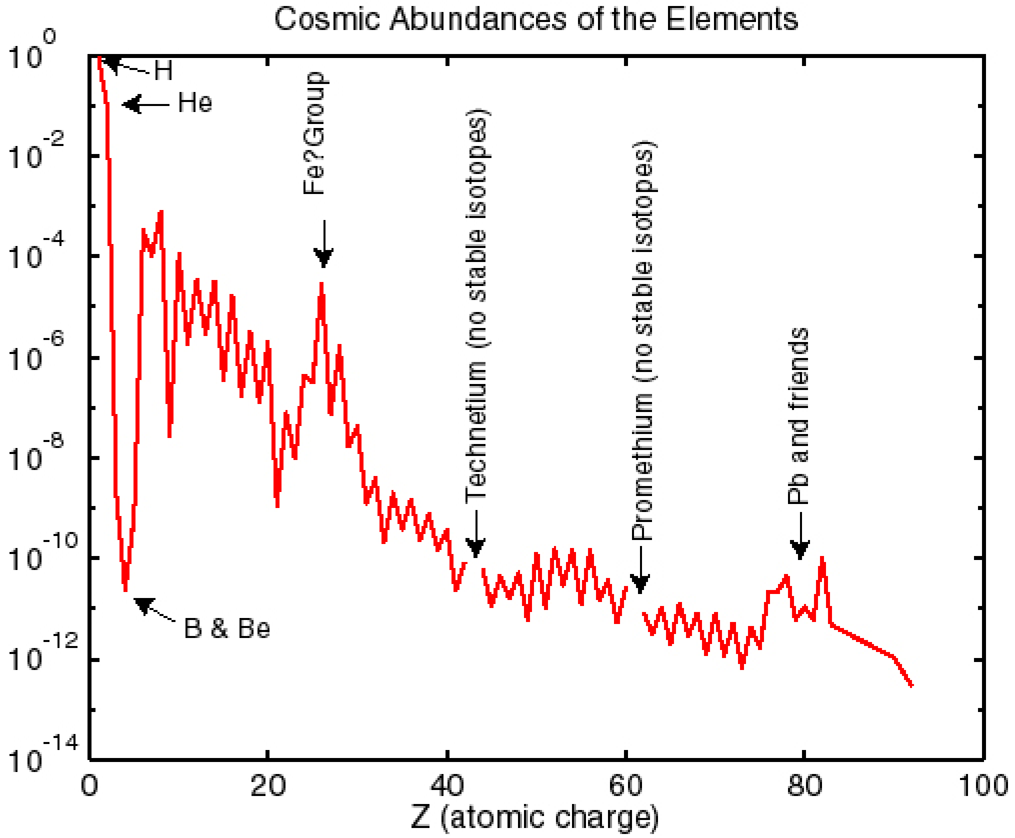
12.744/12.754 The Basic Rules, Nuclear Stability, Radioactive Decay and Radioactive Dating

What we see in the earth and oceans is the product of the "cosmic" abundance (i.e. the original) pattern of elements, and the chemical and physical processes that form and maintain the oceans and earth as whole. There is a very distinct flavour to the cosmic abundance of the elements that we can explain on the basis of their origin, and from the fundamental nature of nuclear structure and stability.



- There are four fundamental forces of nature, their respective strengths span 39 orders of magnitude
    1. Strong nuclear (short range, between nucleons) **Strength = 1**
    2. Electromagnetic (infinite, but shielded) **Strength = 10^{-2}**
    3. Weak nuclear force (very short range between leptons, beta decay) **Strength = 10^{-13}**
    4. Gravity (infinite range, cannot be shielded against) **Strength = 10^{-39}**
- The two most significant advances in modern physics (and science in general) this century are relativity and quantum mechanics. The former provided a fundamental linkage between matter, energy and gravity. The latter rationalized atomic (and ultimately nuclear) structure, and the behaviour of subatomic particles. The two in combination played a fundamental role in building the foundations for our understanding of the cosmos around us, and its origins. **Mass is energy**, energy is mass: $E=mc^2$. What does this mean? One gram of matter yields the following amount of energy:

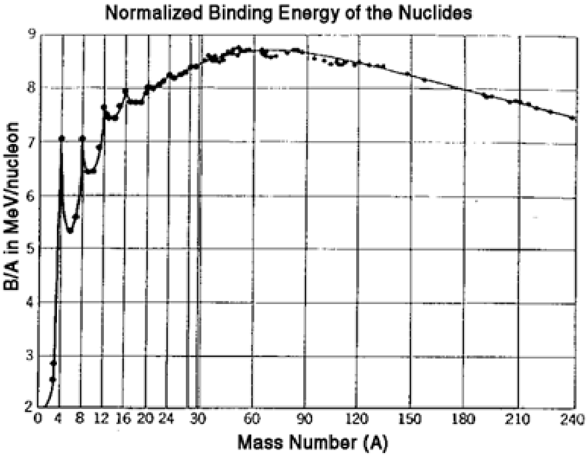
  $$E = mc^2 = 1\,g \times \left(3\times10^8\,m/s\right)^2 \approx 10^{14}\,J \approx 2\times10^{13}\,cal$$

  The nuclear mass of hydrogen (i.e., a proton) is 1.007277 AMU. Combining four hydrogen atoms together to make a helium nucleus, it weighs 4.00150 AMU, rather than 4.0291 AM. The whole is lighter than the sum of the parts. This difference is liberated in the form of **fusion energy**.

  $$\Delta m = 0.02761 AMU \approx 10^{-12}\,cal$$

  That doesn't sound like much, but converting 1 gram of H_2 in this fashion yields a respectable 10^{11} calories.
- **Nuclear stability and mass are related**. More stable nuclei are heavier (binding energy is converted to mass) $E=mc^2$ so that 1 A.M.U. = 930 MeV. That is, 1 MeV is about 0.1% of an A.M.U.

- You can calculate the total binding energy of the nucleus from its mass (using $E=MC^2$) and divide by the number of nucleons in the nucleus (neutrons and protons) to produce an **average binding energy vs mass curve**. The higher the number, the more stable the nucleus is.

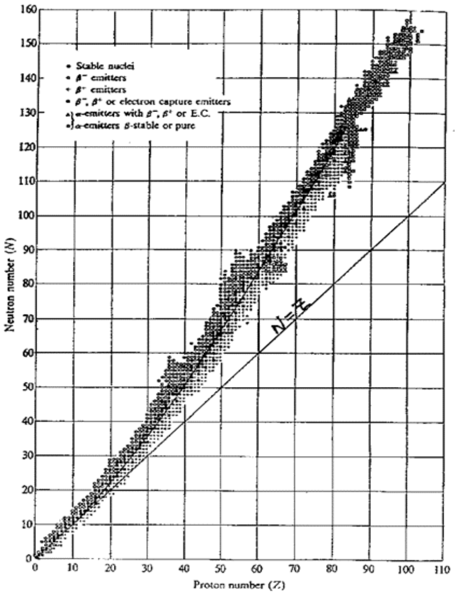


  - There are several **factors affecting nuclear stability**, many of which are reflected in the abundance curve:
    1. attraction between nucleons, the more the merrier, so B.E. increases with A initially
    2. spin pairing (odd/even) produces zig-zag in B.E., and in cosmic abundances. Also, the abundance of stable isotopes favours even numbers:

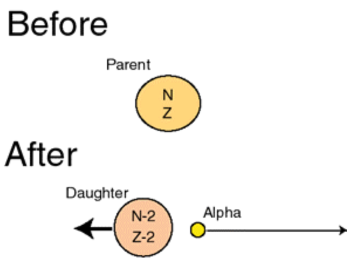
# Protons (Z)	# Neutrons (N)	Atomic Number (A)	# of Stable Isotopes
Even	Even	Even	156
Even	Odd	Odd	50
Odd	Even	Odd	48
Odd	Odd	Even	5

    3. shell structure (magic numbers) shows preferred numbers (2,8,20,28,50,82, 126) with very high abundances for isotopes or elements with N or Z equal to those closed shell numbers.
    4. surface tension (unrequited bonds): smaller nuclei are less stable because they have high surface/volume ratios.
    5. coulomb repulsion works against larger nuclei, because you pay a price stuffing charge into a small volume. Although you always gain by adding more nucleons, you have to add protons and neutrons in *roughly* equal proportions. The coulomb repulsion gives an ultimate limit to nuclear size.

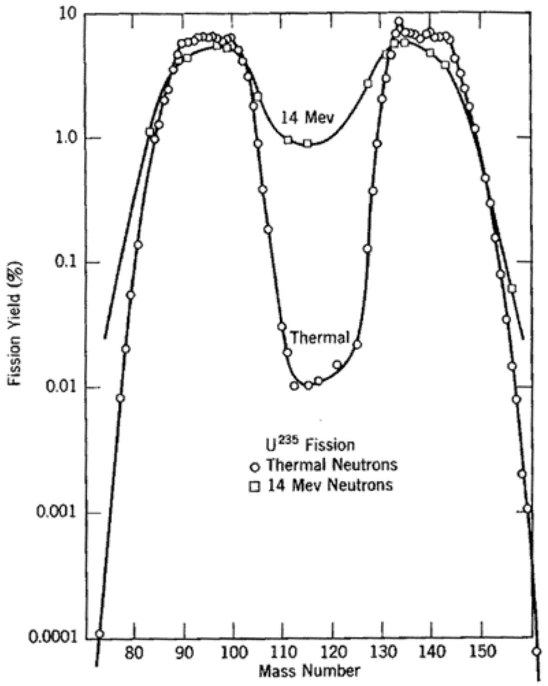
    The binding energy curve peaks at Fe, Ni region: these are the most stable nuclei.

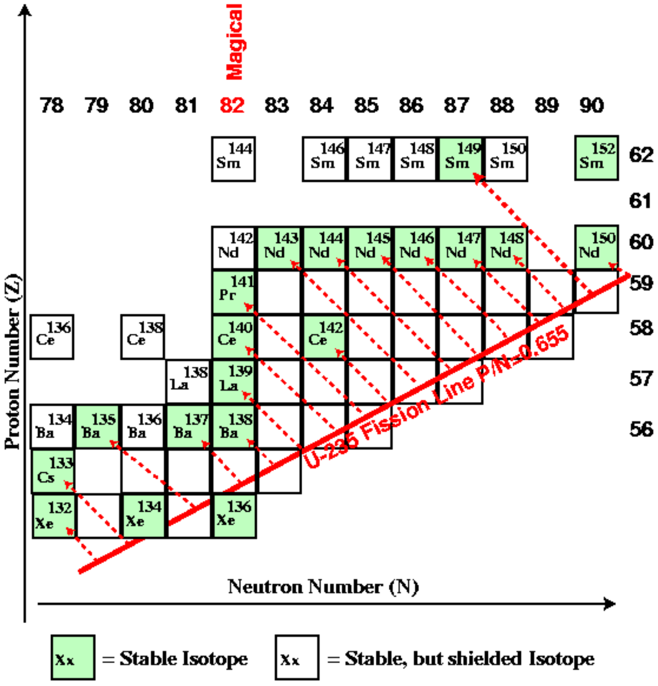


  - **Neutrons, protons and isotopes**
    Nuclei consist of a mix of neutrons and protons, roughly in equal proportions. There exists a valley of stability in the N vs. Z plot which drifts away from N=Z toward N>Z at higher masses. In general, the further away from this valley you get, the more unstable you become, and the shorter your half-life against decay.
  - **Radioactive decay (alpha, beta, gamma, & fission)**
    The general form of a radioactive decay is that a parent nucleus transforms to a daughter with the emission or adsorption of one or more particles. All decays are quantum-mechanical transitions, and are thus characterized by a unique single energy. If there are only two daughter particles, then each will have a single energy. There is a general inverse relationship between the energy of decay, and the half-life. There are four basic kinds of radioactive decay, used to transition to lower energy (more stable states):



1. **Alpha decay** (alpha particles emitted only from the heaviest nuclei), dA=-4, dZ=-2. It is mono-energetic (although a nucleus could decay by a number of different alpha decays). Involves the strong nuclear force. There is a general inverse relationship between energy of decay and half-life. For an approximately factor of 3 range in energy, there is a 24 order of magnitude range (related to quantum-mechanical tunneling). Usually accompanied by a gamma decay when the daughter nucleus is created in an excited state. Even though an alpha decay has only a single energy, a parent nucleus can decay to a number of ground states in the daughter nucleus.
2. **Beta decay** (electrons/positrons) ejected from nuclei, dA=0, dZ=+-1. It is like a proton becomes a neutron, or vice versa. That is, the atomic number of the nucleus (number of protons plus neutrons) does not change. Beta decay involves the weak nuclear force. It is not mono-energetic, because it involves more than two particles, so there is an infinite number of ways that energy and momentum can be shared among the products. The third particle is a neutrino, which is very weakly interacting, has no charge, and probably has no mass. Despite its weak behaviour, neutrinos are very important particles for cosmology, and are the reason stars die. There is an inverse relationship between beta decay half life and energy, but has much more complex rules of decay associated with spin/parity differences between parent and daughter wave functions.
3. **Gamma decay** (photons) no isotope change, dA=0, dZ=0. It is the relaxation of excited state in the nucleus, usually as a de-excitation after another nuclear decay (e.g., alpha decay). It is mono-energetic (a quantum state change). The character of the emission (electric or magnetic, dipole or quadrupole, etc) is determined by the spin and parity differences between the initial and final wave functions. These rules also play a role in determining the half-life, in addition to the inverse relationship between half-life and energy. Unlike beta and alpha particles, which interact with matter very strongly because they are charged, photons are much more penetrating, and although they are not responsible for isotope changes, are often signatures of such changes.
4. **Fission** is the splitting of largest nuclei due to coulomb repulsion. This generally happens in nature very slowly, with half-lives usually many orders of magnitude greater than the age of the universe. However certain nuclei are susceptible to induced fission (usually from slow neutrons): e.g. ^{235}U. The nucleus, which behaves very much as a liquid drop, fragments asymmetrically, producing a large and a



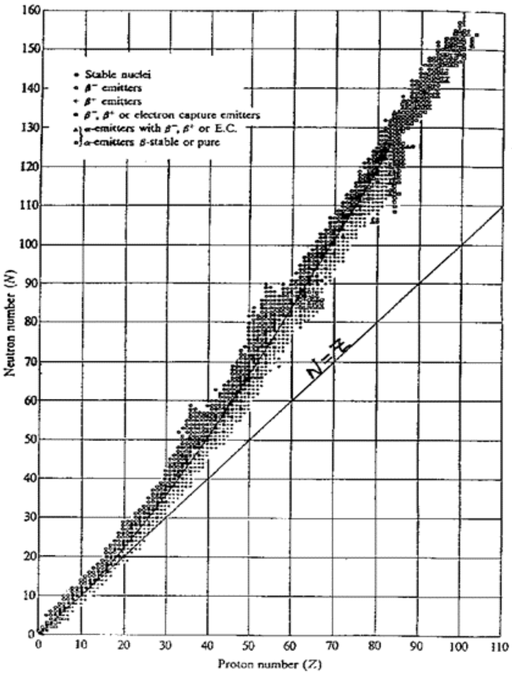


small fragment. The split is statistical, never happening the same way. A two-humped mass distribution of fission products is produced. For U, the humps are centred around masses 90 and 130. Also, because the neutron/proton ratio is different at the high mass end, then the fragments are *off the line of stability*, and are therefore subject to beta decay. These beta decays produce characteristic isotope signatures, because some isotopes are "shielded" from production by isotopes in the way.

Isotopes and radioactive decay

Atoms consist of a central core or "**nucleus**" which contains the bulk of the atomic mass, surrounded by a cloud of electrons. Chemical reactions involve the outer cloud of electrons. Unionised atoms are electrically neutral, with the outer, negative electron cloud perfectly balanced by the inner, positive nucleus. The nucleus consists of an *approximately* equal number of neutrons and protons, collectively referred to as "nucleons". Neutrons have a similar mass to protons, but have no charge. An unionised atom has an equal number of electrons and protons. The number of protons determines what *element* an atom is. For a given element (number of protons), the number of neutrons may vary, and are called ***isotopes***. Not all combinations of neutrons and protons are "allowed", and only a few are ***stable***. The rest "fall apart" or "decay" more or less quickly, depending on how unstable they are. For example,

- Carbon has 2 stable isotopes
  - ^{12}C is more abundant (6 protons, 6 neutrons)
  - ^{13}C is less abundant (6 protons, 7 neutrons)
- Oxygen has 3 stable isotopes
  - ^{16}O is most abundant (8 protons, 8 neutrons)
  - ^{17}O is least abundant (8 protons, 9 neutrons)
  - ^{18}O is intermediate (8 protons, 10 neutrons)
- Fluorine has only 1 stable isotope (^{19}F)
- Tin has 10 stable isotopes



A general rule is there is approximately equal numbers of neutrons and protons, but as you go to larger masses, you require more neutrons than protons for stability.

Nuclear stability is governed by a combination of quantum mechanical rules, nuclear forces, and electrostatic charge. When these are violated, the nucleus is unstable. An unstable nucleus will decay to another isotope. The further it is away the "happy line", the faster it will decay.

The rate at which a nucleus decays is fixed by its internal properties, and is not affected by biological, chemical or physical conditions.

There are three basic modes of radioactive decay. They are

Alpha Decay: Emission of an alpha particle (bare ^{4}He nucleus), done by heavy nuclei only

$${}^{A+N}_{A}P \xrightarrow{\lambda} {}^{A+N-2}_{A-2}D + {}^{4}_{2}He$$

Beta Decay: Emission of an electron, a positron, or electron capture, done by many nuclei

$${}^{A+N}_{A}P \xrightarrow{\lambda} {}^{A+N}_{A+1}D + e^{-} \qquad {}^{A+N}_{A}P \xrightarrow{\lambda} {}^{A+N}_{A-1}D + e^{+} \qquad {}^{A+N}_{A}P + e^{-} \xrightarrow{\lambda} {}^{A+N}_{A-1}D$$

Gamma Decay: Emission of a gamma ray (photon), usually as a result of alpha or beta decay.

The first one results in a change of atomic mass (the total number of neutrons and protons), the latter two do not. The first two result in a change in the element (i.e., number of protons in the nucleus). The last one does not.

The Radioactive Decay Equations

$$\frac{dN}{dt} = -\lambda N \qquad N = N_0 e^{-\lambda t}$$

Activity Definition

$$A = \lambda C$$

Radioactive dating

Radioactive decay rates are unaffected by physical, chemical or biological environmental factors[1]. This constancy makes radioactivity a powerful and reliable tool for timing geochemical events and rates.

Nature provides us with a variety of radioisotopes that we can use to study nature. These radioisotopes are either

- "primordial" (inherited from the material that makes up the earth)
- "nucleogenic" (produced directly or indirectly by the decay of other isotopes)
- "cosmogenic" (produced by cosmic rays bombarding the earth)
- "anthropogenic" (produced by nuclear bombs, accelerators, or reactors)

Primordial radioisotopes obviously will have very long half-lives, generally in excess of many millions of years.

There are three basic approaches to dating. All make use of the immutability of the radioactive decay constant, and rely on the radioactive decay equation somehow.

Simple Dating: Simple dating requires that you know what the original concentration, activity, or isotopic ratio of the radioactive species was. Thus, if you have the radioactive decay equation:

$$N = N_0 e^{-\lambda t}$$

By dividing both sides by N_0 and taking the natural logarithm of both sides, you get

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

where we have used the fact that the natural log of e^x is just x. And noting that $\ln(x) = -\ln\left(\frac{1}{x}\right)$ we finally get

$$\underline{t = \lambda^{-1} \ln\left(\frac{N_0}{N}\right)}$$

In the above equation, N can be either the number of radioactive atoms, its concentration, or the isotopic ratio of the radioisotope relative to a stable isotope of the same element.

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[1] This is because the nuclear forces are much, much stronger than the environmental forces that we are accustomed to. For example, the force that binds protons and neutrons in the nucleus is 10^{39} times stronger than gravity!

The most famous example of simple dating is ***Radiocarbon Dating***. This involves the use of natural radiocarbon (^{14}C), which has a half-life of 5730 years and is produced in the atmosphere by cosmic rays. The bombardment of the atmosphere by cosmic rays maintains a global inventory of about 80 tonnes of natural radiocarbon. This radiocarbon is distributed between the atmosphere, the ocean, and the land biosphere. Once a sample (for example a fossil bone, or a water mass) is isolated from the modern carbon pool, the radiocarbon decays. We normally use the ratio of radiocarbon to normal carbon (^{12}C) to account for chemical or biological processes that move carbon around.

Consider the radiocarbon age of the deep ocean. The average radiocarbon isotope ratio (^{14}C/^{12}C) for the deep ocean is about 83% of the atmospheric ratio. Using our knowledge of the half-life, and the simple dating equation, we obtain

$$t = 8267 \ln\left(\frac{1}{0.83}\right) = 1540\, years$$

This is about the right time-scale for the grand oceanic overturning, but seems just a little long. There is a reason for this, and it's related to the very slow gas exchange rate for carbon isotopes. But if you want the full answer, you'll have to wait for a later lecture!

Parent-Daughter Dating: When you don't know what the initial radioisotope activity should be, either due to spatial or temporal variations in the isotope, things become more difficult. However, when a radioisotope decays to a stable isotope, and that isotope is retained in the "sample", we can reconstruct what the initial activity or concentration was. If we have the parent (*P*) decaying to the daughter (*D*)

$$P \xrightarrow{\lambda} D$$

we rewrite the radioactive decay equation as

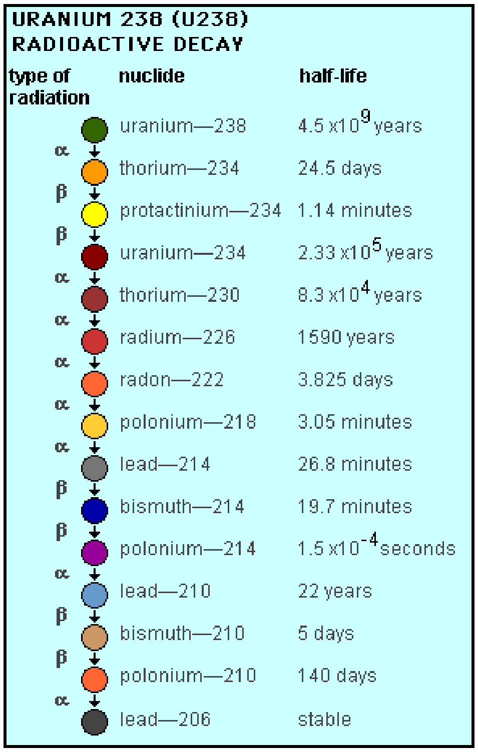
$$P = P_0 e^{-\lambda t}$$

but since the daughter is retained, then it is just the *difference* between the initial and final parent populations

$$D = P_0 - P$$
$$P_0 = P + D$$
$$t = \frac{1}{\lambda}\ln\left(\frac{P_0}{P}\right) = \frac{1}{\lambda}\ln\left(\frac{P+D}{P}\right) = \frac{1}{\lambda}\ln\left(1+\frac{D}{P}\right)$$

Note that although this eliminates the need for knowing the initial parent radioisotope, you must assume that no biogeochemical processes have acted to separate the daughter and parent during this "aging" process. That is, the system is assumed to be "closed".

Secular Equilibrium and Disequilibrium Dating: The heaviest radioisotopes typically do not decay to stable isotopes right away. They generally decay to other radioisotopes that in turn decay, forming a decay chain that ultimately ends up with stable isotopes. All the natural decay chains (there are 4 of them) lead to Pb isotopes.



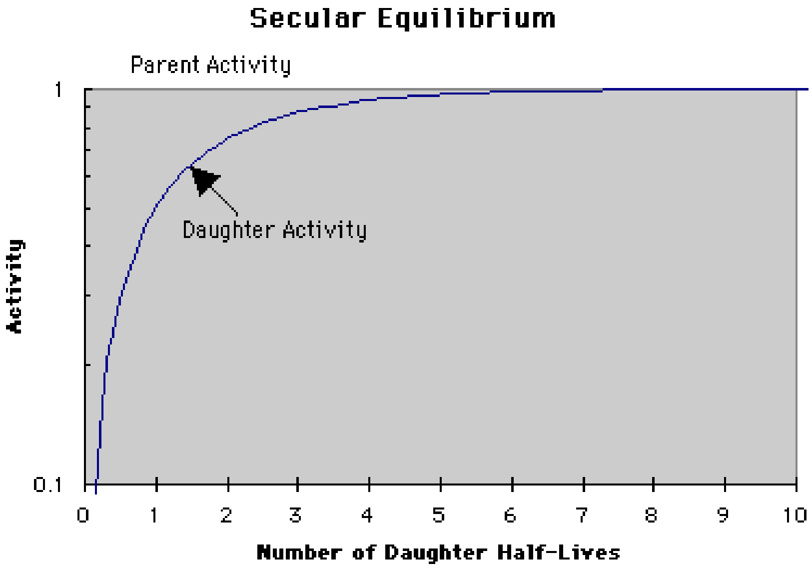
URANIUM 238 (U238)
RADIOACTIVE DECAY

type of radiation	nuclide	half-life
	uranium—238	4.5 x10^9 years
α		
	thorium—234	24.5 days
β		
	protactinium—234	1.14 minutes
β		
	uranium—234	2.33 x10^5 years
α		
	thorium—230	8.3 x10^4 years
α		
	radium—226	1590 years
α		
	radon—222	3.825 days
α		
	polonium—218	3.05 minutes
α		
	lead—214	26.8 minutes
β		
	bismuth—214	19.7 minutes
β		
	polonium—214	1.5 x10^{-4} seconds
α		
	lead—210	22 years
β		
	bismuth—210	5 days
β		
	polonium—210	140 days
α		
	lead—206	stable

Given enough time, these chains reach “secular equilibrium”. That is, for a given radioisotope in the middle of the chain, its concentration (and hence activity) will evolve until the rate at which it is being produced (from the decay of its parent radioisotope) exactly matches the rate at which it is decaying. That is, the abundances of the isotopes will adjust so that

$$A_{daughter} = A_{parent}$$

Now the important point to note is that with any radioactive decay (either α or β) the radioisotope changes its number of protons, and hence becomes another element. This may result in a fundamental change in the isotope’s chemical or biological behaviour. Such changes can lead to a susceptibility to chemical or biological fractionation which can drive the system away from secular disequilibrium.



There are two basic tools that arise from this. The first is that an event, such as formation of magma from a parent igneous rock, or crystallisation of a substance from solution results in a sudden generation of disequilibrium that will then tend back toward secular equilibrium. The equations describing this relaxation are complicated and beyond the scope of this course, but they can be used to determine how long it has been since the event has occurred.

The second involves the fact that the chain may be held out of equilibrium by a competing chemical or biological process. We can then use the immutable radioactive decay constants to determine the rate of the process responsible. A classic example of this is U-Th disequilibrium in the ocean.

Uranium is homogeneously distributed in the ocean, because it is made soluble by carbonate complexation. However, ^{238}U is radioactive, with a half-life of 4.5 x 10^9 years. It decays to ^{234}Th, which has a half-life of about 24 days. Ordinarily, the two should rapidly approach secular equilibrium, with

$$A_{234} = A_{238}$$

However, unlike its parent, ^{234}Th (and indeed Thorium as an element) is extremely particle reactive, and adsorbs rapidly onto biologically produced particles in the ocean. The net result is that the A_{234} is generally *less* than A_{238} because there are two pathways of losing ^{234}Th from seawater: radioactive decay, and particle adsorption. By balancing production and loss, we have:

$$production = loss$$
$$A_{238} = \lambda C_{234} + kC_{234}$$

where the first term on the RHS is the radioactive decay, and the second is the adsorption rate (here assumed to be proportional to the concentration of the Thorium in seawater). The first term is the ^{234}Th activity, and using its definition, we can get

$$k_{adsorption} = \left(\frac{A_{238}}{A_{234}} - 1 \right) \lambda$$

If you think about this equation, it should make sense.