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1) Formulation:

- a) Continuum hypothesis
- b) Kinematics, Eulerian and Lagrangian descriptions. (rates of change)
- c) Definitions of streamlines, streamtubes, trajectories.

2) a) Mass conservation. (For fixed control volume and fixed mass). Careful definition of incompressibility and distinction with energy equation for  $\rho = \text{constant}$ .

b) Newton's law

c) Cartesian tensors, transformations, quotient law for vectors and tensors, diagonalization.

d) Tensor character of surface forces (derivation)  $\Sigma_i = \sigma_{ij}n_j$

e) Momentum equation for a continuum.

f) Conservation of angular momentum, symmetry of stress tensor. Trace of stress tensor, mechanical definition of pressure in terms of trace.

g) Stress, rate-of-strain tensor, isotropic fluids. Relation between mechanical and thermodynamic pressure. Fluid viscosity, kinematic viscosity. Navier-Stokes equations.

f) Navier-Stokes equations in a rotating frame, centripetal and Coriolis accelerations.

g) Boundary conditions, kinematical and dynamical—surface tension.

3) Examples for a fluid of constant density.

a) Ekman layer, Ekman spiral, dissipation in boundary layer, spin-down time, Ekman pumping. Ekman layer with applied wind-stress. Qualitative discussion of Prandtl boundary layer.

b) The impulsively started flat plate. Diffusion of momentum and vorticity.

4) Thermodynamics.

a) State equation, perfect gas, Internal energy.

b) First law of thermodynamics. Removal of mechanical contribution and the equation for internal energy. Dissipation function.

c) Dissipation due to difference between mechanical and thermodynamic pressure.

d) Entropy, enthalpy, temperature equation for a perfect gas. Potential temperature. Liquids.

e) Temperature change in water for the impulsively started flat plate due to friction.

5) Fundamental theorems-Vorticity

a) Vorticity, definition, vortex lines, tubes, non-divergence. Vortex tube strength.

b) Circulation and relation to vorticity.

c) Kelvin's theorem, interpretation. Friction, baroclinicity. Effect of rotation, Induction of relative vorticity on the sphere. Rossby waves as an example.

d) Bathtub vortex.

e) Vortex lines frozen in fluid. Helmholtz' theorem.

f) The vorticity equation, interpretation.

g) Enstrophy.

h) Ertel's theorem of potential vorticity. Relation to Kelvin's theorem. PV in a homogeneous layer of fluid. The status function.

i) Thermal wind. Taylor-Proudman theorem. Geostrophy in atmospheric and oceanic dynamics. Consequences.

j) Simple scaling arguments and heuristic derivation of quasi-geostrophic PV equation.

6. Fundamental Theorems. Bernoulli statements

a) Energy equation for time dependent dissipative motion.

- b) Bernoulli theorem for steady inviscid flow. The Bernoulli function  $B$ .
- c) Crocco's theorem (relation of  $\text{grad } B$  to entropy gradients and vorticity.)
- d) Bernoulli's theorem for a barotropic fluid.
- e) Shallow water theory and the  $pv$  equation and Bernoulli equation. Relation between  $\text{grad } B$  and potential vorticity and stream function.
- f) Potential flow and the Bernoulli theorem.
- g) Surface gravity waves as an example of (f)
- f) Waves, trajectories, streamlines.
- g) Two streams, internal waves with currents. Kelvin-Helmholtz instability. Richardson number Miles-Howard theorem. Effect of surface tension. Physical interpretation of instability.
- h) Brief discussion of baroclinic instability. Wedge of instability, estimate of growth rates.