Energetics of a Turbulent Ocean

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One of the earliest theoretical investigations of ocean circulation was by Count Rumford. He proposed that the meridional overturning circulation was driven by temperature gradients. The ocean cools at the poles and is heated in the tropics, so Rumford speculated that large scale convection was responsible for the ocean currents. This idea was the precursor of the thermohaline circulation, which postulates that the evaporation of water and the subsequent increase in salinity also helps drive the circulation. These theories compare the oceans to a heat engine whose energy is derived from solar radiation through some convective process.

In the 1800s James Croll noted that the currents in the Atlantic ocean had a tendency to be in the same direction as the prevailing winds. For example the trade winds blow westward across the mid-Atlantic and drive the Gulf Stream. Croll believed that the surface winds were responsible for mechanically driving the ocean currents, in contrast to convection. Although both Croll and Rumford used simple theories of fluid dynamics to develop their ideas, important qualitative features of their work are present in modern theories of ocean circulation.

Modern physical oceanography has developed a far more sophisticated picture of the physics of ocean circulation, and modern theories include the effects of phenomena on a wide range of length scales. Scientists are interested in understanding the forces governing the ocean circulation, and one way to do this is to derive energy constraints on the different processes in the ocean. The goal is to use these constraints to understand the forces that drive ocean circulation and the forces that maintain the density stratification. Using Sandstrom's theorem on horizontal convection it is possible to rule out convection as an important source of energy for the ocean circulation. Once this possibility has been ruled out we will consider the kinetic energy budget and study the effects of winds and tides on the ocean circulation [12].

1.1 Ocean Stratification

Figure 1 shows the density stratification in the Pacific Ocean[12]. The upper half kilometer of the ocean is highly stratified. Below this region there are still density changes but not to the same extent, and the density is much more uniform in the abyssal ocean. There is a region of highly dense water near the surface in the poles. This represents cold water



Figure 1: Potential density stratification in the Pacific Ocean. The level sets of pressure are closely packed near the surface where the ocean is highly stratified. In the deep ocean the pressure gradient weakens. Near south the pole the ocean is highly mixed, and there is a tongue of dense water sinking to the bottom.

from the surface, sinking and mixing with water from the abyss. There is a corresponding upwelling in other parts of the ocean that increases the potential energy. This upwelling requires an energy source to drive it.

1.2 The Oceanic Energy Budget

Throughout these notes we will use the Boussinesq approximation:

$$\frac{D\mathbf{u}}{Dt} + f\hat{z} \times \mathbf{u} = -\frac{1}{\rho_0}\nabla p + b\hat{z} + \nu\nabla^2 \mathbf{u} - \nabla\Phi_{Tide}$$
(1)

$$\nabla \cdot \mathbf{u} = 0 \tag{2}$$

$$\frac{D\theta}{Dt} = \kappa_{\theta} \nabla^2 \theta \tag{3}$$

$$\frac{DS}{Dt} = \kappa_S \nabla^2 S. \tag{4}$$

Here D is the advective derivative, θ is the potential temperature, and S is the salinity, b is the buoyancy, ν the molecular viscosity, and Φ_{Tide} the gravitational potential due to the tides. We assume that b is linear in θ and S:

$$b(\theta, S) = g\alpha\theta - g\beta S. \tag{5}$$

We take a linear combination of equations (3) and (4) to produce an expression $\frac{Db}{Dt}$ and eliminate θ and S from the equations:

$$\frac{Db}{Dt} = g\alpha \frac{D\theta}{Dt} - g\beta \frac{DS}{Dt} = \kappa \nabla^2 b.$$
(6)

The quantity $\kappa = \kappa_{\theta} + \kappa_S$. The total kinetic energy of the ocean is proportional to $\frac{1}{2} \int_{Ocean} \mathbf{u}^2 dV$. If take the dot product of the momentum equation by \mathbf{u} and integrate over the volume of the ocean we obtain the term by term kinetic energy budget of the ocean:

$$\frac{1}{2}\frac{\partial}{\partial t}\|\mathbf{u}\|^2 = -\int \frac{1}{2}|\mathbf{u}^2|(\mathbf{u}-\mathbf{u}_s)\cdot\hat{n}dA - \int (\frac{wp}{\rho_0} - \frac{1}{2}\nu\nabla|\mathbf{u}|^2)\cdot\hat{n}dA + \langle wb \rangle - \epsilon - \langle \mathbf{u}\cdot\nabla\Phi_{Tide} \rangle.$$
(7)

Here $\langle \cdot \rangle$ stands for the integral over the volume of the oceans, ϵ stands for $\langle \nu | | | \nabla^2 \mathbf{u} | |^2 \rangle$, the viscous dissipation, and w is the velocity in the \hat{z} direction. If we ignore the effects of viscous dissipation at the boundaries and evaluate the dot products explicitly we find that:

$$\frac{1}{2}\frac{\partial}{\partial t}\|\mathbf{u}\|^2 = -\frac{1}{\rho_0}\int (wp - \tau \cdot b)dA + \langle wb \rangle - \epsilon - \langle \mathbf{u} \cdot \nabla \Phi \rangle.$$
(8)

The terms in this equation are the components of the kinetic energy budget. The $\langle wb \rangle$ represents the conversion of kinetic energy to potential energy. The dissipation ϵ represents the conversion of kinetic energy to internal energy. The other terms represent the work due to pressure, the tides, and friction stress at the boundary.

The potential energy is entirely due to gravity, and we define it to be $\langle -zb \rangle$. If we multiply the buoyancy equation by z we can determine the local potential energy change:

$$\frac{\partial}{\partial t}zb + \nabla \cdot (\mathbf{u}zb + z\kappa\nabla b) = wb - \kappa \frac{\partial b}{\partial z}.$$
(9)

The terms on the right come from moving z inside the divergence. If we integrate this equation and ignore the second term on the right we calculate the potential energy budget:

$$\frac{\partial}{\partial t}PE = \int zF_b dA - \langle wb \rangle + \langle \kappa \frac{\partial b}{\partial z} \rangle.$$
(10)

In this expression $F_b = \kappa \nabla b \cdot \hat{n}$. We recognize the familiar $\langle wb \rangle$ from the kinetic energy budget. There are also two other terms that represent the conversion of internal energy to kinetic energy.

The combination of the potential energy and kinetic energy budgets gives an energy budget for the entire ocean. Kinetic energy is generated by the winds and tides, and by conversion of potential energy. It is dissipated through viscosity into internal energy. The internal energy is generated by heat fluxes and dissipation and is converted into potential energy through buoyancy [12].

1.3 Convection

By eliminating the effects of the winds and the tides we can determine if the buoyancy fluxes are strong enough to drive the thermohaline circulation. We drop the tidal forces



Figure 2: A numerical simulation of horizontal convection. There is a heat source on the edges and a heat sink in the center. Periodic boundary conditions are used. The arrows indicate the direction of the flow. The temperature is constant in most of the fluid and the heat flux is confined to the surface. There is no significant convective flow

and the boundary pressure and stress terms. The circulation in this ocean is driven only by convection. However the configuration of the ocean is different from standard convection because both the heating and cooling occur at the surface of the ocean.

Johan Sandstrom studied horizontal convection in systems where the heating was above and at the same height as the cooling[2]. This mimics the oceanic system where the heating and cooling occur at the same altitude but with a large horizontal displacement. Sandstrom argued that a heat engine does no work if the heating occurs at a higher or equal pressure than the cooling. This argument translates to the ocean, where the term $-\int zF_b dA$, the work done by buoyancy, is either 0 or negative.

The application of Sandstrom's conjecture to the ocean has been validated by other numerical and theoretical work. If we take the limit $\kappa \mapsto 0$ with the ratio $\frac{\nu}{\kappa}$ constant the ocean becomes stagnant and homogeneous. The energy conversion between internal energy and potential energy ceases, and vice versa. Lab experiments and numerical simulations by Rossby show explicitly that the overturning stream function vanishes. Another example is Figure 2, which is a two dimensional numerical simulation of horizontal convection. All of the temperature gradients are confined to the surface and there is no convective flow.

There are actual numerical estimates for the strength of convection in the ocean. For example Faller and Huang[5] have estimated the order of magnitude of energy conversion between internal and potential energy to be:

$$-\rho_0 g^{-1} \int z \bar{F}_b dA \approx \pm .01 TW \tag{11}$$

$$\rho_0 g^{-1} \langle \kappa \bar{\partial}_z b \rangle \approx \pm .001 TW. \tag{12}$$

Similarly $\langle wb \rangle$ is zero because of Sandstrom's theorem. Therefore there is no appreciable energy flow from potential energy to kinetic or from internal energy to potential. This means that the circulation of the ocean is driven by the winds and tides and that this energy is converted into internal energy through viscous dissipation. This internal energy is returned to the atmosphere through heat fluxes.

1.4 Kinetic Energy Sources

If the ocean is not driven by heat it must be driven mechanically. The source of these forces are surface winds and tides. The surface work term can be decomposed into three important components, the geostrophic flow, ageostrophic flow, and surface waves:

$$w_{wind} = \int \tau \cdot \mathbf{u}_g dA + \int \tau \cdot \mathbf{u}_{ag} dA + \int (\tau \cdot \mathbf{u}_w - pw) dA.$$
(13)

The geostrophic flow comes from balancing the Coriolis force with the atmospheric pressure. The wind is a predominantly geostrophic flow, and the geostrophic flow is directed along the level sets of the pressure. The ageostrophic flow is the actual flow minus the geostrophic flow. Scientists have tried to estimate the energy transfer due to each of these terms. The geostrophic flow contributes approximately .8TW, the ageostrophic flow 3TW, and the surface waves 60TW. The actual kinetic energy transfer to the circulation of the ocean below the turbulent boundary layer at the surface is more difficult to determine. It is believed that the surface waves are dissipated in the form of turbulence and that they cannot create kinetic energy from the ageostrophic flow penetrates the boundary layer. It is thought that all of the energy stress from the geostrophic flow is transferred to kinetic energy in the surface boundary layer[1].

The other major source of mechanical energy is the tides. The overall power due to tidal forces is roughly 3.5TW. Most of this, 2.5TW is done in shallow seas against the continental shelves. About 1TW is done against the abyssal topography. It is believed that only 1TW of this is converted into useful kinetic energy in the ocean. This means that the global transfer of power from the winds and tides into ocean kinetic energy is roughly 2.3TW. Therefore we would expect the dissipation of kinetic energy into internal energy to be 2.3TW in order to balance the inputs[3].

2 The Energy Cascade

The ocean contains energy on a wide range of length scales and frequencies. The oceanic kinetic energy is dominated by geostrophic turbulence, i.e. the energy in the mesoscale eddie field. Figure 3 shows two kinetic energy density spectra from moorings in the North Atlantic and the Southern Ocean. Note that the spectra are predominantly red. On average about 90% of the kinetic energy in the ocean is in subinertial frequencies, i.e. timescales longer than the local pendulum day $2\pi/f$. On smaller scales¹ the kinetic energy is dominated by internal waves in the inertial peak and the semi-diurnal lunar tide peak. Only a very small fraction of the kinetic energy lies in the small scale 3 dimensional turbulence. However, the latter is an important pathway to the final dissipation of kinetic energy. The bulk of the potential energy on the other hand is in the planetary scale ocean stratification.

¹We use frequencies and wave numbers somewhat interchangeable in this discussion. This can in general not strictly be justified, though it should be acceptable for a rough qualitative discussion as given here. Spectral energy estimates in wavenumber space are generally much harder to obtain from observational data.



Figure 3: (a) Kinetic energy estimate for an instrument in the western North Atlanticat about 15° N at 500m. This record was described in [6]. (b) Power densityspectral estimate from a record at 1000m at 50.7°S, 143°W, south of Tasmania in theSouthern Ocean [8]. The inertial and principal lunar, semi-diurnal M2, tidal peaks are marked, along with the percentage of kinetic energy in them lying between f and the highest frequency estimate. Least-squares power law fits for periods between 10 and 2 hours and for periods lyingbetween 100 and 1000 hours are shown. The approximate percentage of energy of the internal wave band lying in the inertial peak and the M2 peak are noted.

2.1 Pathways of Kinetic Energy: From Forcing to Dissipation

The generation of kinetic energy below the surface mixed layer acts primarily at planetary scales. Most of the energy is generated by the large-scale wind field and by the tides. Dissipation on the other hand occurs at scales on the order of O(1cm) or O(1s) or less. Thus energy has to be transferred across the spectrum from the largest scales to the smallest scales. How does this happen? It is well understood that a forward energy cascade acts in 3-dimensional isotropic turbulence. I.e. turbulent kinetic energy is transported to smaller scales until it gets finally dissipated. However, the situation in the ocean is much more complicated. There are (at least) two main difference. Due to the presence of stratification and rotation larger scale turbulence in the ocean is obviously not isotropic. Moreover, exchanges between kinetic and potential energy play an important role.

2.1.1 Conversion Between Kinetic Energy and Potential Energy

As argued before, the total conversion between kinetic and potential energy, given by the conversion term $\overline{\langle wb \rangle}$ approximately has to vanish. However, significant conversions can occur locally or over certain spatial and temporal scales. For the following discussion we want to decompose variables into three different scales. The buoyancy field for example is given by

$$b = \overline{b} + b_e + b_t$$
.

The first term on the right hand side describes the buoyancy field associated with the mean circulation, i.e. planetary scales. b_e denotes the contribution of the geostrophic eddy field and b_t describes small scale 3D turbulence. Assuming that this scale separation can strictly be made, the PE \rightarrow KE conversion term can be decomposed into

$$\overline{\langle wb\rangle} = \overline{\langle w_t b_t \rangle} + \overline{\langle w_e b_e \rangle} + \langle \bar{w}\bar{b}\rangle \tag{14}$$

The mean $PE \rightarrow KE$ conversion $\langle \bar{w}\bar{b} \rangle$ is dominated by the generation of potential energy by winds² and by the abyssal overturning circulation which releases potential energy. It can be written as

$$\langle \bar{w}\bar{b} \rangle = \langle \bar{w}_{Ekman}\bar{b} \rangle + \langle \bar{w}_{ag}\bar{b} \rangle = -\int \int (\tau_{wind} \cdot \bar{\mathbf{u}} - \tau_{bottom} \cdot \bar{\mathbf{u}}) dA + \langle \bar{w}_{ag}\bar{b} \rangle \approx -0.8 \pm 0.1 \text{TW} + 0.1 \pm 0.1 \text{TW} + 1.0 \pm 0.5 \text{TW}$$
(15)

The first terms on the RHS describe the production of potential energy by the Ekman pumping/suction due to the global wind field and a probably small sink of PE by Ekman pumping/suction due to bottom friction. Note that the error estimate for the latter is on the order of 100%. The last term describes the production of kinetic energy by subsidence in the (ageostrophic) abyssal overturning circulation. The two contributions are of opposite sign and similar within the error estimates.

²The large scale wind field is often regarded to produce potential energy. This happens due to Ekman pumping which, however, is indeed a conversion from KE to PE. More accurately, the wind field produces kinetic energy of which the major part is instantaneously converted into potential energy by geostrophic adjustment.

The mean currents are associated with strong buoyancy gradients and shear flows which become unstable and generate eddies. This process releases potential energy into geostrophic eddy kinetic energy. Since this is associated with a spindown of the large scale circulation, the second term in (14) can be expressed as an effective eddy stress τ_{eddy} acting on the large scale circulation:

$$\overline{\langle w_e b_e \rangle} = \iiint \mathbf{u} \cdot \partial_z \tau_{eddy} \, dV \approx 0.5 \pm 0.1 \mathrm{TW} \tag{16}$$

Finally turbulent dissipation of kinetic energy results in an increase in potential energy. Osborn (1980) [7] argued that, in a stably stratified fluid, most of the turbulent kinetic energy is dissipated by viscous friction, while a fraction Γ is used to vertically mix the fluid and thus increase the potential energy.

$$\overline{\langle w_t b_t \rangle} = |\Gamma \overline{\langle \nu || \nabla \mathbf{u} ||^2 \rangle}| \le \Gamma \epsilon \approx 0.5 \mathrm{TW},$$

where ν is the vicosity and ϵ is the total dissipation of kinetic energy in the deep ocean, which has to equal the total input of kinetic energy into the latter. The mixing efficiency Γ in the stratified ocean was estimated to be around 20%. Estimates from direct observations [11] suggest

$$\overline{\langle w_t b_t \rangle} \approx -0.4 \pm 0.1 \text{TW}$$
 (17)

Summing up the estimates of equations (15), (16) and (17) we find that the estimated total conversion is about zero. While the sign of the PE \rightarrow KE conversion by the large scale circulation is not clear, the meso-scale eddy field releases PE while turbulent mixing increases PE. The PE balance in the upper ocean (here the upper 1000-2000 meters) is dominated by the potential energy released by eddies which is maintained by the large-scale winds. In the abyss the release of potential energy by the deep overturning circulation is maintained by mixing and by the large scale wind forcing.

2.1.2 The Energy Cascade and Dissipation of Kinetic Energy

In the previous section we argued that kinetic energy is produced from potential energy at the geostrophic eddy scale while it is removed by small scale 3-dimensional turbulence. In this section we try to address the question how the energy is transported to smaller scales where it can finally be dissipated by turbulence.

Kinetic energy in inertial and tidal waves can be transferred to small scales through wave-wave interactions in the internal wave field and can finally be dissipated through breaking of internal waves. The estimated energy dissipated by internal wave breaking is about 1.5 TW.

It can be shown that the viscous dissipation of kinetic energy ϵ is proportional to the enstrophy $Z \equiv \frac{1}{2} |\zeta|^2 = \frac{1}{2} |\nabla \times \mathbf{u}|^2$:

$$\frac{\partial E}{\partial t} + \nabla \cdot (\mathbf{u}p + \mathbf{u}E - \nu\nabla E) = -2\nu Z + F.$$
(18)

In three dimensional turbulence, enstrophy can be generated by vortex stretching

$$\frac{\partial Z}{\partial t} + \nabla \cdot (\mathbf{u}Z + \nu \nabla Z) = \zeta_i S_{ij} \zeta_j - \nu ||\nabla \zeta||^2, \qquad (19)$$



Figure 4: Time mean, spectral kinetic energy flux vs total wavenumber in a homogeneousACC region (rectangles centered at 57S, 120W): black curve using sea surface height (SSH) on a 32×32 grid, red curve using SSH on a 64×64 grid, blue curve using velocity on a 64×64 grid.Positive slope reveals a source of energy. The larger negative lobe reveals a net inversecascade to lower wavenumber. Error bars represent standard error. Taken from [10]

where $S_{ij} = \frac{1}{2}(\partial_{x_i}u_j - \partial_{x_j}u_i)$. The first term on the right hand side describes the change of enstrophy due to vortex stretching.

In two dimensional geostrophic turbulence (i.e. the mesoscale eddy field), however, enstrophy is conserved in the limit that the viscosity goes to zero. We find

$$\frac{\partial Z}{\partial t} + \nabla \cdot (\mathbf{u}Z + \nu \nabla Z) = -\nu ||\nabla \zeta||^2, \qquad (20)$$

where $\zeta = \partial_x v - \partial_y u$ is now a scalar. Since enstrophy is conserved in the limit $\nu \to 0$ the viscous energy dissipation has to vanish in geostrophic turbulence. In 3D turbulence on the other hand enstrophy is generated through vortex stretching and thus viscous energy dissipation remains finite as $\nu \to 0$ (since $Z \to \infty$).

Indeed it can be shown that in 3D isotropic turbulence, energy is transported to smaller scales until it is finally dissipated. The magnitude of the viscosity only determines the scale at which the energy is finally dissipated. In geostrophic turbulence on the other hand energy is transported to larger scales and is thus not dissipated. This "inverse cascade" is generally only limited by the planetary scale or the " β -effect" (see [9] for details). Figure 4 shows an observational estimate of the spectral energy flux in wavenumber space in a region in the Antarctic Circumplolar Current. We can see a weak forward cascade for wavelength of less than about 150km and a dominating inverse cascade on larger scales. The bulk of the ocean kinetic energy resides in the geostrophic eddy field and is thus transported to larger horizontal scales and generally also larger vertical scales, i.e. the eddy field tends to become barotropic. So where does the energy go from here?

2.2 Geostrophic Kinetic Energy Dissipation

Once the Eddies become barotropic, they will lose energy through bottom boundary layer drag, i.e. kinetic energy is dissipated in the bottom Ekman layer. The estimated energy loss through this process is

$$\epsilon_b = \iint \rho C_d |\mathbf{u}|^3 \, dA \approx 0.2 - 0.8 \, \mathrm{TW} \,. \tag{21}$$

Another important mechanism for dissipation of geostrophic KE arises from geostrophic flows over topography. Flow with velocity U impinging over topographic features with wavenumbers between f/U and N/U can cause radiation of internal waves which tend to interact and break within about 1 km above the topography. Observations indeed show strongly enhanced mixing over topography which can be attributed to this process. This is likely to be an important contributor to abyssal mixing. The estimated dissipation of KE due to this process is

$$\epsilon_t = \iiint \rho \nu |\nabla \mathbf{u}|^2 dV \approx 0.2 - 0.4 \,\mathrm{TW}\,,\tag{22}$$

where **u** here denotes the velocity field due to topographically generated internal waves. In the surface boundary layer, particularly in the regions of the boundary currents, the geostrophic velocity field can form very sharp fronts due to frontogenesis. At large Richardson numbers, i.e. weak stratification, these fronts can go unstable and form small scale turbulence which is dissipated in a forward energy cascade. Since this "loss of balance" is confined to the surface boundary layer, the energy loss due to this pathway does not contribute to the mixing necessary to maintain the deep overturning circulation.

2.3 The Oceanic Energy Cycle

As an attempt to summarize the pathways of energy in the ocean, we found that in the upper ocean, the circulation is powered by the winds. Eddies are generated by baroclinic instabilities of the large scale circulation. Eddy-eddy interaction then transfers energy into barotropic motion following the geostrophic inverse energy cascade. Some of the energy in the eddy field is lost by loss of balance in the surface layer, where sharp fronts can evolve that finally become unstable. Another part is lost to bottom friction. Moreover, the geostrophic flow impinging over topography can generate internal waves which interact and break. This results in abyssal mixing that powers the overturning circulation.

A much more complete picture of the oceanic energy cycle is sketched in figure 5. For further details, the reader is referred to [4] and [12].

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Figure 5: Schematic of the estimated pathways and reservoirs of energy in the ocean. All fluxes are given in TW. From [4].

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