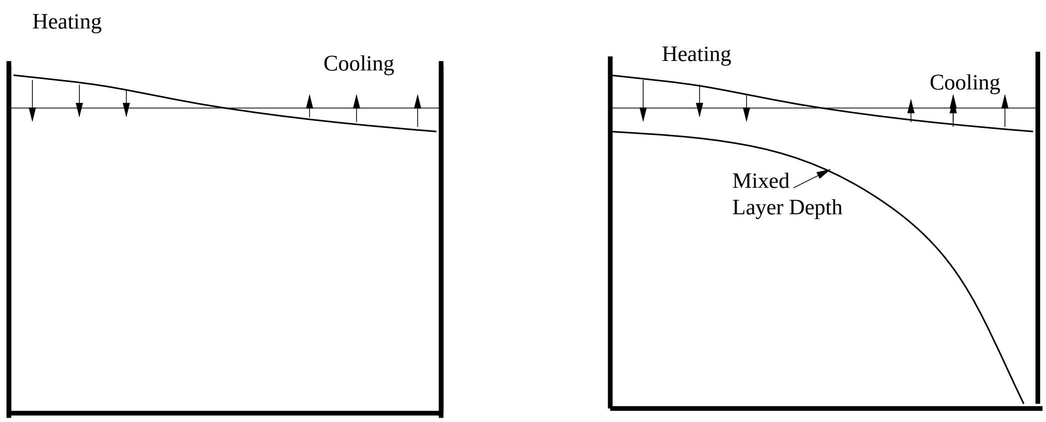


Fig. 3. Application of the Sandstrom' Theorem to the world oceans.

Since heating and cooling apply to the sea surface, practically the same level, circulation is extremely weak, if the system has no additional mechanical energy for supporting the circulation, Fig. 3a. However, circulation is very strong in the world oceans because the effective depth of heating is moved downward due to tidal and wind mixing, Fig. 3b.

It can be shown that heating/cooling an infinitely thin layer on the upper surface do not generate much gravitational potential energy, Fig. 4a. On the other hand, cooling can induce an unstable stratification, and thus induces convective overturning and a loss of gravitational potential energy, Fig. 4b.



a) Heating/cooling an infinitely thin layer do not generate much GPE.

b) Convective adjustment induced by cooling is a sink of GPE.

Fig. 4: Cooling and heating do not generate much gravitational potential energy; convective adjustment is a sink of potential energy.

Although the discrepancy between Sandstrom's theorem and the oceans has been known for many years, the problem remains unsettled. Many people have tried to find a mechanism which makes heating penetrate much deeper than cooling. This attempt has so far failed to yield satisfactory results because cooling-induced convection at high latitudes can easily penetrate to one kilometer (or deeper) below the surface, but it is hard to find a simple mechanism, which makes heating penetrate even deeper.

The difficulty associated with Sandstrom theorem is that ***his argument was entirely based on thermodynamics, without a more rigorous fluid dynamical analysis***. His model involves highly idealized circulation physics, especially as it completely excludes diffusion,friction, and wind stress. As will be shown shortly, including diffusion will substantially change the model's behavior, and a detailed analysis of the dynamic balance in the model answers the questions posed by the old model. We will begin with a detailed analysis of a simple tube model used by many investigators. The essential problem of Sandstrom theorem is the lack of mixing in the model. Diapycnal mixing rate in the ocean is about 1000 times stronger than the molecular mixing rate. As a result of this strong mixing, there is a strong meridional pressure gradient that drives a strong meridional overturning, Fig. 2b.

However, vertical (or diapycnal) mixing in a stratified fluid requires mechanical energy because light fluid is pushed downward and heavy fluid is pushed upward during mixing. Thus, mechanical energy is required for the maintenance of the stratification, and one of the most important issues facing us is the energy source for the mixing and its spatial and temporal distribution.

C. Mixing-induced circulation in a tube model

Thermally driven circulation in the oceans can be idealized in terms of a tube model (Huang, 1999). The model consists of a closed square loop, with two vertical arms of length H and two horizontal arms of length 2B, Fig. 5. The tube has a uniform cross section of unit area, and temperature and velocity are assumed uniform across each section. The cooling source is located at a distance D from the origin (defined as the central point of the low arm) at the right-hand side of the lower arm, and the heating source is located at a directly opposite position at the left-hand side of the upper arm.

When D is larger than B, the cooling source is located at a vertical level of D-B in the right arm. When D is larger than B+H, the cooling source is located in the upper arm. Temperature at the cooling and the heating sources is maintained constant. In this study we will assume the fluid is Boussinesq, so that heating/cooling does not change the volume of the fluid.

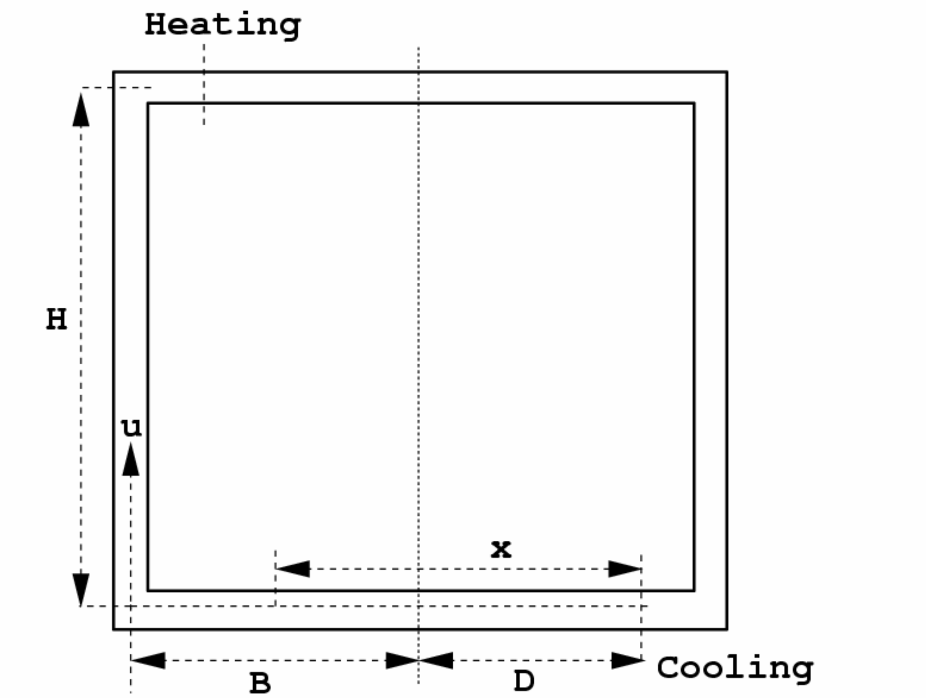


Fig. 5. An idealized tube model for the ocean.

The density (temperature) distribution within the tube in a steady state is governed by a simple balance between density advection and diffusion,

$$u\rho_X = \kappa\rho_{XX} \tag{2}$$

where X is the along-tube coordinate, defined positive clockwise from the cooling source. Introduce the following non-dimensional numbers

$$s = \frac{X}{L}, b = \frac{B}{L}, d = \frac{D}{L}, h = \frac{H}{L}, L = 2B + H$$

Assuming the circulation is steady and clockwise, the density distribution within the loop can be determined. We will label the solution to the left (right) of the cooling source with a superscript $l\,(r)$,

$$\rho^l = \rho_2 - \Delta\rho R_1\left(e^{\alpha s} - 1\right) \tag{3}$$

$$\rho^r = \rho_2 - \Delta\rho R_2\left(e^{-\alpha s} - 1\right) \quad (4)$$

where ρ_2 is the density at the cooling source, ρ_1 is the density at the heating source, $\Delta\rho = \rho_2 - \rho_1$ is the density difference,

$$R_1 = \left(e^{\alpha} - 1\right)^{-1},\ R_2 = \left(e^{-\alpha} - 1\right), \alpha = \frac{uL}{\kappa} \quad (5)$$

In this study our focus is on the model's behavior within a parameter range pertinent to a simple vertical advection and diffusion balance in the oceans; thus, we will use the following parameters: $L = 1km$, $b = 0.25$, $h = 0.5$, the velocity will be in units of $10^{-7} m/s$.

In a steady state, pressure torque is balanced by friction

$$\int_s \rho\mathbf{g}ds = r\rho_0 u \quad (6)$$

where $\mathbf{g}$ is gravitation vector, and r is the friction parameter.

The strength of the circulation strongly depends on the location of the cooling source, Fig. 6. The solutions for $d < 0.5$ (heating source is higher than cooling source) are shown in the left panel. Since mixing rate is normally less than $10^{-4} m^2/s$, only the left part of this panel is relevant to the oceans. For any given d, it is clear that the circulation is almost linearly proportional to the mixing rate κ, but it is insensitive to friction r. Thus, the circulation is mixing controlled and this case resembles the meridional circulation in the oceans.

To obtain a circulation on the order of $10^{-7} m/s$, we need a mixing rate of $10^{-4} m^2/s$. When mixing is very weak, on the order of $10^{-7} m^2/s$ (molecular diffusion), the non-dimensional velocity will be on the order of 10^{-3} (or $10^{-10} m/s$ dimensionally). This is equivalent to $3mm$ displacement per year, and it is probably very difficult to observe. ***Thus, both Sandstrom's observation and Jeffreys's argument are correct, and at the same time both are incomplete and somehow inaccurate***.

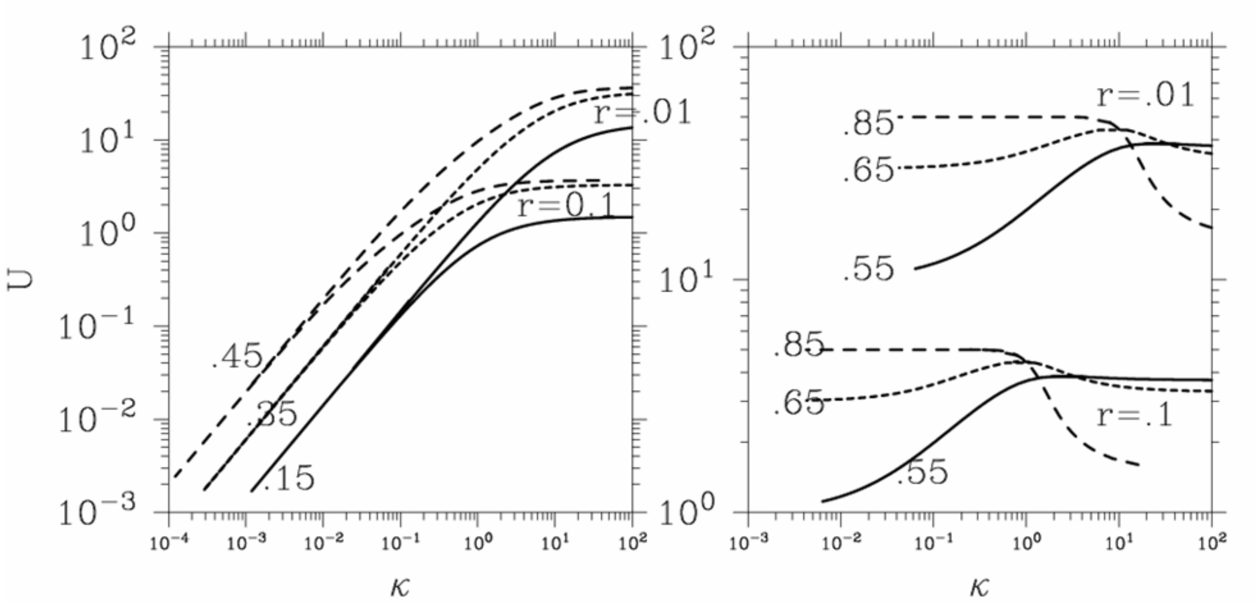


Fig. 6. Efficiency of the tube model; Mixing coefficient in units of $10^{-4} m^2/s$, velocity in units of $10^{-7} m/s$.

The cases of d>0.5 (cooling source is higher than the heating source) are shown in the right panel of Fig. 6. For a given d (the location of cooling source), the circulation is insensitive to the mixing rate. When κ changes 1000 times, the circulation rate changes only slightly. However, the circulation rate is very sensitive to the friction parameter r. As r increases ten times, the circulation rate also changes about ten times. Thus, the circulation is frictionally controlled when the cooling source is at a level higher than the heating source.

2. Where does the "Sandstrom Theorem" stand?

A. Pure thermal-driven circulation

A close examination reveals that circulation driven solely by thermal forcing can be classified into the three following types.

Type 1: The heating source is placed at a pressure level higher than the cooling source; it is well known that there is strong circulation.

Type 2: The heating source is placed at a pressure level lower than the cooling source. Sandström (1908) carried out such a laboratory experiment and reported that there was no circulation in the final steady state (see Defant, 1961, p. 491). However, Jeffreys (1925) argued that wherever there is horizontal density difference, there should be circulation; but he did not state how fast the circulation is. Although Type 2 thermal circulation can be weak, the circulation is detectable.

Type 3: Heating and cooling sources placed at the same pressure level (also referred to as horizontal differential heating or horizontal convection). This type of heating/cooling resembles situations in the ocean which is primarily heated and cooled from the upper surface, neglecting the penetration of solar radiation and geothermal heating. Rossby (1965) carried out a series of laboratory experiments in thermal circulation under horizontal differential heating, and reported steady circulation that occupied the entire depth of the tank. Results from lab experiments and numerical experiments by Rossby (1965, 1998) show steady circulation can be induced by horizontal convection.

B. The Paparella and Young Theorem

Paparella and Young (2002) theorem can be adapted for flow governed by two-dimensional, non-Boussinesq equations, including conservation of mass, momentum, and thermal energy, plus the equation of state (Wang and Huang, JFM, 2005)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{7}$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \rho \mathbf{g} + \mu \nabla^2 \mathbf{u} \tag{8}$$

$$\rho c_p \frac{DT}{Dt} = \kappa \rho c_p \nabla^2 T - p \nabla \cdot \mathbf{u} + \Phi \tag{9}$$

$$\rho = \rho_0 \left[1 - \alpha (T - T_0)\right] \tag{10}$$

where ρ (ρ_0) is density (mean density) of the fluid, g is gravitation acceleration, T is temperature, T_0 is a constant reference temperature, p pressure, c_p the specific heat, α is the thermal expansion coefficient, and Φ the dissipation function

$$\Phi = \mu \left[2\left(\frac{\partial u}{\partial x}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + \left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)^2 \right] + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)^2 \tag{11}$$

where $\lambda = -\frac{2}{3}\mu$ is commonly accepted as a parameter for most fluids.

Taking the dot product of equation (8) with the velocity vector leads to the conservation equation of mechanical energy

$$\frac{\partial}{\partial t}\left[\rho E_k + \rho g z\right] + \nabla\cdot\left[\left(\rho E_k + \rho g z + p\right)\mathbf{u}\right] = p\nabla\cdot\mathbf{u} + \mu\nabla^2 E_k + \mu\left\|\nabla\mathbf{u}\right\|^2 \tag{12}$$

where $E_k = \frac{1}{2}\mathbf{u}\cdot\mathbf{u}$ is the kinetic energy per unit mass and $\left\|\nabla\mathbf{u}\right\|^2 \equiv \nabla u\cdot\nabla u + \nabla w\cdot\nabla w$ is the deformation of velocity.

While in a steady state no mechanical energy enters/leaves the system through the boundaries, averaging Eq. (12) over the volume leads to a simple balance between the pressure work and dissipation

$$\left\langle p\nabla\cdot\mathbf{u}\right\rangle - \mu\left\langle\left\|\nabla\mathbf{u}\right\|^2\right\rangle = 0 \tag{13}$$

where $\langle\ \rangle$ denotes the an ensemble average.

Assuming $\alpha = const$, and $p = p_0 + p'$, $p' = -\rho_0 g z$, to a very good approximation, (7) is reduced to a balance between the first two terms

$$\nabla\cdot\mathbf{u} = -\frac{1}{\rho_0}\frac{D\rho}{Dt} = -\alpha\frac{DT}{Dt} = -\alpha\kappa\nabla^2 T \tag{14}$$

thus,

$$\left\langle p\nabla\cdot\mathbf{u}\right\rangle = -\rho_0 g\alpha\kappa\left\langle z\nabla^2 T\right\rangle \tag{15}$$

Using the Green formulae, $\left\langle z\nabla^2 T\right\rangle$ is reduced to

$$\left\langle z\nabla^2 T\right\rangle = \frac{1}{DL}\iint z\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right)dxdz = -\frac{\overline{T}_{top} - \overline{T}_{bottom}}{D} \tag{16}$$

where D is the depth of the water, $\overline{T}_{top}$ and $\overline{T}_{bottom}$ are the mean temperature at the top and bottom boundary respectively. For the case of heating/cooling from the upper surface, the mean temperature at the upper boundary $T_{bop} = \left(T_h + T_c\right)/2$, and the bottom temperature cannot be lower than T_c, i.e., $\overline{T}_{bottom} \geq T_c$, and; thus, Eq. (16) is reduced to

$$\left\langle z\nabla^2 T\right\rangle \geq -\frac{\Delta T}{2D} \tag{17}$$

where $\Delta T = T_h - T_c$, and the mean mechanical energy generation rate is

$$\frac{1}{\rho_0}\langle p\nabla\cdot\mathbf{u}\rangle \le \frac{\kappa\alpha g\Delta T}{2D} = \frac{\kappa g'}{2D} \tag{18}$$

This relation provides an upper limit for the mechanical energy convertible from thermal energy: $\frac{1}{\rho_0}\langle p\nabla\cdot\mathbf{u}\rangle DL \le \frac{\kappa g'}{2}L$, which is independent of the water depth.

From Eq. (13) the mean dissipation rate within the cell should be

$$\nu\left\langle \|\nabla\mathbf{u}\|^2 \right\rangle = \frac{\kappa g'}{2D} \tag{19}$$

A direct application of Paparella and Young formulae provides an upper limit for the mechanical energy convertible from surface thermal forcing in the world ocean. Assume the equator-pole temperature difference is $\Delta T \simeq 30^o C$, $\overline{D} \simeq 3.75km$ is the mean depth of the world ocean, $\alpha \simeq 2\times10^{-4} K^{-1}$, $\kappa \simeq 1.5\times10^{-7} m^2 s^{-1}$, so the energy conversion rate is $\langle p\nabla\mathbf{u}\rangle \sim \frac{\rho_0 g\alpha\kappa\Delta T}{2\overline{D}} \simeq 1.2\times10^{-9} W/m^3$. The total volume of the ocean is $1.324\times10^{18} m^3$, so the total conversion rate is about $1.5\times10^9 W$. This is 1000 times smaller than the rate of tidal dissipation. Since the total amount of heat flux going through the oceans is about $2\times10^{15} W$, the efficiency of the ocean as a heat engine is about 7×10^{-7}.

C. The extension of the Paparella and Young Theorem ?

Some of the essential points in this theory require further exploration:

i) Mixing coefficient near the upper/lower boundary remains the same as the molecular value. This may not be true in general. For example, if the ocean is heated from below and cooled from above, it is well known that the thermal boundary layer on the sea floor may be turbulent, if the Rayleigh number is larger than the critical value. On the other hand, the surface mixed layer is actually a boundary layer dominated by strong surface waves and turbulent, and the consequence of such non-molecular mixing remains to be explored.

ii) Thermal expansion coefficient is assumed constant. In reality, it is non-constant. Modification due to the nonlinearity of the equation of state remains unknown.

iii) Within the ascending (descending) plum over (below) the heating (cooling) source, the hydrostatic approximation made in the derivation may not be valid.

3. Lab experiments testing “Sandstrom theorem”

A) Previous results from lab and numerical experiments

Most previous results suggested that circulation in lab experiments under the horizontal differential heating occupies the whole depth of the tank, or the so-called fully-penetrating flow. The next figure gives some numerical experimental results.

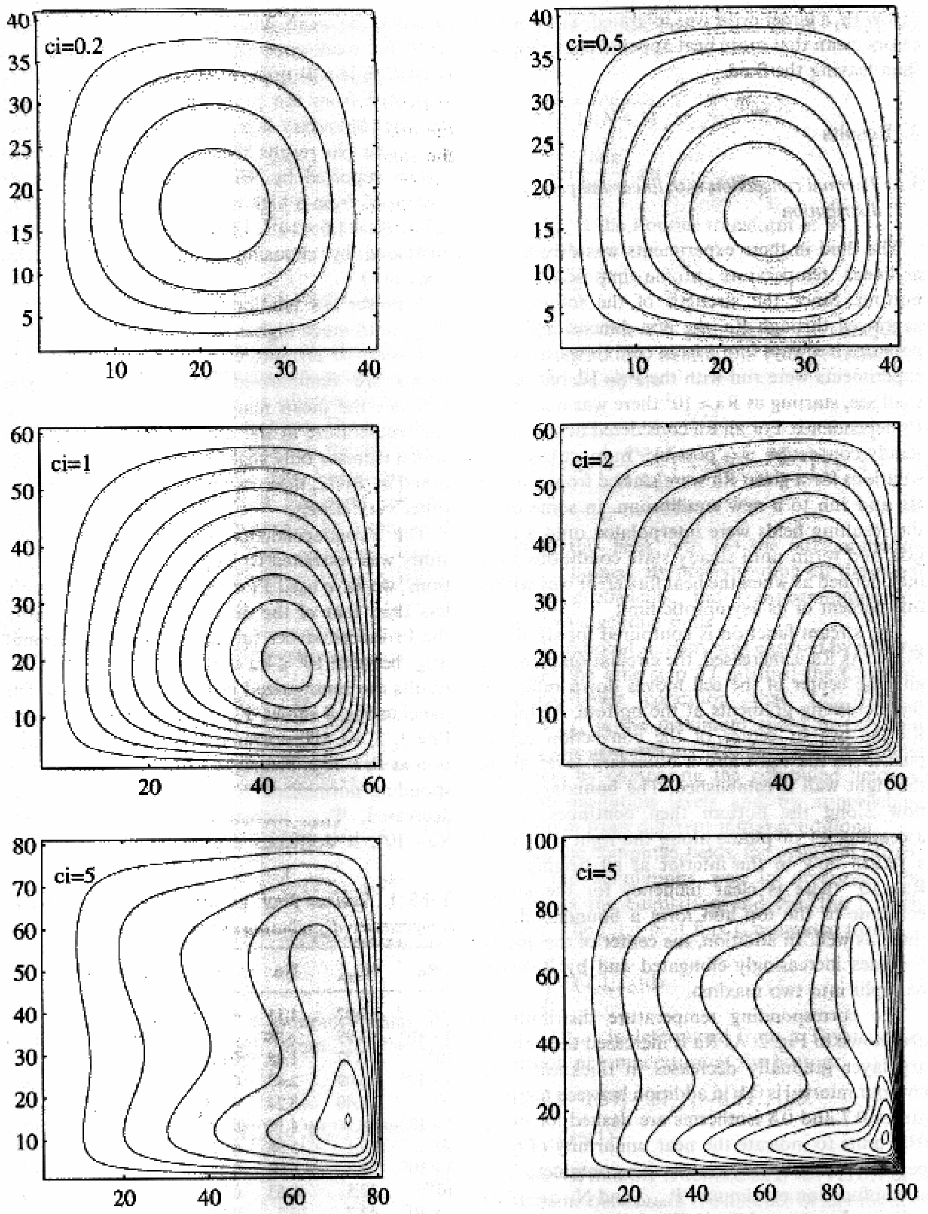


Fig. 1. $\Psi(x, y)$ at Rayleigh numbers 10^3, 10^4, 10^5, 10^6, 10^7, 10^8 (left to right, top to bottom). The axes indicate the number of grid points in the horizontal vertical. ci refers to the contour interval used. See Table 1 for further details.

Fig. 7. Streamfunction maps from numerical experiments for Rayleigh number (10^3 – 10^8) (Rossby, 1998).

B) New experimental results

The basic ideas of the Sandstrom theory have recently been tested through lab experiments. Circulation driven by horizontal differential heating is studied, using a double-wall plexiglass tank (20x15x2.5 cm^3) filled with salt water (Wang and Huang, 2005). In instances of heating/cooling from above and below, results indicate that there is always a quasi-steady circulation. In contrast to most previous results from experimental/numerical studies, circulation in these new experiments appears in forms of a shallow cell adjacent to the boundary of thermal forcing.

The typical quasi-steady mean velocity field for Cases 1 to 4 is shown in Fig 8. The temperature difference of the thermal sources of these cases is the same, $\Delta T = 18.5°C$. It is observed that the flow pattern produced by heating/cooling from the top is a mirror image of heating/cooling from the bottom. The pattern in Figs. 8c and 8d is quite different from Fig. 8a and 8b. In general, circulation for Case 3 is much stronger than Case 1; the circulation cell is rather tall near the heating end, but it is shallow and intense near the cooling end. Case 4 (Fig. 8d) is quite different from other cases. The circulation cell for this case is confined to the right half of the tank and no longer occupies the whole length of the tank. In addition, the circulation is separated from the bottom boundary, except near the heating end. Most importantly, the strength of the circulation is greatly reduced. Nevertheless, circulation in all cases is visible to the naked eyes.

The results indicate that there is always a relatively stable steady circulation which occurs within a shallow depth of the fluid, or the so-called partial cell. Although there is no external mechanical energy, the circulation driven by horizontal convection exists. Thus, the Sandstrom theorem is inaccurate. However, the circulation driven by horizontal convection is quite weak. If results from the lab experiments can be stretched and applied to the oceans, then surface thermal forcing alone can drive a circulation, which is so weak and may not be able to penetrate to the deep ocean. Of course, there are major differences between the lab experiments and the oceans, such as the huge difference in the Rayleigh number, the Reynolds number, and the effect of rotation.

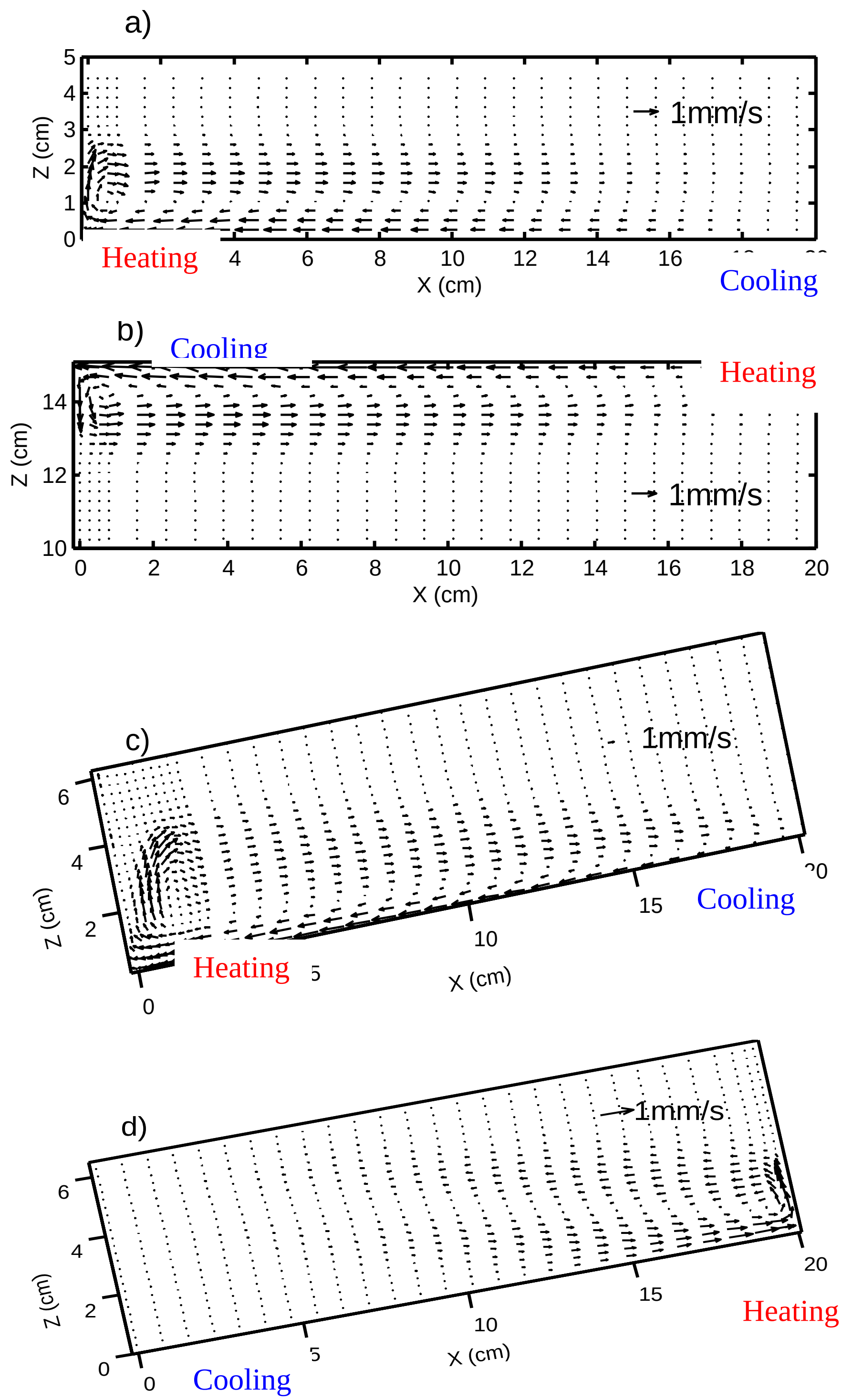


Fig. 8. Time-mean circulation driven by purely thermal forcing obtained from lab experiments (Wang and Huang, 2005).

The experimental results for the case with horizontal differential thermal forcing (Case 1 and 2) fit a classical scaling 1/5-power law by Rossby (1965), i.e., the non-dimensional streamfunction maximum obeys $\Psi \sim Ra^{1/5}$, where the horizontal Rayleigh number is defined as $Ra_L = \frac{g' L^3}{\nu\kappa}$, where $g' = g\alpha\Delta T$ (ΔT is the temperature difference of the cooling and heating sources), κ and ν are molecular diffusivity and viscosity of the fluid respectively. Alternatively, one can also use the vertical Rayleigh number $Ra_\delta = \frac{g'\delta^3}{\nu\kappa}$, where δ is the thickness of the velocity boundary layer adjacent to the heating/cooling surface. The non-dimensional stream- function maximum is defined as $\Psi = \tilde{\Psi}/\kappa$, where $\tilde{\Psi}$ is the maximum of the dimensional streamfunction defined as $\tilde{\Psi} = max\left|\int_0^\delta \overline{u}(z)dz\right|$. However, for the case with a slanted bottom, the circulation seems obey different power laws, Fig. 9. These results suggest that circulation driven by differential heating can be very complicated, and very sensitive to the slope of the boundary where the heating/cooling applies.

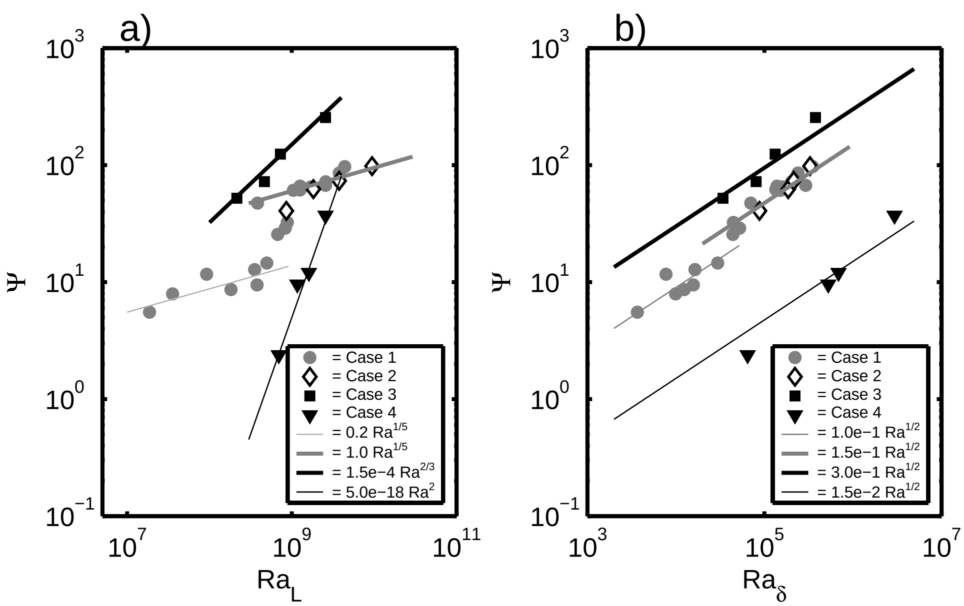


Fig. 9. The relationship between non-dimensional streamfunction maximum, Ψ , and Rayleigh number. a) horizontal Rayleigh number; b) vertical Rayleigh number.

Experimental results indicate that for the range of Rayleigh number larger than 5×10^8 (in this study) the ratio between energy dissipation rate and energy generation rate seems consistent with the theoretical upper limit predicted by theory of Paparella and Young (2002). However, for the case of low Rayleigh number, the discrepancy is quite large. There are many potential sources of error. For example, the assumption of hydrostatic pressure may introduce errors, but no estimations available now. Thus, the reason for the discrepancy between the theory of Paparella and Young and our lab experiments remain unclear.

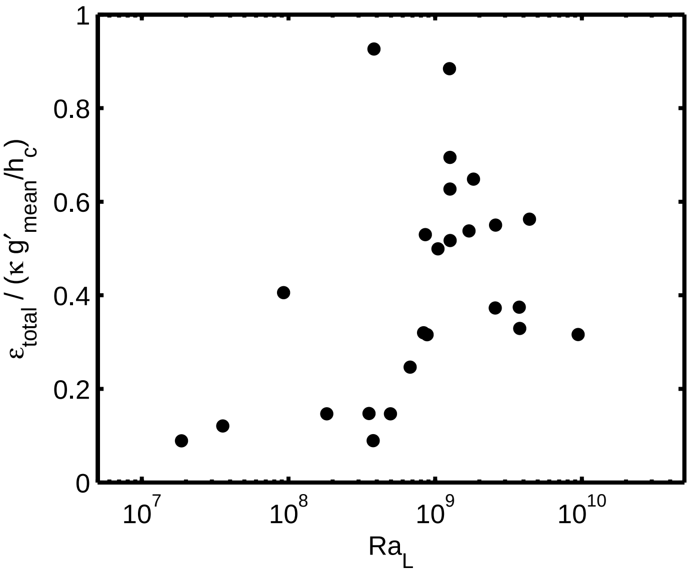


Figure 10. Ratio of total energy dissipation rate and buoyancy flux diagnosed from experiments with horizontal differential heating (Case 1 and 2).

C) Challenges posed from lab experiments

Experiments always pose many exciting questions. In this case, we have lab experimental results which are quite different from previous results. There are many unsolved questions:

a) Why there is the partial-penetration circulation, different from previous results?
b) Can we numerically reproduce results obtained from lab experiments?
c) Can we work out the scaling laws for the case with slopping bottom?

4. The essential limitation of the "Sandstrom Theorem"?

The essential limitation of the Sandstrom theorem is that the theorem omits the important role of wind stress. Wind stress drives strong circulation in the world oceans, including Ekman transport in the upper ocean and the Antarctic Circumpolar Current. Wind energy is the most important contributor to the mechanical energy balance in the world ocean. Any model or theory of the oceanic general circulation without wind contribution is incomplete, and their results should be interpreted with caution.

By the traditional definition, thermohaline circulation is the circulation driven by the density difference of the water. Thus, anything that affects the density also affects the thermohaline circulation, such as surface heat flux and freshwater flux. Furthermore, mixing in the ocean interior turns out to be one of the most important factors that controls the thermohaline circulation. In addition, wind stress also can affect the thermohaline circulation. Separating the oceanic circulation into the so-called wind-driven circulation and thermohaline circulation is an idealization that was designed to help us to understand the complicated processes, and such a separation should never be taken literally.

Surface forcing, such as heat flux and freshwater flux, are the necessary preconditions for the thermohaline circulation, i.e., without the surface thermohaline forcing, there will be no density difference in the water column and hence no thermohaline circulation. However, surface forcing alone does not really control the direction of the meridional overturning cell or the strength of the circulation. Recent studies indicate that the strength of the meridional overturning cell is controlled by several factors:

1) Wind forcing in the Southern Ocean. Under the current geometry of the land-sea distribution, the Antarctic Circumpolar Current plays a quite unique role in controlling the global thermohaline circulation, Toggweiler and Samuels (1998).

2) The amount of mechanical energy available for mixing (Munk and Wunsch, 1998). In fact, simple scaling indicates that the meridional overturning rate is linearly proportional to the amount of mechanical energy available for mixing (Huang, 1999).

3) Wind stress at other location can also affect the thermohaline circulation.

In fact, the system seems so complicated that any simple theory can hardly provide a complete picture of the circulation. This is one of the grand challenges for physical oceanography, waiting for us to resolve.

References:

Colin de Verdiere, A., 1993: On the oceanic thermohaline circulation. *Modeling Oceanic Climate Interactions*. J. Willebrand and D. L T. Anderson, Eds., D. Reidel, 151-183.

Defant, A., 1961. Physical Oceanography, Vol. 1, Pergamon Press, 728 pp.Gill, A. E., 1977: The hydraulics of rotating-channel flow, *J. Fluid Mech*., **80**, 641-671.

Dutton, J., 1986: *Dynamics of Atmospheric Motion*. Dover, 617pp.

Hodske, C. L., T. Bergeron, J. Bjerknes, and R. C. Bundgaard, 1957. *Dynamic Meteorology and Weather Forecasting*, American Meteorological Society, Boston, 800 pp.

Huang, 1999: Mixing and energetics of the oceanic thermohaline circulation. *J. Phys. Oceanogr*., 29, 727-746.

Jeffreys, H., 1925: On the motions produced by differences of temperature and humidity. *Quarterly Journal of the Royal Meteorological Society*, **51**, 347-356.

Munk, W. H. and C. Wunsch, 1998: The Moon and mixing: abyssal recipes II, DeepSea Res., I, 45, 1976-2009.

Paparella, F. and W. R. Young, 2002: Horizontal convection is non turbulent. *J. Fluid Mech.*, **205**, 466-474.

Rossby, T., 1965. On thermal convection driven by non-uniform heating from below; An experimental study. *Deep-Sea Res.*, **12**, 9-16.

Rossby, T., 1998. Numerical experiments with a fluid heated non- uniformly from below. *Tellus*, **50A**, 242-257.

Sandström, J. W., 1908: Dynamicsche Versuche mit Meerwasser. *Annalen der Hydrographie under Martimen Meteorologie*, **36**, 6- 23.

Sandström, J. W., 1916: Meteorologische Studien im schwedischen Hochgebirge. Goteborgs K. *Vetenskaps-och Vitterhetssamhalles Handl*., Ser. 4, **22**, No. 2, 48pp..

Toggweiler and Samuel, 1998: On the ocean's large-scale circulation near the limit of no vertical mixing. *J. Phys. Oceanogr*., **9**, 1832-1852.

Wang W. and R. X. Huang, 2005: An experimental study on thermal circulation driven by horizontal differential heating, *Journal of Fluid Mechanics*, **540**, 49-73.