How Rossby Waves Break. Internal Wave – Mesoscale Eddy – Zonal Mean Interactions

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ABSTRACT

The issue of internal wave–mesoscale eddy–zonal mean interactions in Physical Oceanography is revisited. Implicit in this work is the concept that adiabatic pseudomomentum flux divergences in the wave field serve as interior sources and sinks of eddy potential vorticity. The exchange of internal wave momentum with eddy potential vorticity is an intrinsic component of linear wave propagation in geostrophically balanced mean fields. The key element in producing permanent changes in potential vorticity of the mesoscale eddy field is recognizing weak non-linearity within the internal wavefield to be an irreversible process.

Relatively minor changes to an extant theoretical analysis of internal wave–mesoscale eddy interactions supports the interpretation that the mesoscale eddy field acts as a source of internal wave energy and pseudomomentum. For the Garrett and Munk spectral description of the background internal wavefield, this interaction can be cast as a horizontal viscosity of 50-100 m² s⁻¹ and an effective vertical viscosity of $-8 \times 10^{-3}$ m² s⁻¹ acting on the mesoscale eddy field. The prediction for the horizontal viscosity is consistent with observations of eddy–internal wave coupling ($\nu_h \approx 50$ m² s⁻¹) estimated from current meter array data. The prediction for the effective vertical viscosity is very sensitive to how energy is distributed in the spectral domain: observations indicate an effective vertical viscosity of $3 \times 10^{-3}$ m² s⁻¹. The difference between observed and predicted exchange coefficients is attributed to differences between the observed internal wave spectrum and the Garrett and Munk model.
1. Introduction

a. Preliminaries

Winds and air-sea exchanges of heat and fresh water are ultimately responsible for the basin-scale currents, or general circulation of the oceans. In order to achieve a state where the energy and enstrophy (vorticity squared) of the ocean is not continuously increasing, some form of energy dissipation is required to balance this forcing. While the above statement may seem obvious, little is known about how and where this dissipation occurs.

In the atmospheric context, radiative damping (Stone 1972) is typically invoked to provide damping of planetary scale motions. Much of the atmosphere is sufficiently close to radiative equilibrium, in which heating associated with incoming solar radiation is balanced by cooling associated with black body radiation, that thermal perturbations associated with eddy motions are damped.

Such a process does not operate within the ocean and we are left searching for other mechanisms by which planetary waves could be damped. If one assumes the ocean interior is adiabatic, the energy sinks would necessarily be located at the boundaries. The oceanic eddy field could be closed by some combination of bottom Ekman layers, eddy-mixed layer interactions, and an interior enstrophy cascade. Despite the intellectual prejudice that views the oceanic interior as adiabatic and inviscid, or perhaps more precisely because, it seems prudent to enquire whether interior processes could provide a damping of both enstrophy and energy. The intent herein is to revisit the issue of internal wave - mesoscale eddy coupling (Müller 1976) as the oceanic equivalent of radiative damping.

b. Eddy-Wave Coupling as an Eddy Dissipation Mechanism

A convenient starting place to examine eddy-wave coupling is to invoke a decomposition of the velocity \( \mathbf{u} = (u, v, w) \), buoyancy \( b = -g \rho / \rho_0 \) with gravitational constant \( g \) and density \( \rho \) and pressure \( \pi \) fields into a quasi-geostrophic ‘mean’ (\( \bar{\gamma} \)) and small amplitude internal wave (\( \gamma' \)) perturbations of any variable \( \gamma \) on the basis of a time scale separation: \( \gamma = \bar{\gamma} + \gamma' \) with \( \bar{\gamma} = \tau^{-1} \int_0^\tau \gamma \, dt \) in which \( \tau \) is much longer than the internal wave time scale but smaller than the eddy time scale.

1) The Potential Vorticity Balance

After employing this averaging process to the equations of motion, the (quasi-geostrophic) potential vorticity equation becomes [Müller (1976)]:

\[
(\partial_t + \bar{u} \partial_x + \bar{v} \partial_y) (\partial_x^2 \Phi + \partial_y^2 \Phi + \partial_z \left[ \frac{f^2}{N^2} \partial_z \Phi \right] + \beta y) = \\
\partial_x \left[ \partial_x u' w' + \partial_y v' w' + \partial_z (v' w' + \frac{f}{N^2} b' u') \right] \\
- \partial_y \left[ \partial_x u' w' + \partial_y u' v' + \partial_z (u' w' - \frac{f}{N^2} b' v') \right] + Q
\]

(1)
in which $Q$ represents modification of the mean buoyancy profile through diabatic processes, $\Phi$ is the geostrophic streamfunction ($\Phi = \pi / f$ with Coriolis parameter $f$) and pressure $\pi$ is defined in the absence of the internal wavefield. The factor $\nabla_h$ is the 2-D horizontal gradient operator. The eddy-wave coupling defined here is assumed to be adiabatic, so that $Q = 0$.

2) THE INTERNAL WAVE BALANCE

The evolution of the internal wavefield is governed by a radiation balance equation:

$$\mathcal{L} n = S_o[n]$$  \tag{2}$$

in which $\mathcal{L}$ is the Liouville operator $\mathcal{L} = \partial_t + (C_g + \mathbf{u}) \cdot \nabla_x + r \cdot \nabla_k$ with group velocity $C_g = \nabla_k \omega$ and refraction rate $r$ given by the ray equations: $r = d\mathbf{k}/dt = -\nabla_x \sigma$. The term $S_o$ represents the generation, transfer, and dissipation of wave action ($n$). The variable $n$ represents the action spectrum, $n = e/\omega$, in which $e$ is the energy spectrum, $\omega = \sigma - k \cdot \mathbf{u}$ is the intrinsic frequency which relates eulerian frequency $\sigma$ to the wavevector $k = (k, l, m)$ through a dispersion relation, $\omega^2 - f^2 = N^2 k_h^2 / m^2$ in the hydrostatic approximation, with horizontal wavenumber $k_h^2 = k^2 + l^2$. This description assumes the wave phase varies much more rapidly than the background velocity field and stratification profile. In the absence of sources, sinks and nonlinearity, (2) states that the action flux is nondivergent:

$$\int d^3k \mathcal{L} n = \nabla \cdot \int d^3k (C_g + \mathbf{u}) n(k, x, t) = 0.$$

3) THE COUPLED SYSTEM

Assuming phase conservation (or equivalently a scale separation and local plane wave solution) Müller (1976) demonstrates that the source terms in the potential vorticity equation can be cast as the flux of pseudomomentum:

$$\partial_x [\partial_x u^i u'^j + \partial_y u^i v'^j + \partial_z (v'^i w'^j + f N^2 b'^j u'^i)]$$

$$- \partial_y [\partial_x u^i u'^j + \partial_y u^i v'^j + \partial_z (u'^i w'^j - f N^2 b'^j v'^i)] =$$

$$- \nabla_h \times \nabla^j \cdot \int d^3k n(k, x, t) k^i C^j_g$$  \tag{3}$$

in which superscripts indicate vector components and summation over repeated indices is implied. The integral on the right-hand-side of (3) represents the flux of pseudomomentum. The pseudomomentum flux is also referred to as the Eliassen-Plam flux (Eliassen and Palm 1961) and identified as the flux of angular momentum (Jones 1967), though formulations for zonal or axisymmetric flows dominate the literature. Since the horizontal wavevector evolves following a wave-packet in non-axisymmetric background flows according to the ray equations, it follows almost trivially from (1) and (3) that a momentum
flux divergence will induce a potential vorticity perturbation, and that this can be accomplished adiabatically. There are subtleties, though: Bühler and McIntyre (2005) provide a conservation statement involving pseudomomentum and vortex impulse defined as the rotated dipole moment of the Lagrangian mean potential vorticity. The point of writing the action spectrum in (3) with an explicit spatial dependence is to emphasize the result applies to a slowly varying wavepacket: a plane wave solution extends to infinity and has no perturbation potential vorticity.

A complicating factor is that the vorticity perturbation induced by a wave packet in the linear analysis is reversible in the sense that the mean state is unchanged after it’s passage. Müller (1976) provides the insight that, if the action of nonlinearity is to relax the wavefield back to an isotropic state, it is possible for the associated vorticity perturbation to become permanent: nonlinearity enables a net transfer of energy and vorticity between mesoscale eddies and internal waves. This insight is at the heart of the calculation presented below.

The problem examined by Müller (1976) is the specific example of an isotropic background internal wavefield interacting with the mesoscale eddy field. It is not the most straightforward example of the internal wave–mean flow interaction process. The scenario discussed by Müller assumes that anisotropic conditions created by eddy straining will be relaxed back to isotropic conditions by nonlinearity in the internal wavefield. It is this irreversible piece of the wave–mean interaction that Müller seeks to capture. It represents a frictional process acting to dissipate eddy potential vorticity perturbations (1).

The quasi-geostrophic potential vorticity (1) and radiation balance equation (2) form a coupled system. Müller closes the system by invoking perturbation expansions associated with $\mathcal{L}$ and $n$:

$$\mathcal{L}_0 + \delta \mathcal{L} \left[ n^{(0)} + n^{(1)} + \ldots \right] = S[n^{(0)}] + \frac{\delta S}{\delta n}[n^{(1)} + n^{(2)} + \ldots] ,$$

in which $\mathcal{L}_0 = \partial_t + \mathbf{C}_g \cdot \nabla x + \mathbf{r}^{(i)} \cdot \nabla x$, $\delta \mathcal{L}$ is the perturbation introduced by the mean flow and $\delta S/\delta n$ denotes the functional derivative. The zeroth order equation describes the generation, propagation, interaction, and dissipation processes that set up the background internal wavefield. The first-order equation describes perturbations induced by wave–mean interactions and the relaxation of those perturbations by nonlinearity. The formal solution for $n^{(1)}$ is:

$$n^{(1)} = -D^{-1} [\delta \mathcal{L} n^{(0)}]$$

in which $D^{-1}$ is the functional inverse of $D = \mathcal{L}_0 - \delta S/\delta n$. The keys to recognizing the importance of nonlinearity are (a), if $S = 0$, the average perturbation is $n^{(1)} = 0$, and (b) nonlinear transfers conserve energy ($\omega n$) and pseudomomentum ($k n$), not their spatial fluxes. Wave propagation in geostrophic background flows is based upon a nondivergent action flux. In conserving pseudomomentum, nonlinearity serves as a nonconservative process relative to the issue of linear wave propagation.

Müller assumes the zeroth order state is independent of horizontal azimuth, and the effect of the first-order wave fluxes on the mean is estimated by substituting the first-order
wavefield \((n^{(1)})\) into the mean source terms. These are formally written as:

\[
\mathcal{F}^{(1)ij} = \int d^3k \ f^{ij} \ D^{-1}[k^\alpha \frac{\partial}{\partial k^m} n^{(0)} \frac{\partial}{\partial x^m} \overline{u^\alpha}] \\
\mathcal{M}^{(1)\beta} = \int d^3k m^{\beta} \ D^{-1}[k^\alpha \frac{\partial}{\partial k^m} n^{(0)} \frac{\partial}{\partial x^m} \overline{u^\alpha}].
\]

Expressions for \(f^{ij}\) and \(m^{\beta}\) are algebraic functions of frequency and wavenumber and are given in Müller (1976). The notation attempts to follow that of Müller (1976) and summation over indices \(m\) and \(\alpha\) is implied. The usage of \(f, \beta\) and \(m\) in (6) and (7) should not be confused with the definitions herein.

The end products of the analysis are predictions for correlations between momentum flux cospectra \([C(m, \omega) + iQ(m, \omega)]\), power spectra \([P(m, \omega)]\) and the mesoscale gradients (Müller 1976):

\[
C_{u'u'}(m, \omega) + iQ_{u'u'}(m, \omega) \quad \text{and} \quad \overline{\tau_x + \tau_y} \equiv S_s \\
P_{u'u'}(m, \omega) - P_{v'v'}(m, \omega) \quad \text{and} \quad \overline{u_x - v_y} \equiv S_n \\
[C_{u'w'}(m, \omega), C_{v'w'}(m, \omega)] \quad \text{and} \quad [\overline{\tau_z}, \overline{\tau_z}] \\
[C_{u'v'}(m, \omega), C_{v'u'}(m, \omega)] \quad \text{and} \quad [\overline{b_x}, \overline{b_y}].
\]

A zero correlation is implied between

\[
P_{u'u'}(m, \omega) + P_{v'v'}(m, \omega) \quad \text{and} \quad \overline{\tau_x - \tau_y} \equiv \zeta .
\]

Integration over wavenumber and frequency returns a simple characterization of the coupling as a diffusive process, for which:

\[
-2u'w' = \nu_h(\overline{\tau_x + \tau_y}), -u'w' = \nu_o \overline{\tau_z}, -u'u' = \nu_h \overline{u_x}, -v'v' = \nu_h \overline{v_y}, -u'b' = K_h \overline{b_x}, \text{ and } -v'b' = K_h \overline{b_y}.
\]

c. Forward

The disconcerting part of the story is that the quantitative predictions made by Müller are inconsistent with current meter observations obtained as part of the Polymode program. In one case (Ruddick and Joyce 1979), the observed correlation between \(\overline{u'w'}\) and eddy shear \(\tau_z\) was more than an order of magnitude smaller than the prediction. In another (Brown and Owens 1981), the observed correlation between \(\overline{u'v'}\) and eddy strain \(S_s\) was more than an order of magnitude larger.

It turns out that there is some uncertainty in Müller’s estimation of the correlation coefficients (McComas and Bretherton 1977). This knowledge, in combination with the simple picture provided by Bühler and McIntyre, prompted revisiting Müller’s analysis and the Polymode observations. The Polymode data are reviewed in Section (2). Müller’s formulation is reviewed in Section (3). A summary and discussion are presented in Section (4).
2. The LDE Data Set, Revisited

a. Coherence Estimates

Moored current and temperature measurements were made for 15 months in the main thermocline of the Gulf Stream Recirculation Region near 31°N, 69°30′W to assess the energetics and dynamics of the mesoscale eddy field as part of the Polymode Local Dynamics Experiment (LDE) [Bryden (1982), Brown and Owens (1981), Brown et al. (1986)]. The issue of eddy-internal wave coupling was recently revisited using these data, Polzin (2005). The observed correlations between frequency cospectra and eddy gradients have been extracted from that analysis and are presented here.

The coherence functions (Fig. 1) between horizontal fluxes \[ P_{u'u'}(m, \omega) - P_{v'v'}(m, \omega) \] and mesoscale rate of strain estimates \[ \overline{\tau}_x - \overline{\tau}_y \equiv S_n, \overline{\tau}_x + \overline{\tau}_y \equiv S_s \] are similar: correlations are small for near-inertial \( (\omega < 2f) \) frequencies, the semi-diurnal frequency is anomalous and continuum frequencies typically exhibit correlations between 0.05 and 0.15 with a possible increasing trend with increasing frequency.

The coherence functions (Fig. 2) between vertical fluxes \[ C_{u'u'}(m, \omega) - \frac{f}{N^2} C_{v'v'}(m, \omega), \]
\[ C_{u'u'}(m, \omega) + C_{u'v'}(m, \omega) \] and vertical gradients \[ \overline{\tau}_z \] are somewhat smaller and thus both components have been averaged to improve statistical reliability. Correlations are approximately 0.02 at frequencies exceeding 10 cpd and oscillate about zero for frequencies between 2 and 10 cpd. Large negative values are noted at near-inertial frequencies. The vertical flux of horizontal momentum associated with near-inertial frequencies, though, is relatively small.

Integration of the cospectra over the frequency domain provide effective viscosity estimates of \( \nu_h \approx 50 \text{ m}^2 \text{ s}^{-1} \) and \( \nu_v \approx 3 \times 10^{-3} \text{ m}^2 \text{ s}^{-1} \), Polzin (2005), their Fig.s (4&6). Notably, the near-inertial contribution to the vertical viscosity is less than 20% of the frequency integrated cospectrum. The effective viscosities and implied exchanges of energy and momentum are interpreted by Polzin (2005) as indicating that (i) internal wave – mesoscale eddy coupling through horizontal interactions plays an \( O(1) \) role in the energy budget of the internal wavefield and is a significant sink of eddy energy, and (ii) eddy-internal wave interactions play an \( O(1) \) role in the eddy potential enstrophy budget.

b. Spectra

The Garrett and Munk spectral description of the background internal wavefield, specifically the GM76 model [Cairns and Williams (1976)], will be used to make a prediction for the observed coherences. A significant difference between predicted and observed vertical coherences will be found and it is conjectured that this difference lies in the departure of the observed spectra from the GM model. Such a conjecture, though, begs knowledge of the observed spectra. As part of an ongoing analysis of historical data sets, Lvov et al. (2005) argue for a regional characterization of the background internal wave spectrum, rather than the universal GM model. Within the Southern Recirculation Gyre, spectral levels in the frequency domain are lower than the high frequency part of the GM model,
Frequency spectra tend to be whiter, vertical wavenumber spectra tend to be redder and the vertical wavenumber/frequency spectrum is not separable. To be definite, data obtained as part of the Frontal Air-Sea Interaction Experiment (FASINEX) are shown to demonstrate these characteristics.

FASINEX was designed to investigate the response of the upper ocean to atmospheric forcing in the presence of oceanic fronts. An array of surface moorings with Vector Measuring Current Meters (VMCMs) was deployed in the subtropical convergence zone of the Northwest Atlantic (27° N, 70° W) from January to June of 1986, Weller (1991) and Eriksen et al. (1991). Frequency spectra at depths of 700 m are defined by a power law $\omega^{-s}$ of $s = 1.75$, Fig. (3). Vertical profiles of horizontal velocity and density were obtained during February-March using the High Resolution Profiler (HRP), Polzin et al. (1996). The vertical profiles revealed a complex pattern of variability associated with the frontal velocity structure in the upper 250 m. Here we report results concerning data from depths of 250-1000 m, Fig. (4). Sampling took place over several degrees of latitude and longitude, centered 1° north of the moored array. It is assumed that the sampling is random relative to the underlying eddy field and so is not spatially biased. The high frequency internal wavefield in the main thermocline is known to exhibit an annual cycle with maximum energy in the late winter in this region, Briscoe and Weller (1984). It is not clear how this annual cycle would appear in the finestructure data, but the FASINEX data were obtained during the more energetic part of the annual cycle.

The Fasinex horizontal velocity vertical wavenumber spectra are fit with a parametric spectrum having an asymptotic roll-off of $m^{-2.3}$ and low-wavenumber roll-off $m_o$ equivalent to mode-15, Fig. (4). The fit indicates a spectrum redder than GM76 ($m^{-2}$) having a narrower bandwidth ($m_o$ equivalent to mode-3). Note also that the spectra are nonseparable: ratios of kinetic to potential energy are larger at 50 m vertical wavelength than 500 m, indicating increased near-inertial frequency content at 50 m. Polzin et al. (2003) argue that the decreasing ratio of kinetic to potential energy at even smaller scales is associated with an increasing vortical contribution.

3. A Revisionist Discussion of Müller (1976)

Formal inversion of $\delta S/\delta n$ [(4) and (5)] is intractable if nonlinearity is assumed to be represented by resonant wave–wave interactions. Further progress is possible by interpreting $\delta S/\delta n$ as a relaxation time scale:

$$D^{-1}[Q] = \tau_R(k) Q .$$

Nonlinearity is assumed to force anisotropic conditions introduced by wave–mean interactions back to the isotropic zeroth order wavefield on a time scale $\tau_R(k)$. This assumption permits the explicit evaluation of the viscosity and diffusion coefficients. By further assuming $\tau_R(k)$ is independent of $k$, Müller integrates by parts and changes to a $(k; l, \omega)$ representation. A slightly different sequence of approximations is invoked here.
The primary shortcoming of Müller is that it was written from the perspective that the thermocline was characterized by a diapycnal diffusivity of $K_{\rho} = 1 \times 10^{-4} \text{ m}^2 \text{s}^{-1}$, which implies a time scale of 5-10 days for nonlinear interactions to drain energy out of the background internal wavefield. Several decades of research has since demonstrated that the background diffusivity is $K_{\rho} = 1 \times 10^{-5} \text{ m}^2 \text{s}^{-1}$, with corresponding time scale of 50-100 days. This is crucial for Müller’s calculation. Müller’s scheme invokes the ability of eddies to create anisotropic conditions out of an isotropic background wavefield. Nonlinearity is then invoked to relax the perturbed internal wavefield back to an isotropic state, and it is this relaxation that creates a permanent exchange of pseudomomentum for potential vorticity. With order of magnitude larger relaxation times, larger scale internal waves can propagate through an eddy-wave interaction event with minimal permanent exchange of pseudomomentum and vorticity. These propagation effects are substantial.

The intervening 30 years of research also suggests estimating the relaxation time from a cascade representation of nonlinearity \(^1\) in the internal wavefield (Polzin (2004a)):

\[
\frac{\partial E^\pm(m,\omega,\theta)}{\partial t} \pm \frac{\partial [C_g E^\pm(m,\omega,\theta)]}{\partial x} + \frac{\partial F^\pm_e(m,\omega,\theta)}{\partial m} = \frac{1}{2m} [F^+_{e}(m,\omega,\theta) - F^-_{e}(m,\omega,\theta)].
\]  

Energy densities associated with opposing wavevectors $k$ and $-k$ is represented by $E^+$ and $E^-; E(m)$ is the integral: $E(m) = \int_{-N}^{N} \int_{0}^{\pi} (E^+(m,\omega,\theta) + E^-(m,\omega,\theta)) \, d\omega \, d\theta$ over frequency $\omega$ and horizontal azimuth $\theta$; superscripts denote the sign of the vertical wavenumber and azimuthal angle. Frequency and azimuthal domain cascades are neglected here. The frequency cascade is believed to be of secondary importance [Polzin (2004a)] and observational records do not contain information from which an azimuthal cascade could be defined. If the azimuthal cascade sets a shorter timescale, it would be more appropriate here.

The factor $F^\pm_e$ represents the transport of energy density, $E^\pm$, to higher vertical wavenumber and is given by:

\[
F^\pm_e(m,\omega,\theta) = a \, m^4 \, N^{-1} \, \phi(\omega) \, E^\pm(m,\omega,\theta) \, E(m),
\]  

with $a = 0.20$ and

\[
\phi(\omega) = [\left(\omega^2 - f^2\right)/\left(N^2 - \omega^2\right)]^{1/2}.
\]

The functional representation denoted by $\phi$ implies increasing transport with increasing wave frequency, as suggested by the observations. The transport magnitude set by $a = 0.20$ is taken from the validation studies of Polzin et al. (1995) and Gregg (1989).\(^2\)

Nonlinearity conserves energy and pseudomomentum. For planetary waves, the zonal component of pseudomomentum ($ke/\omega$) is sign definite (since $\omega = -\beta k/(k^2 + l^2 + \frac{f^2}{N^2} m^2)$)

\(^1\)The time scales to be inferred from resonant interaction theory (McComas and Müller 1982) are not reliable as they were derived in limits in which the underlying interaction matrix is asymptotically incorrect Lvov et al. (2005).

\(^2\)there is a typo in (Polzin et al. 1995) that leads to $a$ being misquoted as $a = 0.1$ in (Polzin 2004a).
and conservation of pseudomomentum is equivalent to enstrophy \([ (k^2 + l^2 + f^2) e ]\) conservation. The conservation of two quadratic quantities gives rise to two distinct cascades: a forward enstrophy cascade and an inverse energy cascade.

For internal waves, pseudomomentum is a signed quantity. The cascade representation in (10) conserves pseudomomentum by backscattering wave energy between two waves of similar, but oppositely signed wavevectors, at a rate in proportion to the spectral transport of energy to smaller scales. Backscattering appears as the righthand-side of (10).

The anisotropic relaxation time scale to be inferred from (10) and (11) is:

\[
\tau_R^{-1} = \frac{-1}{E^+ - E^-} \frac{\partial (E^+ - E^-)}{\partial t} = a m^3 N^{-1} \phi(\omega) E(m). \tag{12}
\]

With this time scale, propagation effects are obviously important. An order of magnitude estimate of nonlocal effects is provided by Müller:

\[
D^{-1} \approx D_{eff}^{-1} = \tau_R / [1 + (\tau_R/\tau_p)^2]. \tag{13}
\]

The factor \(\tau_p\) represents a propagation time scale:

\[
\tau_p = L^i / C_g^{(0)i}, \tag{14}
\]

in which \(L^i\) are the length scales of the mesoscale eddy field. Owens (1985) estimates a zero crossing of the transverse velocity correlation function of 100 km from the LDE current meter data. The longitudinal velocity correlation function falls off more slowly and thus the longitudinal length scale is not resolved. For the purpose of producing theoretical estimates of the coherence functions, the horizontal propagation time scale will be naively estimated with \(100 \leq L \leq 200\) km. Wunsch (1997) finds that gradients of low frequency velocity are largely confined to the first several vertical modes, and thus a vertical scale \(H\) representative of the thermocline depth \(H = 700\) m is assumed for the vertical propagation time scale.

\section*{a. The Horizontal Dimension}

In the hydrostatic approximation, the horizontal viscosity becomes:

\[
\nu_h = -\frac{1}{8} \int d^3k \frac{\omega^2 - f^2}{\omega^2} \frac{\omega k_h \tau_R}{1 + (\tau_R/\tau_p)^2} \frac{\partial n_{\lambda}^{(0)}}{\partial k_h}. \tag{15}
\]

The zeroth order wavefield is represented using the Garrett and Munk (GM76) distribution,

\[
e^{(0)}_2(m, \omega) = \frac{B}{(m_o^2 + m^2)^{1/2}} \frac{2f}{\pi} \frac{1}{\omega(\omega^2 - f^2)^{1/2}}, \tag{16}
\]

with a slight modification. A low-wavenumber roll-off \(m_o\) equivalent to mode-4 (not mode-3) is used to obtain both an appropriate energy \((30 \times 10^{-4} \text{ m}^2 \text{ s}^{-2} \text{ at } N_o = 3 \text{ cph})\) and high vertical wavenumber shear spectral density \((7N^2/2\pi \text{ s}^{-2} \text{ rad m}^{-1})\). See the appendix of
Gregg and Kunze (1991) for details. The relation between the 2-D energy spectrum and a 3-D isotropic action spectrum dictates:

\[ n_3^{(0)}(m, \omega) = e_2^{(0)}(m, \omega)/k_h \omega, \]  

(17)

so that (15) becomes:

\[ \nu_h = \frac{f N^2}{4\pi a} \int_{f}^{N} \int_{0}^{\infty} \frac{[4 \omega^2 - 3 f^2] d\omega dm}{\omega^3 m^3 \left[ \omega^2 + \left( \frac{N^2 (m_o^2 + m^2)}{\text{Lam}_4^2 B} \right) \right]^2}. \]  

(18)

Numerical evaluation returns

\[ \nu_h \approx 50 \text{ m}^2 \text{s}^{-1} \]  

(19)

for parameters (31° latitude, \( N=2.6 \text{ cph}, L=100 \text{ km} \)) appropriate to the Polymode LDE data. The parameter regime is such that numerical evaluation also suggests an approximate linear dependence upon \( N \), spectral amplitude and eddy length scale \( L \). The viscosity coefficient depends only weakly upon the strength of the cascade process \( (a) \) and the Coriolis parameter \( f \). These dependencies come with a note of caution: they may pertain only to the GM spectrum. Müller’s estimate was \( \nu_h \approx 7 \text{ m}^2 \text{s}^{-1} \).

Direct comparison with the observed coherence functions can be had by multiplying the integrand of (18) by the appropriate strain estimate (\( |S_n| = 1.41 \times 10^{-6} \text{ s}^{-1} \) or \( |S_s| = 1.07 \times 10^{-6} \text{ s}^{-1} \)) and dividing by the GM–\( P^{1/2}_{uv} P^{1/2}_{vu} \) power spectral density functions. So constructed, the model coherence function estimates mimic the observed estimates (Fig. 1) reasonably well. Coherence function estimates tend to zero as \( \omega \) approaches \( f \) and attain levels of approximately 0.1 within the continuum frequency band. The observed cospectral estimates tend to increase towards higher frequency whereas the theoretical prediction decreases. Coherence function levels using \( L = 100 \) and \( L = 200 \) km tend to bracket the observations.

The agreement is impressive given the relatively unsophisticated treatment of the relaxation mechanism. It is difficult to further refine the model without having a 2-D vertical wavenumber–frequency spectrum on which to base the calculations.

b. The Vertical Dimension

In the hydrostatic approximation, the effective vertical viscosity becomes:

\[ \nu_v = \frac{f^2 N^2}{N^2} K_h = \frac{1}{2} \int d^3k \frac{\omega^2 - f^2}{\omega^2} \frac{\omega k_h^2}{m} \frac{\tau_R}{1 + (\tau_R/\tau_p)^2} \frac{\partial n_3^{(0)}(k_h, m)}{\partial m}. \]  

(20)

For a GM wavefield (16), this translates into:

\[ \nu_v = \frac{f^2}{\pi a} \int_{f}^{N} \int_{0}^{\infty} \frac{(\omega^2 - f^2)(2m_o^2 \omega^2 - 3(m_o^2 + m^2) f^2)}{\omega^3 m^3 (m_o^2 + m^2) \left[ \omega^2 + \left( \frac{N^2 (m_o^2 + m^2)(\omega^2 - f^2)^{1/2}}{\text{Ham}_4^2 B} \right)^2 \right]} d\omega dm. \]  

(21)
Direct comparison with the observed coherence function estimate is possible by multiplying the integrand of (21) by the observed rms shear, \([(\bar{\tau}^2_z + \bar{\tau}^2_z)/2]^{1/2} = 1.42 \times 10^{-4} \text{ s}^{-1}\). The resulting coherence estimates are near zero at high frequencies and negative at near-inertial, and qualitatively mimic the observations, Fig. (2). The resulting integral is distinctly negative,

\[ \nu_v + \frac{f^2 K_h}{N^2} = -8 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}, \]

rather than the observed positive value, \[ \nu_v + \frac{f^2 K_h}{N^2} = 3 \times 10^{-3} \text{ m}^2 \text{ s}^{-1} \] (see also Polzin (2005), their Fig. 6). The difference between observed and predicted exchange coefficients is associated with small but distinctly positive coherence function estimates at high frequency, Fig. 2, and dominance of the \[ u'w' \] and \[ v'u' \] cospectra by high frequency contributions.

The near zero values of the coherence function predictions using the GM76 spectrum are easily appreciated. In the high-frequency, high-vertical wavenumber limits, a power-law characterization of the 2-D energy spectrum and resulting 3-D action spectrum results in:

\[ e_2(\omega, m) \propto \omega^{-s} m^{-t} \rightarrow n_3(k) \propto \left(\frac{m}{k_h}\right)^s \left(\frac{1}{m}\right)^t \frac{1}{k_h^t}. \]

For the GM76 spectrum, \((s, t) = (2, 2)\) and \(n_3(k) \propto k_h^{-4} m^0\), and thus flux-gradient representations in the vertical wavenumber domain will predict minimal spectral transports (McComas and Bretherton 1977).

The observations and theoretical estimates of the vertical exchange coefficient are both much smaller than Müller’s prediction of \( \nu_v \approx 0.45 \text{ m}^2 \text{ s}^{-1} \). Müller’s large estimate is a product of neglecting propagation effects for buoyancy frequency waves of large vertical scales.

c. Future Directions

Müller’s representation of eddy-internal wave coupling can be succinctly characterized as downgradient transport of wave action in the spectral domain under a relaxation time scale approximation. The magnitude of the coupling coefficients (effective viscosities) are thus a function of the spectral parameters: spectral amplitude, high wavenumber/frequency power law specifications and low wavenumber roll-off; and external parameters, i.e. mesoscale velocity gradient variances and characteristic spatial scales. Variability in both fields is apparent [e.g. Lvov et al. (2005) and Zang and Wunsch (2001)].

It should not be of great surprise if variability of the two fields is directly related. Polzin (2005) present evidence that the background internal wavefield in the Southern Recirculation Gyre of the Gulf Stream is forced primarily by the eddy-internal wave coupling mechanism discussed here. To the extent this is true, the source function \( S^{(0)}(\omega, m) \) for the zeroth order \( n^{(0)} \) wavefield becomes:

\[ \int^N_f \int^\infty_0 S^{(0)}(\omega, m) \, d\omega \, dm = (\nu_v + \frac{f^2 K_h}{N^2})(\bar{\tau}^2_z + \bar{\tau}^2_z) + \nu_h(\bar{\tau}^2_x + \bar{\tau}^2_y + \bar{\tau}^2_x + \bar{\tau}^2_y) \] (22)

The source function is evaluated for the GM76 spectrum in Fig. (5) using parameters for the LDE data: \((H, L, \bar{\tau}^2_z + \bar{\tau}^2_z, \bar{\tau}^2_x + \bar{\tau}^2_y + \bar{\tau}^2_x + \bar{\tau}^2_y) = (700 \text{ m, } 100 \text{ km, } 4.1 \times 10^{-8} \text{ s}^{-2},...
For these parameters, eddy-wave coupling represents a strong sink of near-inertial energy at large vertical wavelengths. If employed in a time stepping algorithm for the radiation balance equation (2), such waves would quickly disappear and the resulting spectrum would be non-separable and in much better agreement with the observations, Figs (3-4). In particular, note that the source/sink boundary corresponds to the roll-off in the observed vertical wavenumber spectrum, Fig. (4).

An obvious conjecture is that observed non-separability and variability in frequency-wavenumber domain powers can be explained as a balance between this spectral source function and nonlinear transports in the zeroth order radiation balance equation (2).
the right-hand-side of the potential vorticity equation (1) is non-zero only at critical levels. Here the rate of strain associated with a mesoscale eddy field provides the mechanism for momentum and energy exchange without the requirement of diabatic effects or resonance conditions.

This work represents a quantitative interpretation using a semi-empirical approach. The intent is to identify the essential pieces of the internal wave–mesoscale eddy–zonal mean problem in order to formulate questions for further research. Some such issues are presented in the following discussion.

a. A Simple Interpretive Context

Bühler and McIntyre (2005) point to an analogy between internal wave propagation and the problem of particle pair separation in 2-D turbulence. In this relative dispersion problem, particle pairs undergo exponential separation when the rate of strain exceeds relative vorticity:

$$S_s^2 + S_n^2 > \zeta^2$$

with $S_s \equiv \overline{v}_x + \overline{v}_y$ the shear component of strain, $S_n \equiv \overline{u}_x - \overline{u}_y$ the normal component and $\zeta \equiv \overline{v}_x - \overline{u}_y$ relative vorticity. Equation (23) is simply the Okubo-Weiss criterion [e.g., Provenzale (1999)]. Bühler and McIntyre (2005) argue that the problem of small amplitude waves in a larger scale flow field is kinematically similar to particle pair separation under the hypothesis of action conservation. In this case the ray equations lead to exponential increase/decrease in the density of phase lines, i.e., an exponential increase/decrease in horizontal wavenumber, $k_h = (k^2 + l^2)^{1/2}$, Fig. (6). Vorticity simply tends to rotate the horizontal wavevector in physical space. This provides a simple picture of pseudomomentum flux divergence associated with an internal wave packet interacting with an eddy strain field.

Bühler and McIntyre (2005) further argue for a scenario which they term ‘wave capture’. Simply put, exponential growth of the horizontal wave number implies exponential growth or decay of vertical wave number in the presence of geostrophic shear. Those waves with growing vertical wavenumber will tend to be trapped (captured) within the extensive regions of the eddy strain field. Wave dissipation is regarded as a necessary consequence in that work. While wave dissipation ultimately includes wave breaking and diabatic processes, pseudomomentum “dissipation” is triggered by the backscattering of waves associated with momentum conservation in a cascade representation of adiabatic nonlinearity within the internal wavefield.

1) COHERENT VORTICES

One of the surprises of the Local Dynamics Experiment was the prevalence of coherent submesoscale lenses, Elliott and Sanford (1986). These lenses typically had temperature-salinity properties that distinguished them as having relatively long (several-year) life spans. These submesoscale features had horizontal scales of $L = 15$ km, significantly smaller than the mesoscale field ($\approx 100$ km). Such life spans are not consistent with a viscous decay
and \( \nu_h = 50 \text{ m}^2\text{s}^{-1} \), which provides a temporal decay scale \( L^2/\nu_h \approx \) of less than 1 month.

An explanation is provided by numerical simulations of 2-D turbulence. The coherent vortices that evolve out of either freely decaying or forced simulations typically have a core with intense relative vorticity, and a thin outer ring of intense strain. In terms of particle dispersion, the vortex cores can trap and transport particles over anomalously large distances, Provenzale (1999). If one is to take the analogy provided by Bühler and McIntyre (2005) literally, the vortex cores, which tend to rotate rather than stretch phase lines, would not be prone to momentum exchanges with the internal wavefield and thus would not be prone to viscous decay. The existence of long-lived, coherent submesoscale vortices is consistent with the proposed model of wave-eddy interaction.

2) Kunze’s Dispersion Relation

Müller’s perturbation expansion of the radiation balance equation assumes advection by the mean flow contributes at zeroth order and nonlinearity at first order. This expansion predicts energy to be uncorrelated with relative vorticity (8). This is not the only approach to characterizing wave-mean interactions. Kunze (1985) produces a dispersion relation by taking the determinant of the linearized equations of motion. The resulting expression was used in conjunction with ray-tracing arguments to explain an observed correlation between energy and relative vorticity in an upper ocean frontal regime. Applications of this approach have been to explain wave trapping in which nominally subinertial \((f + \zeta/2 < \omega < f)\) waves are not free to propagate away from their generation region and the background flow is symmetrical. In Müller’s approach, the generation process is not specified but all waves are assumed to lie within \( f < \omega < N \) and the advective perturbation to the intrinsic frequency is small. In principle, mean gradients could be included within the framework of a perturbation expansion to the radiation balance equation. But use of Kunze’s dispersion relation is not straightforward. Polzin et al. (1996) point out that Kunze’s dispersion relation cannot be obtained as part of a systematic perturbation expansion unless the near-inertial limit is formally invoked.

b. Zonal-Mean Interactions

The content of Andrew and McIntyre’s generalized Eliassen and Palm flux theorem is that a residual meridional circulation in an erstwhile zonal mean background requires an Eliassen-Palm flux divergence, which in the context of linear planetary wave theory is directly proportional to the rate of enstrophy dissipation, Andrews et al. (1987). The work here regarding eddy-internal wave coupling thus bears directly upon the mean circulation in that problem.

In the context of residual mean theory for a zonal jet, the residual mean circulation
\((\overline{u}^*, \overline{w}^*)\) is given by Andrews et al. (1987):

\[
\begin{align*}
\overline{u}_t - f_0 \overline{u}^* - \overline{X} & = \rho_0^{-1} \nabla \cdot F \\
\overline{b}_t + \overline{w}^* \overline{b}_{oz} - \overline{B} & = 0 \\
\overline{v}_y + \overline{w}_z & = 0
\end{align*}
\]

with \(\overline{\cdot}\) indicating a zonal average, \((\cdot)^*\) the residual field, \(F\) the Eliassen-Palm flux, \(\overline{X}\) mean frictional stress or mechanical forcing and \(\overline{B}\) mean diabatic buoyancy forcing.

A number of investigators (e.g. Olbers et al. (2004); Kuo et al. (2005)) have taken this approach for idealized applications to the Southern Ocean. These applications typically assume \(\overline{X}\) and \(\overline{B}\) represent windstress and buoyancy fluxes acting only within a surface diabatic layer, and that \(\nabla \cdot F = \overline{X} = 0\) and \(\overline{B} = (K_\rho \overline{b}_z)_z\) given by a weak background diffusivity of \(K_\rho \approx 1 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}\), thereby arriving at the result that the residual circulation is dominated by a closed cell in which the surface fluxes act: planetary wave damping occurs in a surface diabatic layer.

The internal wave mesoscale eddy coupling problem examined here suggests the assumption of an inviscid interior may not be justified. Under the quasi-geostrophic approximation,

\[
\rho_0^{-1} \nabla \cdot F = \overline{q'} \overline{\nu'}
\]

in which the right-hand side represents the flux of quasi-geostrophic perturbation potential vorticity. From the enstrophy equation:

\[
\frac{\partial \overline{q'^2}}{\partial t} + \nabla \overline{q'} \cdot \overline{\nu'} = D.
\]

Polzin (2005) used the LDE data to demonstrate that the enstrophy dissipation rate \(D\) is the same order of magnitude as the observed downgradient potential vorticity flux (Brown et al. 1986) in the Southern recirculation Gyre.

Moored observations from the Antarctic Circumpolar Current suggest a significant interior EP flux divergence, Phillips and Rintoul (2000), in contrast to the adiabatic, inviscid assumptions characterizing the recent theoretical work. This flux divergence requires [from (24) and (25)] an interior dissipation mechanism - such as eddy-wave coupling.

c. Issues of Variability

Variability of the viscosity coefficients acting on the mesoscale field will depend upon variability in the background wavefield. Such variability exists (Lvov et al. (2004)). A possible implication of this work is that the variability in the oceanic internal wavefield is related to variability in the eddy field through (22). In terms of understanding the geographic variability of viscosity coefficients, there is a simplicity if the energetics of the internal wave field are dominated by an interior coupling to the mesoscale eddy field, as appears to be the case in the Southern Recirculation Gyre, Polzin (2005). This represents the dynamic balance advocated by Müller and Olbers (1975), albeit at somewhat reduced interaction rates.
The internal wave–eddy coupling becomes more complicated if the geostrophic flow field locally forces the zeroth order wavefield though quasi-stationary internal lee wave generation. This scenario may be important as the North Atlantic Current crosses the Mid-Atlantic Ridge, as the Antarctic Circumpolar Current transits through the Drake Passage or in the lee of the Kerguelen Plateau. An further complication is posed by non-equilibrium (vertically decaying) internal wave states associated with tidal forcing at mid-ocean ridges [Polzin (2004b)].

I leave the reader with the following questions:

- How spatially variable are these effective viscosities? Are results from the Gulf Stream Recirculation applicable to more climatologically sensitive regions such as the Southern Ocean [(Naveira-Garaboto et al. 2004), (Polzin and Firing 1998)] or the Greenland–Iceland–Norwegian Seas, Naveira-Garaboto et al. (2005)?

- The behavior of simplified oceanic general circulation models depends upon how viscous damping is implemented [(Cessi and Ierley 1995), (Fox-Kemper and Pedlosky 2004)]. What happens to the behavior of the general circulation as the thermocline tightens, the eddy scale decreases, or the rms eddy velocity increases?

- Can we really claim to understand how the climate system works if GCMs are simply tuned to today’s conditions and do not address the proper sub-grid scale physics?
Acknowledgment. Much of the intellectual content of this paper evolved out of discussions with R. Ferrari.
REFERENCES


Figure Captions

Fig. 1. Coherence functions created by averaging (a) $-\text{sgn}(S_n)[P_{uu} - P_{vv}]/P_{1/2}^{1/2}$ and (b) $-\text{sgn}(S_s)C_{uv}/P_{1/2}^{1/2}$ with $C_{uv}$ being the real part of the $uv$ cross-spectrum. Figure extracted from Polzin (2005), see that work for further description of the data analysis. Over plotted as thick lines are GM76 based estimates for the coherence functions using (18). The upper estimate employs an eddy decorrelation scale of $L = 200$ km, and the lower a decorrelation scale of $L = 100$ km.

Fig. 2. Coherence function created by averaging $-\text{sgn}(\tau_x)[C_{uw} - fN^{-2}C_{ub}]/T(\omega)P_{1/2}^{1/2}P_{1/2}^{1/2}$ and $-\text{sgn}(\tau_x)[C_{vw} + fN^{-2}C_{ub}]/T(\omega)P_{1/2}^{1/2}P_{1/2}^{1/2}$ with $\text{sgn}(x)$ representing the sign of $x$. The factor $C_{xy}$ represents the real part of the $xy$ cross-spectrum. The transfer function $T(\omega) = (\omega^2 - f^2)/(\omega^2 + f^2)$ accounts for cancelation of the Reynold’s stress by the buoyancy flux and renders the denominator consistant with the numerator. Figure extracted from Polzin (2005), see that work for further description of the data analysis. Over plotted as thick a line is a GM76 based estimate for the coherence functions using (21).

Fig. 3. FASINEX frequency spectra (thin lines) of horizontal kinetic ($E_k$) and potential ($E_p$) energy from the main thermocline (700 m). Thick curves represent fits of $E(\omega) = A/\omega^q(\omega^2 - f^2)^{1/2}$ with $q = 0.75$ and returning a frequency integrated kinetic energy estimate 20% of the GM model. In contrast, the frequency integrated observed kinetic energy estimate is approximately half (48%) the GM specification. The thick vertical line represents the buoyancy frequency cut-off. Figure extracted from Lvov et al. (2005), see that work for further data analysis details.

Fig. 4. FASINEX-vertical wavenumber spectra of horizontal kinetic and potential energy, $N$-scaled and stretched under the WKB approximation to $N_o = 3$ cph. These data were obtained 100-200 km north of the FASINEX moored array. Sampling was intermittent in both space and time. Note that the low-wavenumber spectral estimates have typically smaller ratios of $E_k$ and $E_p$. The thick line represents a fit of $B/(m^2 + m^2)^{1.15}$ with $m_*$ corresponding to mode-15 to the kinetic energy spectrum. Figure extracted from Lvov et al. (2005), see that work for further data analysis details.

Fig. 5. The spectral source function defined as the integrand on the right-hand-side of (22), evaluated for the GM76 spectrum. The thick line separates positive source values from negative. Negative values of the source function are, effectively, energy sinks.
FIG. 6. Phase lines (dashed) of waves being passively advected by three realizations of a steady mean flow having streamlines denoted by the solid contours: (a) a spatially constant deformation strain. (b) a spatially constant shear strain. (c) a spatially constant vorticity. Parcel velocities induced by internal waves are normal to wave crests (lines of constant phase). The tendency of mesoscale strain to create anisotropic wavefields and internal wave stresses by preferentially orienting the phase lines along the extensive axis of a strain field can be inferred [(a) and (b)]. Similarly, the tendency of vorticity to not result in wave stresses can also be inferred (c).
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Fig. 2. Coherence function created by averaging $-\text{sgn}(\overline{w}_z)[C_{uw} - fN^{-2}C_{vb}] / T(\omega)P_{uu}^{1/2}P_{ww}^{1/2}$ and $-\text{sgn}(\overline{w}_z)[C_{uw} + fN^{-2}C_{vb}] / T(\omega)P_{vv}^{1/2}P_{ww}^{1/2}$ with $\text{sgn}(x)$ representing the sign of $x$. The factor $C_{xy}$ represents the real part of the $xy$ cross-spectrum. The transfer function $T(\omega) = (\omega^2 - f^2)/(\omega^2 + f^2)$ accounts for cancelation of the Reynold’s stress by the buoyancy flux and renders the denominator consistent with the numerator. Figure extracted from Polzin (2005), see that work for further description of the data analysis. Over plotted as thick a line is a GM76 based estimate for the coherence functions using (21).
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