4.3 Oblique Shocks and Expansions Fans: The Supercritical Marine Layer.

The marine layer is a relatively dense and well-mixed layer of moist air that lies above the sea surface and is often capped by a strong inversion in temperature and humidity. In the North Pacific the layer can extend all the way from California to Hawaii and its thickness can increase over that distance from around 600m or less to 2000m. The physical properties of the layer are particularly well observed along the Northern California coast (e.g. Dorman 1985, 1987; Winant et al. 1988 and Dorman et al. 1999). During the summer upwelling season, the North Pacific High drives equatorward winds along the coastline. The winds are intensified to the west of the 1000m high coastal mountain range, an effect that extends 100km or so offshore. Wind speeds near the inversion level can reach values of up to 30m/s. and Froude numbers can exceed unity. During such periods of ostensibly supercritical flow, irregularities in the coastline can produce dramatic changes in the wind speed and layer thickness. In one configuration (Figure I2a,b) the winds accelerate and the layer thins as it passes Point Arena, where the coastline abruptly veers to the southeast. Speeds of 20m/s are reached and the layer thickness decreases form 600 to 300m. The contours of constant wind speed, which are roughly perpendicular to the coastline near Point Arena, become more oblique as Stewarts Point is approached. Similar behavior has been observed along Peru's coastline by Freeman (1950), who likens the acceleration and thinning with an *expansion fan*, a phenomena well documented by aeronautical engineers. The fan is sometimes marked by clearing as the high-speed air descends (Figure 4.3.1). Between Stewarts Point and Bodega Bay, where the coast veers slightly southward, the wind speed diminishes and the layer thickens in what has been described as an *oblique hydraulic jump*. Different visualizations of marine layer jumps (Figures 4.3.2 and 4.3.3) show the abrupt and sometimes wavy character of the transition.

The subsidence associated with the Pacific high-pressure system creates a particularly sharp interface between the cold and moist marine layer and the overlying warm and dry air. It is therefore natural to treat the entire layer as a 'slab' and to use the shallow-water equations as a model. Expansion fans and oblique hydraulic jumps are not admitted in the long-wave limit of these equations and we must therefore allow full freedom in the two horizontal dimensions. If the flow is assumed steady and supercritical, the method of characteristics can be used to obtain solutions. A complete derivation of the characteristic equations appears in Appendix B. The present section contains a non-rigerous discussion of characteristic curves, oblique jumps and expansion fans; a formal application appears in Section 4.4. Both discussions will ignore the effects of rotation, since this considerably simplifies the discussion of characteristics and still allows for a description of the basic phemonena. The neglect of rotation will, however, preclude any discussion of the decay of features in the offshore direction.

It should also be noted that the supercritical mode of the marine layer is just one of several observed configurations. Another is the 'gravity current' mode, in which the layer moves northward along the coast with a distinct leading edge (Figure I2c). This type of flow is discussed in Section 4.5.

The method of characteristics for the steady-shallow water equations in two dimensions has origins in the theory of gas dynamics and compressible flow (Courant and Friedrichs, 1976). The methodology is can be applied in regions of the flow field where

$$F = \frac{(u^{*2} + v^{*2})^{1/2}}{gd^*} > 1 \qquad . \qquad (4.3.1)$$

F is clearly a Froude number based on the full flow speed and the region over which (4.3.1) holds is sometimes called *supercritical*. This usage differs from that of our previous applications in the long-wave limit, where the entire cross-section of a flow is judged subcritical or supercritical depending on whether a long normal mode could propagate in two or one directions. The appropriate Froude number in those cases depends on the flow across the whole cross section. The Froude number defined in (4.3.1) in relevant to the free, locally generated disturbances.

If (4.3.1) holds, the influence of a localized forcing is limited to a downstream subregion of the flow field. The governing equations in this case are hyperbolic and can be solved using the method of characteristics (Appendicies B and C). This property can be demonstrated by considering a uniform southward current with velocity v_o^* and depth d_o^* (Figure 4.3.4). A localized disturbance to the flow introduced at point p will spread out in a widening circle as it is advected downstream. The radius of the circle will grow at rate $(gd_o^*)^{1/2}$ while the center of the circle will move southward at speed v_o^*. If $F>1$, the disturbance will spread within a cone of influence that spans the angle $2A$, where

$$A = \sin^{-1}(F^{-1}) . \qquad (4.3.2)$$

The angle A and the edges of the cone are analogous to the Mach angle and Mach lines of supersonic flow. In shallow water theory A is referred to as the Froude angle. If $F<1$, the disturbance circle spreads upstream and downstream, carrying the influence to all parts of the flow field. The steady shallow water equations in this case are elliptic and the method of characteristics fails.

A related feature distinguishing two-dimensional flows with $F>1$ from those with $F<1$ is that only the former can support a stationary, free disturbance. It is left as an exercise to show that for the uniform southward flow considered above, a small-amplitude, stationary disturbance with horizontal structure $e^{i(k^*x^*+l^*y^*)}$ can exist provided

$$\frac{l^*}{(l^{*2} + k^{*2})^{1/2}} = \sin(A) = \frac{1}{F} < 1 . \qquad (4.3.3)$$

There are two groups of waves (corresponding to $\pm k^*$), each with crests and troughs tilted at the Froude angle with respect to the background flow direction (Figure 4.3.5a). We denote the corresponding lines of constant phase by C_+ and C_- and note that they are aligned at the same angles are the edges of the wedge of influence in Figure 4.3.4. In both cases the alignment is such that the normal component of velocity equals the intrinsic propagation speed $(gd^*)^{1/2}$ of a gravity wave. As F approaches unity from above, the dashed and solid lines become perpendicular to the background flow. The flow is now one-dimensional and hydraulically critical in the sense explained in Chapter 1. For $F<1$ the stationary disturbances cease to exist. In the next section, we will show that the Froude lines are also characteristic curves for the steady flow.

It can also be shown (Exercise 1) that disturbance energy propagates along the characteristic curves in the downstream sense. Stationary disturbances generated by coastline irregularities to the east of the flow should therefore be carried away from the coast along the C_- lines. Suppose that the coastline veers away from the upstream flow direction (Figure 4.3.5b) and that the background flow adjusts so as to run parallel to the coast with a new velocity and depth. The new Froude angle A_1 between the disturbance phase lines and the coast will depend on the new value of F_1, which cannot be calculated without further analysis. However, we have already seen that a supercritical channel flow accelerates and shoals (Section 1.4) when the channels widens. F_1 might therefore be expected to exceed its upstream value F_o and (4.3.2) then implies $A_1<A_o$. One can infer an expansion fan, a family of fanning wave crests and troughs, in the intervening region (Figure 4.3.5b).

Where the coastline turns back into the flow (Figure 4.3.5c), one might expect the Froude number to decrease and A to increase, giving rise to intersecting crests and troughs and perhaps a shock. A simple model that allows prediction of the angle β of the oblique shock is sketched in Figure 4.3.6. The coastline is assumed to turn into the upstream flow at an angle α and the flow upstream and downstream of this point is assumed to be parallel to the coast. The matching conditions across the shock were developed in Section 3.5.2. For example, equations (3.5.2) and (3.5.6) expressing the continuity of normal flux and tangential velocity lead to

$$v_o * d_o * \sin\beta = v_1 * d_1 * \sin(\beta-\alpha)$$

and

$$v_o * \cos\beta = v_1 * \cos(\beta-\alpha)\,,$$

so that

$$\frac{d_o *}{d_1 *} = \frac{\tan(\beta-\alpha)}{\tan(\beta)}\,. \qquad (4.3.6)$$

A third constraint based on the above relations plus the balance of flow force across the jump (equation 3.5.5) is

$$\frac{d_o *}{d_1 *} = \frac{2}{-1 + \sqrt{1 + 8\frac{v_o^{*2} \sin^2 \beta}{g d_o *}}} \qquad (4.3.7)$$

(see Exercise 2). Eliminating d_o*/d_1* between (6) and (7) gives

$$\tan(\beta - \alpha) = \frac{\tan\beta}{-1 + \sqrt{1 + 8\frac{v_o^{*2} \sin^2 \beta}{g d_o *}}}, \qquad (4.3.8)$$

allowing determination of the jump angle given α and the upstream flow.

Exercises

1) For 2-dimensional plane waves in a uniform flow with velocity (0,-v_o*), derive the dispersion relation

$$\omega^* = -l^* v_o^* \pm (g d_o^*)^{1/2} (k^{*2} + l^{*2})^{1/2}$$

and deduce the condition (2) that the waves be stationary. For stationary disturbances, show that the group velocity is

$$\mathbf{c_g} = \frac{(gd^*)^{1/2} k^*}{l^* (k^{*2} + l^{*2})^{1/2}} \left(\pm l^* \mathbf{i} - k^* \mathbf{j} \right)$$

and that energy therefore propagates along the lines of constant phase (characteristic curves) C_+ and $C_{-,,}$ and in the downstream direction of these lines.

2) Prove equation (4.3.7). (Hint: show that equation 1.6.8 holds for the oblique jump if F_u is interpreted as the upstream Froude number based on the normal component of velocity.)

Figure Captions *(note that high resolution versions of the following figures are available)*

Figure 4.3.1 Aircraft photo, facing to the North, showing Cape Mendocino. The area of clear air corresponds is an expansion fan in the lee of the Cape. (Enhanced version of photo by Dr. Clive Dorman). *This photo has not appeared in any publications. High resolution version: CapeMendocino.enhanced.tif*

Figure 4.3.2   Possible hydraulic jump near Point Arena, with the viewer facing southeast. (Photograph by Dr. John Baine.) *need to verify with John Baine that this jump is indeed situated near Point Arena. High resolution version:* shock_front.rotated.tiff

Figure 4.3.3. Image of a hydraulic jump near Point Sur based on LIDAR, a laser device that points upward. Air Density variations cause the light to reflect back, similar to radar. The bottom of the air temperature inversion causes strong backscatter and is indicated by yellow-green boundary. (From Dorman et al. 1999). *Original figure is in ptSurHyJump.jpg*.

Figure 4.3.4 Wedge of influence and the Froude angle *A*.

Figure 4.3.5 (a) Cross-waves in a supercritical flow. The crests and troughs are characteristic curves. (b) Expansion fan caused when the coastline veers away from the upstream flow. (c) Oblique hydraulic jump caused the by the coastline veering into the flow.

Figure 4.3.6 Oblique hydraulic jump at a corner.

Figure 4.3.1

Figure 4.3.2

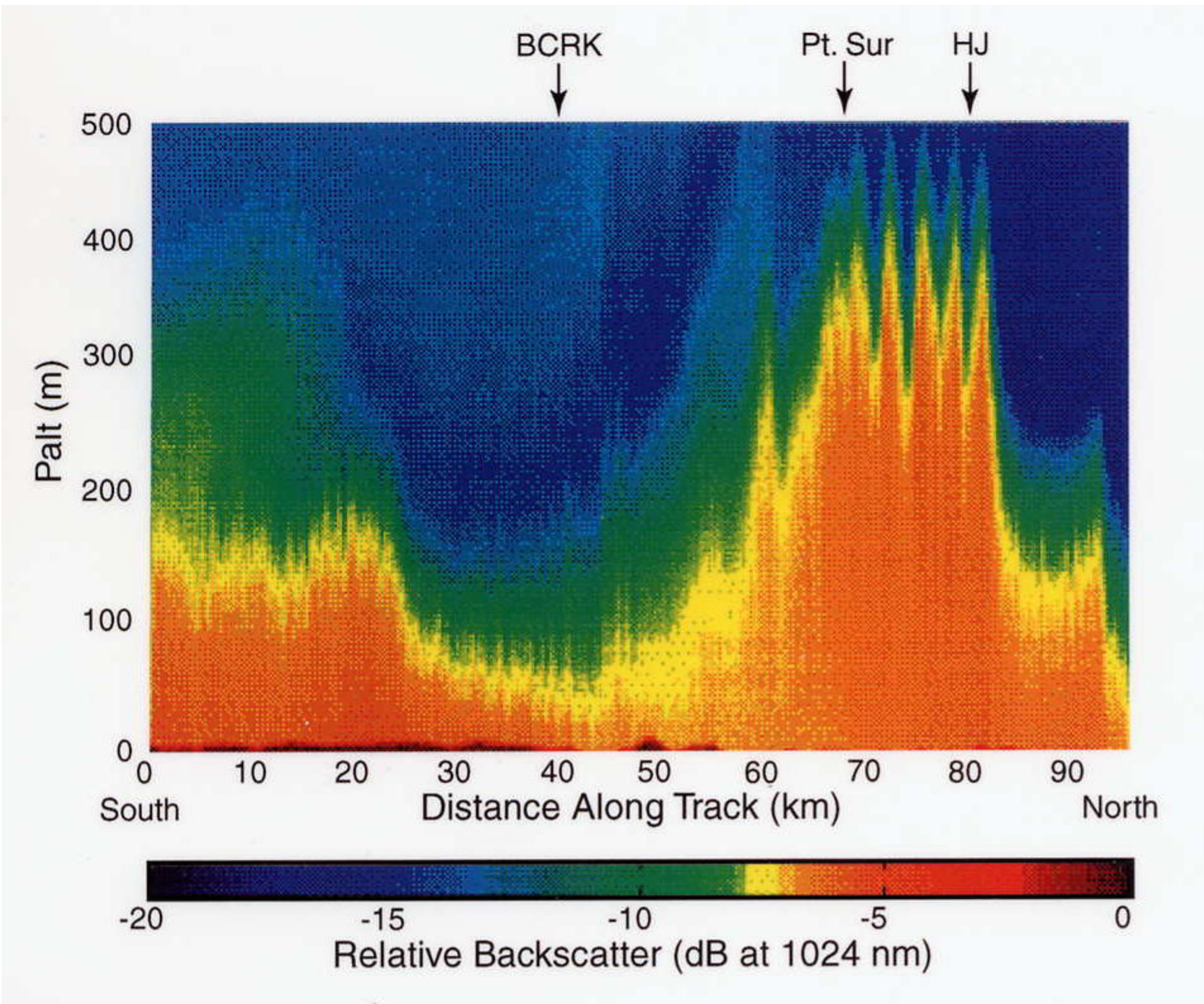
BCRK
Pt. Sur
HJ
500
400
300
200
100
0
Palt (m)
0
10
20
30
40
50
60
70
80
90
South
Distance Along Track (km)
North
-20
-15
-10
-5
0
Relative Backscatter (dB at 1024 nm)

Figure 4.3.3

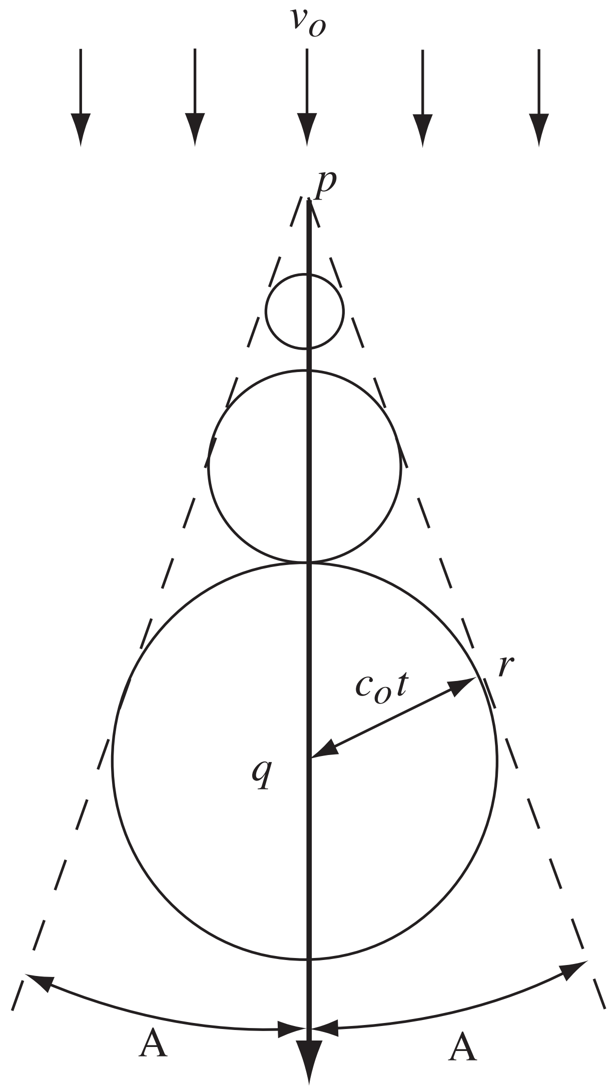

Figure 4.3.4

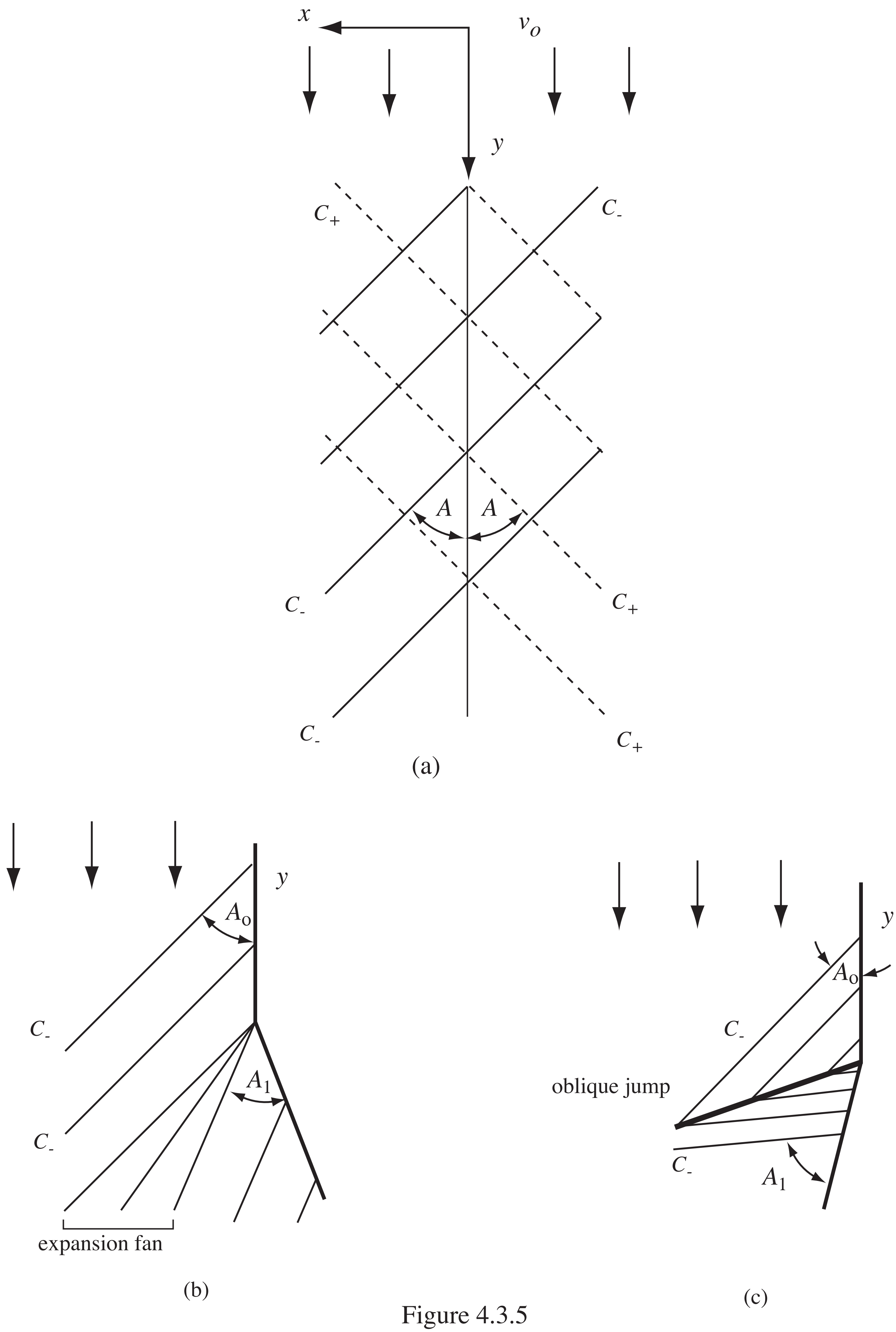
x
v_o
y
C_+
C_-
A
A
C_-
C_+
C_-
C_+
(a)
y
A_o
C_-
A_1
C_-
expansion fan
(b)
y
A_o
C_-
oblique jump
C_-
A_1
(c)

Figure 4.3.5

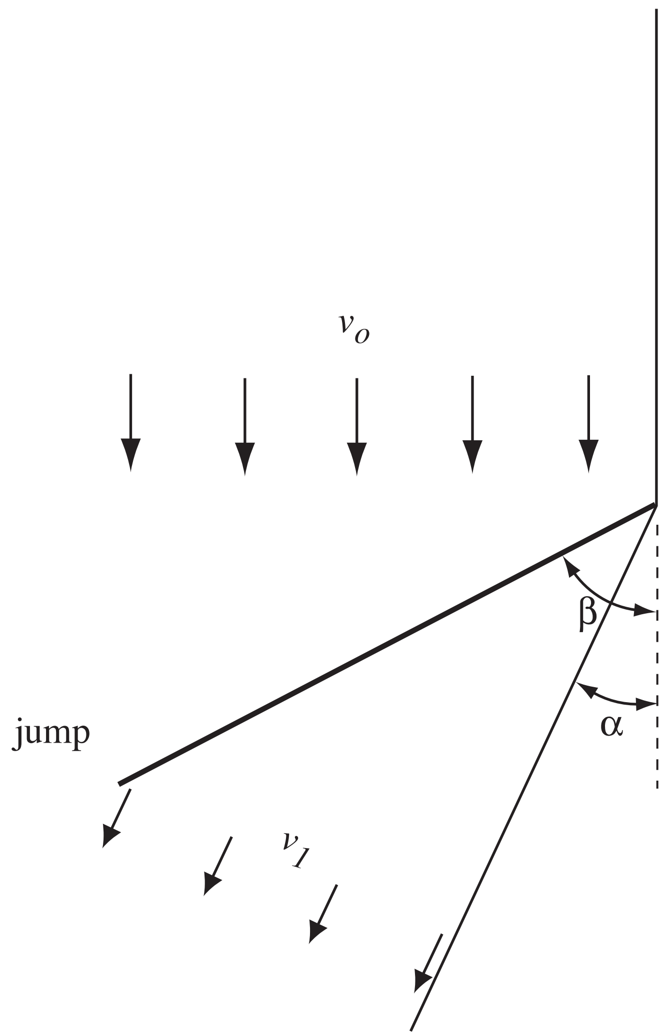

Figure 4.3.6