PERFORMANCE EVALUATION OF SNR PREDICTION SCHEMES
IN ACOUSTIC COMMUNICATION SYSTEMS
USING VARIABLE-RATE MODULATION

Paolo Casari\textsuperscript{a,c}, Beatrice Tomasi\textsuperscript{a}, Konstantinos Pelekanakis\textsuperscript{b}, Mandar Chitre\textsuperscript{b}, Michele Zorzi\textsuperscript{a,c}

\textsuperscript{a} Department of Information Engineering, University of Padova
via Gradeno 6/B, 35131 Padova, Italy
\textsuperscript{b} Acoustic Research Laboratory, Tropical Marine Science Institute
National University of Singapore, Singapore, 119223
\textsuperscript{c} Consorzio Ferrara Ricerche, via Saragat 1, 44100 Ferrara, Italy

Contact author: Paolo Casari, via Gradeno 6/B, 35131 Padova, Italy,
fax: +390498277699, E-mail: casarip@dei.unipd.it
E-mail of all authors:
{casarip,tomasibe,zorzi}@dei.unipd.it , {costas,mandar}@arl.nus.edu.sg

Abstract: In this paper, we consider a variable-rate communication system, for which the digital modulation format in use can be varied across subsequent transmissions, i.e., by employing higher spectral efficiency schemes whenever the Signal-to-Noise Ratio (SNR) so allows. In order to adapt to the evolution of the SNR, the transmitter must be informed on the channel quality via feedback from the receiver. However, in multiuser networks, the amount of feedback may account for a significant portion of the injected traffic, and thus be subject to interference, both from data and from other feedback packets.
To alleviate this problem, we propose the usage of SNR prediction schemes, which relieve the need for frequent feedback by predicting the evolution of the SNR time series over a window of pre-defined length. We compare several schemes and discuss their capability to improve the communications performance in terms of throughput efficiency and outage probability.

Keywords: Variable-rate modulation, SNR prediction, LPF, Kalman filter, RLS, Markov modeling, throughput efficiency, outage, performance comparison.
1. INTRODUCTION AND MOTIVATION

Variable-rate modulation (VRM) is an established technique in terrestrial radio communications [1], where it is employed to adapt the robustness of the transmission system to the current channel state. VRM makes use of feedback from the receiver node (and possibly past knowledge of the channel SNR evolution) in order to choose the modulation scheme, e.g., by avoiding that spectrally efficient schemes are used in the presence of a very low SNR, which would result in frequent transmission errors. Similarly, VRM also tries to ensure that the employed modulation fully exploits the current channel capabilities, by not choosing low-rate modulation formats in the presence of sufficiently high SNR. A successful VRM scheme would therefore improve the throughput of the communication system by matching the transmission scheme to the channel, possibly requiring the least frequent feedback.

The capabilities of a good VRM system can be of great help for underwater acoustic communications. In fact, the acoustic channel state is often subject to changes due to several environmental effects, such as sound speed variations over the water column (which induce different refraction effects and ultimately change multipath patterns), wind-induced surface wave profiles, internal waves [2], as well as seasonal and day/night cycles. VRM schemes can track such changes via feedback from the receiver. This feedback, however, must be sufficiently frequent. Previous work [3] has shown that if the Signal-to-Noise Ratio (SNR) over a link is known only once every 15 s (the time separation between subsequent transmissions in the dataset considered there) it is likely that the wrong modulation is chosen, unless the average channel SNR is sufficiently high, typically over 30 dB. In a typical network, several entities may access the channel using some sort of medium access control (MAC) scheme, and then carry out transmissions, possibly employing a form of error control such as Automatic Repeat re-Quest (ARQ) [4], [8]. In this context, ARQ feedback messages can also serve to report SNR information to be used for adapting the transmit modulation scheme.

It has been shown that sending feedback very often may not be beneficial in multiuser networks, and that employing non-error-controlled schemes may actually yield better throughput and delivery ratio performance in several scenarios [7]. In order to meet the channel knowledge requirements of VRM schemes, while still keeping the amount of feedback to a minimum, in this paper we leverage on standard predictive filtering techniques as well as simple statistical models for the evolution of the SNR time series, and study the performance of a VRM scheme as a function of the feedback delay. In the periods between subsequent feedback packets, a prediction technique is applied to infer future SNR values based on the past knowledge of the SNR process. We compare the performance of the system in terms of the probability of outage events (defined as the choice of a modulation scheme not robust enough with respect to a bit error rate constraint), which reflect the capability of the scheme to actually follow the behavior of the SNR time series, and in terms of the throughput efficiency (defined as the average number of bits correctly transmitted per channel use, given the length of the transmit packet). We carry out our evaluation on the SPACE'08 dataset, collected off the coast of Martha’s Vineyard, MA, USA, in a very shallow water environment.

2. SYSTEM DESCRIPTION AND CONSIDERED PREDICTION TECHNIQUES

In the following evaluation, we will focus on a single transmitter-receiver link where VRM is employed at the transmitter side. The modulation schemes we focus on are Binary Phase Shift Keying (BPSK), Quaternary Phase Shift Keying (QPSK), 16-, 64- and 256-Quadrature Amplitude Modulation (QAM), which have a spectral efficiency of 1, 2, 4, 6 and 8 bits per
channel use, respectively. We assume that each modulation should be used only when the fore-
sen bit error rate (BER), given the available estimate of the future SNR value, is less than
$10^{-4}$, which corresponds to the following vector of minimum SNR thresholds in dB:
$\theta^{dB} = [8.44, 11.7, 18.4, 24.4, 30.4]$, where the first element in the vector corresponds to the
minimum SNR that, under an Additive White Gaussian Noise (AWGN) assumption, ensures a
BER of less than $10^{-4}$ to the BPSK modulation, the second to QPSK, and so forth. An SNR of
less than 8.44 dB means that the system is certainly in outage, as even the most robust modula-
tion is unable to achieve the BER objective. However, we note that the thresholds above have
not been chosen based on packet error rate. Therefore, the transmission of short packets may be
successful even if the system is in outage, depending on how small the actual SNR value is
with respect to the threshold of the modulation scheme in use.

Call $x_n$, the $n$th sample of the time series of the SNR. The objective of prediction algorithms
is to provide an estimate $\hat{x}_{n:M}$ of the SNR at time $n + M$, $M = 1, 2, \ldots$, given the past (known)
samples of the time series, $x_n$, $x_{n-1}$, $x_{n-2}$, etc.

The first predictor we consider is an order-16 linear predictive filter (LPF), which basi-
cally models the SNR series as the output of a 16-pole filter whose input is white noise. The
coefficients of the filter are derived using the Yule-Walker equations [5]. This predictor is sim-
ple, but makes the implicit assumption that the time series to predict has stationary statistics. In
addition, the Yule-Walker equations take into account the estimated correlation of the signal,
which decreases as the signal is sub-sampled, making future prediction increasingly noisier as
$M$ increases.

The second predictor we consider is again a linear filter, whose taps are computed using the
Recursive Least Squares (RLS) algorithm [5] with 5 taps, and a forgetting factor of 0.9. Unlike
the LPF, the RLS algorithm makes no implicit assumptions that the signal is stationary,
and adapts instead the filter taps based on the error incurred by previous predictions.

The third predictor is a Kalman filter which assumes the SNR time series to have locally
constant first-order derivative, i.e.,
$$
\begin{align*}
\hat{x}_{n+1} &= x_n + MT \cdot \hat{x}_n \quad \text{for } k = n, n-M, n-2M,
\end{align*}
$$
where $T_s$ is the sampling interval of the SNR series. The Kalman filter takes as input an esti-
mate of the variance of the noise affecting the state observations [6]: a lower value makes the
filter trust the measured SNR (and effectively turns it into a follower, i.e., a filter that almost
replicates the previous known SNR value at any given time), whereas a higher value makes the
filter rely more on the model. In the following we employed a value of 0.5, which strikes a bal-
bance between prompt reaction to variations and adherence to the state evolution model.

The fourth prediction technique is based on a more conservative hysteresis behavior: basi-
cally, two sets of thresholds are defined, $\theta_{UP}$ and $\theta_{DOWN}$, the first containing the SNR values to
be overcome in order to switch to a more spectrally efficient modulation, the second containing
the SNR values beneath which the modulation is switched down to a more robust one. In the
following, these thresholds are set to $\theta_{UP} = 1.25 \theta$ and $\theta_{DOWN} = 1.1 \theta$, where $\theta$ contains the li-
clear-scale values of the elements of the vector $\theta^{dB}$ introduced at the beginning of this Section.
These values will be shown to increase the resilience of the system to spurious SNR variations,
while still allowing a sufficiently prompt adaptation of the modulation scheme. However, the
scaling coefficients could be increased arbitrarily to provide greater robustness at the price of a
higher probability that the channel is not fully exploited.

As a further note, there are two ways to provide the transmitter with the feedback it needs to
choose the modulation: i) every time a feedback packet is sent, the receiver includes a trace of
all SNR values measured since the last feedback packet was sent, so that the transmitter can
run the predictive algorithms and tune the modulation correspondingly; ii) the receiver runs the predictive algorithms and informs the transmitter about the sequence of modulations to be chosen until the next feedback. The first strategy requires longer feedback packets, whose size may grow considerably if the time period between subsequent feedbacks is long (as increasingly more SNR samples should be inserted into the packet); in turn longer periods would also make feedback more prone to channel errors. Strategy ii) requires much shorter feedback packets, as the receiver only has to state which modulation the transmitter should use. The number of schemes is typically limited, and can be addressed using just a few bits (3 per transmission, in our case). Thus, even in case of very long feedback intervals (e.g., one feedback packet every 20 data packets), the feedback payload would be only 60 bits long. In turn, strategy ii) is more prone to losses, as no prediction is performed at the transmitter side.

The fifth and last prediction technique we consider is based on a **Markov model**. This model contains as many states as the elements of the vector $\theta^{db}$ plus one. Call $s(t)$ the sequence of the states of the Markov chain. We have $s(t) = i$ if and only if $\theta^{db}(i-1) < \text{SNR}(t) \leq \theta^{db}(i)$, where $\theta^{db}(0) = 0$. The prediction performed by the Markov chain is based on the maximization of the function $\phi(\tilde{s}, s)$ which is equal to the rate of the modulation corresponding to $\tilde{s}$, $R(\tilde{s})$, if the channel is able to support it, i.e., if $\tilde{s} \leq s$, and to 0 otherwise (in this case, we assume that modulation cannot be supported by the channel, hence the transmission would be in error). Note that $\phi(\tilde{s}, s)$ is one possible definition of the throughput efficiency of a VRM system. In this paper, throughput efficiency is defined based on the packet error rate rather than the outage probability, as $G(\tilde{s}, s) = R(\tilde{s})(1 - \text{BER}_{\tilde{s}}(\gamma))^\gamma$, where $\text{BER}_{\tilde{s}}(\gamma)$ is the bit error rate of the modulation related to state $\tilde{s}$ if the actual signal-to-noise ratio is equal to $\gamma$, and an independent bit error model is assumed to compute the probability that a packet is correct. The prediction of the following state is formulated as an optimization problem, i.e., the next state $\hat{s}$ is chosen among all possible states $\tilde{s}$, given the previous state $i$, such that the expected throughput is maximized: $\hat{s} = \arg\max_{\tilde{s}} E_{\tilde{s}|i}[\phi(\tilde{s}, s)|i]$; substituting the definitions above yields $\hat{s} = \arg\max_{\tilde{s}} R(\tilde{s}) P[\tilde{s} \leq s | i]$, where the probability that the predicted state is less than or equal to the actual state can be obtained from the transition probability matrix of the Markov chain.

3. THE SPACE’08 DATASET

The dataset we consider in this paper is SPACE’08, which has been collected between October 18 and 27, 2008 at the Martha’s Vineyard Coastal Observatory, operated by the Woods Hole Oceanographic Institution, MA. The experiment performed for the collection of the dataset involved the transmission of 3-minute signals at a carrier frequency of 11.5 kHz frequency once every two hours, from a single location. We consider one of the six receivers that were deployed, namely S4, positioned at 200 m from the transmitter in the Southwest direction. Given the very shallow water flat-bottom environment, the time-variations of the channel were mainly induced by the wind-driven surface roughness and the underwater currents.

The channel impulse responses were estimated every 15 ms from the received signals, which consist of a number of sequentially transmitted $m$-sequences, each composed of 4095 BPSK-modulated symbols sent at a rate of 6.5 kbps. The SNR estimates are computed as the $L_2$-norm of each impulse response, scaled to simulate higher or lower power, and averaged over 60 ms (i.e., 4 samples), in order to simulate SNR estimation over a typical preamble length. In particular, the stability of the SNR traces makes it difficult that more than two SNR thresholds in $\theta^{db}$ are crossed in any experiment, and correspondingly, only 3 of the available
modulation schemes are effectively employed. In the following, we will scale the SNR so that the limited dynamics of the channel traces employed in our analysis allow the usage of the BPSK, QPSK and 16-QAM modulations. We assume that packet transmissions take place at the pace of one every $T_s = 1.2$ s (which includes the transmission time and propagation delay) and correspondingly subsample the time series.

All algorithms described above (with the exception of the thresholds with hysteresis), require a training period: the LPF needs it to estimate the autocorrelation of the time series, the RLS for making the filter weights converge, the Kalman filter to perform some first prediction-correction cycles and the Markov model to estimate the transition probability matrix of the Markov chain. For this reason, for each time series taken from the dataset (and lasting around 170 s), we take the first 60 s (i.e., $60/T_s = 50$ samples) for training. In the following Section, we compare the prediction techniques in terms of the average throughput efficiency (defined in terms of the packet error rate as per the $G(s)$ function introduced above), as well as the outage probability. Unless otherwise stated, the packet length is set to 256 bits, which makes the threshold BER of $10^{-4}$ conservative, as discussed in Section 2.

4. RESULTS

In Fig. 1, we evaluate the capability of the predictors to follow the SNR time series, for two different values of the delay between subsequent feedbacks, 4.8 s (1 feedback packet every 4 data packets, dark gray dashed lines) and 24 s (1 every 20 packets, light gray dashed lines). Fig. 1(a) refers to the LPF predictor, whose filter taps are derived via the Yule-Walker equations: we observe that the filter under-estimates the absolute value of the time series; however, for a low delay it still can follow its general trend, whereas for larger delays the estimates of the correlation between farther samples become increasingly worse, and

![Fig 1. Example of prediction of an SNR time series from the SPACE’08 dataset. (a) LPF predictor (Yule-Walker); (b) RLS predictor; (c) Kalman filter.](image-url)
the filter expectedly predicts an increasingly low value. On the contrary, on this specific time series, the RLS predictor can follow the average trend of the signal, due to the adaptation of the filter taps and to the fact that it does not need the predicted signal to have stationary statistics. However, instantaneous prediction can still be off the ground truth by 1 to 2 dB: in our case, this is expected not to play a major role, as the distance between the thresholds in the vector $\theta^{dB}$ introduced above is larger. The performance of the Kalman filter in Fig. 1(c) shows a similar behavior, with the exception of some overshoots in the long delay case. The reason is that the actual SNR evolution model is unknown, hence we have to resort to basic regularity assumptions, such as a fixed first-order derivative of the process. Hence, when the estimated variation of the signal is large (typically close to the limits of the prediction window, and in any event when the training signal becomes sparser due to undersampling), the Kalman filter predicts a significantly higher SNR with respect to the actual value (in some realizations, the overshoot can be even larger).

We compare the throughput efficiency performance of the different policies in Figs. 2 and 3 across different experiments, for a feedback delay of 1.2 s (1 feedback packet every data packet) and 12 s (1 every 10), respectively. The policies introduced above are also compared against the perfect channel state information (CSI) case, where the SNR is known without error and therefore the chosen modulation always respects the BER constraints; a simple policy is also added which assumes the last known value of the SNR to be valid until new feedback arrives (Hold).

Fig. 2. Throughput efficiency for all policies for a feedback delay of 1.2 s.

Fig. 3. Throughput efficiency for all policies for a feedback delay of 12 s.
Fig. 4. Throughput efficiency for all policies as a function of the feedback delay.

Fig. 5. Outage probability for all policies as a function of the feedback delay.

From the pictures above we observe first of all that even such a simple heuristic policy as the hysteresis predictor improves significantly over the Hold strategy: this result is due to the more conservative switching thresholds of the former, which avoid, e.g., having to hold a spectrally efficient modulation when the channel does not support it, while waiting for the next feedback packet to arrive. The LPF, RLS and Kalman predictors show even higher throughput, very close to that of the perfect CSI case. Actually, especially in the longer feedback delay case in Fig. 3, the Kalman filter shows a slightly higher throughput than with perfect CSI: this is due to the chosen SNR thresholds, which correspond to a BER of $10^{-4}$, and to the comparatively short packet length of 256 bits. In fact, with such parameters a transmission employing the wrong modulation has a chance to succeed. In this regard, it is interesting to compare the behavior of the Kalman and Markov predictors: since the latter chooses modulations conservatively, with the objective to both maximize throughput and minimize the outage probability, the prediction results in QPSK being used almost always (with a throughput efficiency equal to 2), even when a stronger modulation could be supported.

We conclude the evaluation with Figs. 4 and 5, detailing a comparison of, respectively, throughput efficiency and outage probability as a function of the feedback delay. Fig. 4 confirms the behavior of the Kalman predictor observed previously, i.e., that increasing feedback delay leads to choosing the transmit modulation optimistically. Conversely, Fig. 5 confirms the capability of the Markov model to avoid outage events with high probability, as opposed to the Kalman predictor, whose number of outage events increases for increasing feedback delay. We
note that this is not necessarily due to the technique per se: part of the reason lies in the SNR evolution model, which does not reflect the behavior of the SNR as the time series is strongly subsampled in order to predict future SNR values. We also note that the LPF achieves an even lower outage probability, but recall that this result depends on the decreasing behavior of its SNR estimates as the feedback delay increases, see Fig. 1(a).

5. CONCLUSIONS

In this paper, we compared the performance of several prediction techniques applied to Variable-Rate Modulation (VRM) schemes, in order to improve the throughput efficiency of the transmission and possibly decrease the probability of outage events, when a modulation scheme not supported by the current SNR is employed. Our schemes can be applied to the time series of the SNR estimated at the receiver, which are quite straightforward to collect from the preambles of the received packets.

Our results showed that a simple Kalman predictor and a 5-tap RLS predictor perform quite well, and can achieve a throughput efficiency similar to that of the case of perfect channel state information at the transmitter. In case the optimization of the system throughput is given lower priority than the need to avoid outage events, the Markov model analyzed in this paper achieves very good results: this effect is due to its design, that limits the probability to choose a modulation not supported by the channel given the predicted SNR values.

ACKNOWLEDGEMENTS

The authors would like to thank Dr. James Preisig and the Woods Hole Oceanographic Institution for making the SPACE’08 data available, and Dr. Simone del Favero for useful discussions on the Kalman filter approaches.

This work has been supported in part by the US Office of Naval Research under grants no. N00014-10-10422, by the Italian Institute of Technology under the “ProjectSeed” program (NAUTILUS project), and by the Aldo Gini Foundation, Padova, Italy.

REFERENCES