

# Massive iceberg discharges as triggers for global climate change

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**Observations of large and abrupt climate changes recorded in Greenland ice cores have spurred a search for clues to their cause. This search has revealed that at six times during the last glaciation, huge armadas of icebergs launched from Canada spread across the northern Atlantic Ocean, each triggering a climate response of global extent.**

IN 1988 Hartmut Heinrich published an article summarizing his study of a curious set of sedimentary layers in cores from the Dreizack seamounts in the eastern North Atlantic<sup>1</sup>. His conclusion was that these layers record the melting of six great armadas of icebergs which flooded the northern Atlantic during the last glaciation. Each of these bursts produced a prominent sediment layer rich in Canadian-derived ice-rafted debris and poor in foraminifera shells. When Heinrich's paper was published, the attention of the palaeoclimate community was directed towards the climate response to the Earth's orbital cycles, so his discovery went largely unnoticed. Only when a nearly identical record was published<sup>2</sup> from a core located several hundred kilometres away, with a claim that the fresh water generated by the melting of these ice armadas might have disrupted deep-water formation in the northern Atlantic, did the palaeoclimate community recognize the possibility that Heinrich's layers were more than a curiosity. Suddenly attention was refocused on the coupling between surging ice sheets and northward heat transport by the Atlantic's thermohaline circulation.

It was already known from the Greenland ice cores<sup>44,45</sup> that during the last glacial period the northern Atlantic region experienced repeated large and abrupt climate changes. So far only one mechanism—changes in ocean circulation—has been put forward to explain these jumps. The idea behind this mechanism is that high-latitude climate is strongly influenced by heat released from the sea. The amounts and routings of this heat are closely tied to the global pattern of thermohaline circulation. One such route, a conveyor-like circulation operating in the Atlantic, is particularly important. The heat it carries maintains the anomalously warm climate enjoyed by western Europe. The abrupt climate changes revealed in the Greenland record seem to be telling us that the conveyor has turned on and off. Modelling efforts reveal that such changes can be triggered by inputs of fresh water. One potential source of fresh water is that released by the melting of icebergs, such as the discharges recorded in Heinrich's layers (now known as Heinrich events).

Here I review the evidence that Heinrich events are indeed connected with rapid climate variations in the North Atlantic region. The timing of the events is in striking coincidence with the pattern of climate fluctuations documented from ice cores. But the mechanism that drives the events remains a matter of debate. One possibility is that the iceberg discharges reflect an internal oscillatory feature of the dynamics of the Laurentide ice sheet, from which the icebergs come. Alternatively, Heinrich events might represent a response to, rather than a cause of, climate change—external factors may induce global cooling, causing the ice sheets to surge. Although the true picture is likely to emerge only when our understanding of ocean circulation, ice-sheet dynamics and detailed regional palaeoclimate has improved substantially, there can be no doubt that Heinrich events provide a vivid illustration of the way in which the cou-

pled influence of the atmosphere, geosphere and cryosphere may be a dominant control on the Earth's climate.

## Anatomy of Heinrich layers

Although glacial sediments of the northern Atlantic are uniformly rich in ice-rafted debris, that contained in Heinrich layers differs in three distinct ways. First, 20% of the sand-sized fragments are detrital limestone (a constituent nearly absent in ambient glacial sediment)<sup>3</sup>. Second, the clay-sized minerals in Heinrich layers have twofold higher K–Ar ages (~1,000 million years) than those for their ambient glacial equivalents<sup>4,6</sup>. Third, Heinrich layers are devoid of the basal-derived clay minerals so abundant in ambient glacial sediment<sup>4,6</sup>. Coupled with the observation that the detrital layers thin by more than an order of magnitude from the Labrador Sea to the European end of the 46° N iceberg route<sup>3</sup>, these composition differences point strongly to a Canadian origin for the Heinrich ice armadas.

Surprisingly, the dominant feature of Heinrich layers is not a greatly enhanced abundance of ice-rafted debris, but rather a dearth of foraminiferal shells<sup>2,3</sup>. The abundance of these shells drops from its usual glacial value of thousands per gram to hundreds or less per gram in the Heinrich layers. Dilution with ice-rafted debris cannot be the sole cause of this reduction because the geographical extent of sediment sparse in foraminifera exceeds that of the Canada-derived detritus<sup>2,3</sup>. Rather, the low abundance of foraminifera must reflect, in part, a dramatic decrease in marine productivity. Heinrich events occurred when the northern Atlantic was at its coldest, as signalled by the geographical extent of the polar foraminifera, *Neogloboquadrina pachyderma* (left coiling) which extended all the way to 40° N. The depleted <sup>18</sup>O content in the few foraminifera present in Heinrich layers suggests the presence of a low-salinity lid<sup>3</sup>. These observations suggest episodes of extensive sea-ice cover similar to that of today's Arctic.

## Timing of Heinrich events

Perhaps the most startling aspect of the timing of these events is their occurrence at major climate boundaries<sup>7</sup>. As shown in Fig. 1, Heinrich event number 6 marks the transition from the relatively warm northern Atlantic of the last interglacial (that is, marine stage 5) to the cold conditions prevailing during the last glacial (marine stages 4, 3 and 2). Heinrich event 1 marks the onset of the termination which brought the last glacial to a close. There is unpublished evidence (J. McManus, personal communication) of a seventh Heinrich event right at termination of the penultimate glaciation (that is, marine stage 6). The coincidence between Heinrich events and the major changes in the temperature regimen of the northern Atlantic leads to the speculation that fresh water released as the result of the melting of Heinrich's icebergs disrupted deep-water formation, thereby permitting switches between glacial and interglacial modes of thermohaline circulation.



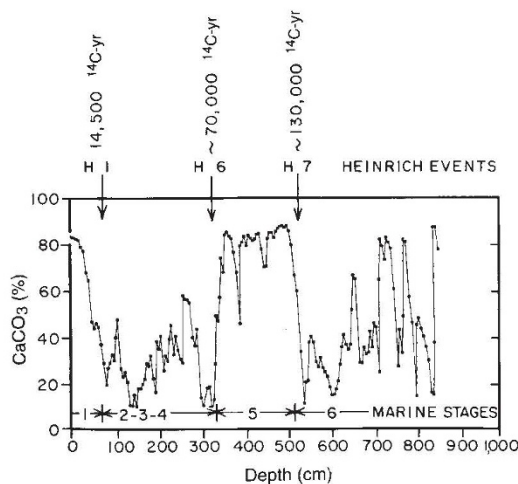


FIG. 1 Depth dependence of CaCO<sub>3</sub> content in core V28-82 from the northern Atlantic (50° N, 24° W) showing an alternation between coccolith-rich interglacial sediment (stages 1 and 5) and ice-rafted debris-rich glacial sediment (stages 2-4 and 6). Heinrich layers mark both the transitions from glacial to interglacial conditions and from interglacial to glacial conditions<sup>7</sup>.

Another intriguing feature of the timing of the Heinrich events is their spacing at intervals of roughly 10,000 years (ref. 8). This spacing suggests forcing associated with the alternation of the relative strength of the seasonality in the two polar hemispheres brought about by the Earth's precession. Perhaps this alternation influences the dynamics of the tropical atmosphere in such a way that the water-vapour burden of the temperate latitudes is changed. But the significance of this observation is not clear, given that the spacing between Heinrich events decreases through the course of the last glacial period from about 13,000 years to about 7,000 years.

**Relationship to Dansgaard-Oeschger cycles**

The glacial portion of the Greenland ice-core record is dominated by rectangular climate cycles averaging a few thousand years in duration<sup>9-11</sup>. These so-called Dansgaard-Oeschger (D-O) cycles are characterized by abrupt jumps in temperature, dust content, ice accumulation rate, and concentration of methane and perhaps carbon dioxide. It is puzzling that Heinrich events are not prominent in this record. But as shown by Bond *et al.*<sup>12</sup>, there is a tie between Heinrich events and D-O cycles. Bond *et al.* have shown that the ice armadas were launched during the most intense cold phase of a package of several D-O cycles, and that each Heinrich event was followed by a prominent warming which initiated a new package of D-O cycles (see Fig. 2). This bundling of progressing colder D-O cycles into subsets gives rise to what has come to be called Bond cycles.

The last of the D-O cold events is the Younger Dryas (YD). Its abrupt end ushered in the Holocene, a period of stable warm climate which has now lasted for about 11,500 calendar years (10,000 <sup>14</sup>C-years). The climate signal of the YD can be seen clearly in records from throughout the northern Atlantic basin. Bond's discovery<sup>3</sup> of detrital carbonate in marine sediments of YD age from the northern Atlantic suggests that it may also have an affinity to the Heinrich events.

**The global imprint**

Evidence is rapidly accumulating that D-O cycles, Bond cycles and the YD all have global signatures (Fig. 3). Analyses of the air trapped in Greenland ice have revealed minima in methane concentrations associated with the cold phases of D-O cycles (including the YD event)<sup>13</sup>. As the major source for this gas during glacial time is thought to have been tropical wetlands, climate effects not directly attributable to changes in the north-

ward transport of heat by the Atlantic's conveyor must have been operative. Wet events in the pollen record from Lake Tulane in Florida appear to correlate with Heinrich events<sup>14</sup>. At least four maxima of the extent of Andean mountain glaciers in the lowlands of Chile have been identified by Denton and colleagues (personal communication). Radiocarbon ages for the last three of these maxima correspond within a few hundred years to the times of Heinrich events 3, 2 and 1. Denton and Hendy have also convincingly documented that mountain glaciers in the Alps of New Zealand reached an intermediate maximum precisely at the time of the onset of the YD<sup>15</sup>. Evidence of the YD has also been reported in foraminiferal records from the Sulu Sea<sup>16</sup>, pollen records from British Columbia<sup>17</sup>, lake levels in Africa<sup>18</sup> and oxygen and hydrogen isotope ratios in Antarctic ice<sup>19</sup>. These data are summarized in ref. 20. So no longer can it be assumed that the influence of these events was restricted to the northern Atlantic basin, weakening the case that they are tied exclusively to the heat release to the atmosphere in association with the formation of deep water in the northern Atlantic<sup>21</sup>.

**The trans-US wet event**

A possibly important clue regarding the role of Heinrich armadas in perturbing ocean circulation comes from the sequence of events following the last of the six Heinrich outbursts. It seems that close to the time of this event, ice caps throughout the world reached prominent maxima<sup>22</sup>. Then shortly after Heinrich event 1, these glaciers went into rapid retreat (Fig. 4). This retreat is heralded by the basal sediments of the numerous small lakes in Switzerland's major Alpine valleys which document that by about 14,000 <sup>14</sup>C-years ago the valleys had become ice-free<sup>23</sup>. However, pollen in the sediments of these lakes reveals that conditions remained sufficiently cold to suppress the re-appearance of trees<sup>24</sup>. Then at about 12,700 <sup>14</sup>C-years ago, a sudden warming occurred, not only in Switzerland, but throughout the northern Atlantic basin. Climate proxies from oxygen isotope<sup>25,26</sup>, beetle<sup>27</sup> and pollen<sup>26</sup> records show that within a period of a few decades, warm climates prevailed. This warming has been widely attributed to a turn-on of the Atlantic's thermohaline circulation<sup>21</sup>.

But this raises a difficult question. What transpired during the 1,400 or so years between the launch of Heinrich armada no. 1

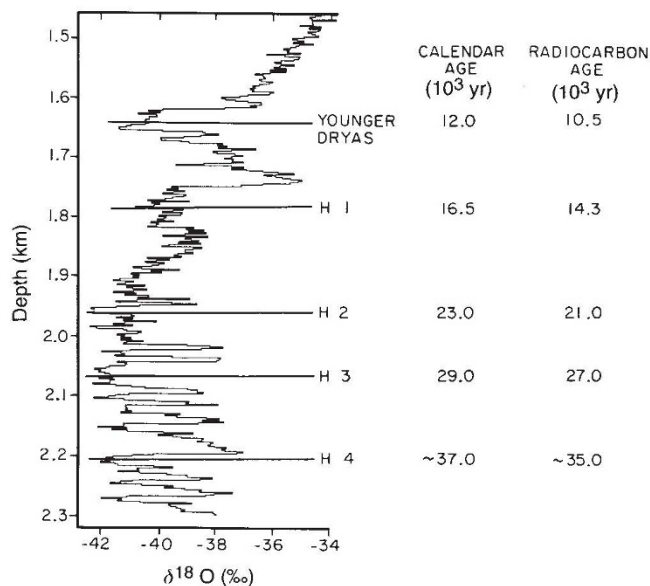


FIG. 2 Bond's placement of Heinrich events<sup>11</sup> in the Summit Greenland GRIP ice-core oxygen-isotope record<sup>9</sup>. Heinrich events occur in the last cold phase of a series of Dansgaard-Oeschger cycles and precede a major interstitial warm pulse<sup>11</sup>. ( $\delta^{18}\text{O}$  is the relative deviation of the <sup>18</sup>O/<sup>16</sup>O ratio in a sample from that in standard mean ocean water.)



and the resumption of thermohaline circulation? A major clue comes from the global pattern of precipitation. Closed basin Lake Lahontan in the Great Basin of the western United States achieved its maximum size (Fig. 5) during this time interval and then underwent an abrupt major dessiccation just after 13,000 <sup>14</sup>C-years ago<sup>28,29</sup>. As suggested by the pollen records from Lake Tulane, Florida<sup>14</sup>, and from Brown's Pond, Virginia<sup>30</sup>, the southeastern United States was also unusually wet between 14,000 and 13,000 <sup>14</sup>C-years ago. This pattern of excess precipitation is similar to that experienced during El Niño periods<sup>31</sup>.

Ocean models offer a possible explanation for this curious interval. They show that sudden freshwater releases into polar regions can easily disrupt the existing pattern of thermohaline circulation. Whereas in some situations, the ocean immediately switches to an alternate thermohaline mode<sup>32-34</sup>, in others, the model ocean lapses into what Manabe calls a "drop dead" mode with no deep-water formation<sup>35,36</sup>. In either case, after a thousand or so years, the model's conveyor circulation can suddenly resume. Thus it is tempting to speculate that the interval between ~14,000 and ~12,600 <sup>14</sup>C-years ago was a "drop dead" ocean circulation phase which ended with a sudden rejuvenation of the Atlantic's conveyor. Of course, this line of speculation carries with it the implication that a lapse in deep-water formation somehow gives rise to wet events in North America akin to those characterizing El Niño intervals<sup>31</sup>. The nature of this connection remains a mystery.

**Surges or fast-flowing ice streams?**

The most obvious explanation for Heinrich events is that they result from surges of the ice stream that drained the Hudson

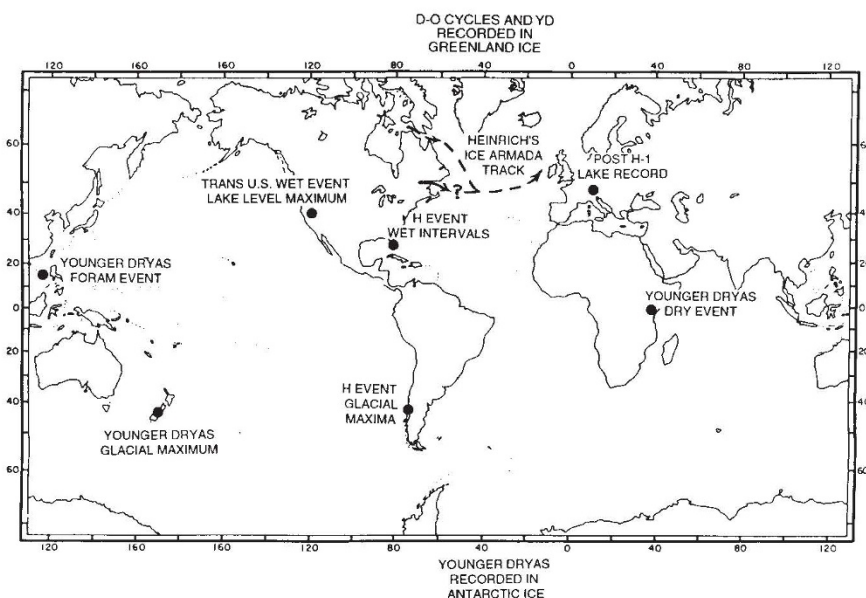


FIG. 3 Map showing the location of radiocarbon-dated records which, taken together, demonstrate the global signature of Heinrich events (H event) and the related Dansgaard-Oeschger (D-O), Bond cycles and the Younger Dryas.

Bay portion of the Laurentian ice sheet. Debris-laden basal ice issued forth from the Hudson Straits and gradually melted as it was carried by the prevailing currents across the Atlantic (along a track 10° wide in latitude centred at 46° N). Such an origin can adequately account for all we currently know about Heinrich layers. MacAyeal<sup>37,38</sup> has proposed a binge-and-purge model for the Hudson Bay lobe of the Laurentian ice sheet to account for these surges. He and Alley have shown that this model has the added attraction of explaining how the large amount (0.1–1.0 km<sup>3</sup>) of debris making up the Heinrich layers became frozen into the cap's basal ice<sup>39,40</sup>.

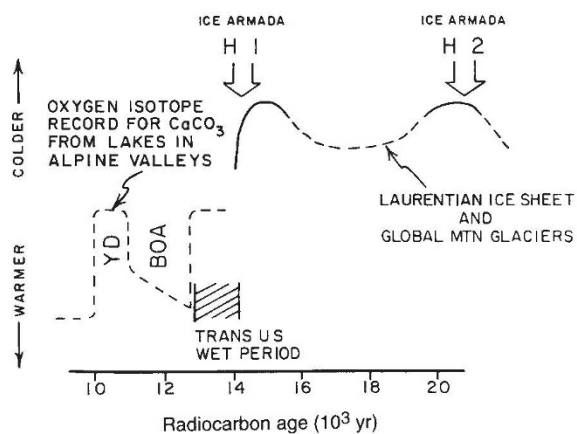


FIG. 4 Events of the last deglaciation: glaciers throughout the world achieved maxima close to 20,000 and to 14,500 radiocarbon-years (<sup>14</sup>C-yr) ago. Based on the radiocarbon dates in hand, Heinrich armadas of icebergs were launched into the Atlantic close to the times of these glacial maxima. Following the second glacial maximum, the world's glaciers began a rapid retreat. In Europe this retreat opened the major Alpine valleys by 14,000 <sup>14</sup>C-yr ago allowing lakes to form. However, ~1,400 years passed before a second phase of warming occurred which allowed trees to colonize these valleys. This warming, which marked the onset of the northern Atlantic basin's Bolling-Allerod (BOA) warm period, occurred abruptly. The interval between the first Heinrich event (H 1) and the onset of the BOA corresponds to the time of an extreme wet interval across the southern tier of the United States (trans-US wet period).

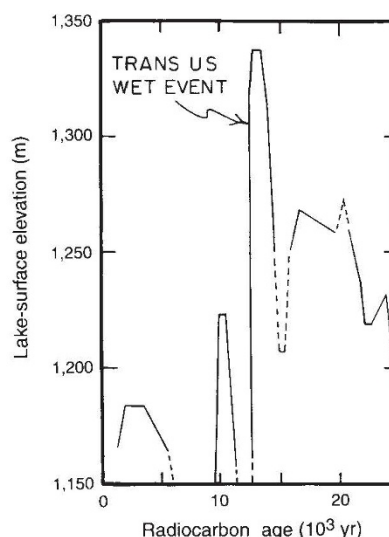


FIG. 5 The level of Lake Lahontan in the Great Basin of the western United States reached a brief late-Quaternary maximum, which dates between 12,700 and 14,500 <sup>14</sup>C-years ago (that is, 14,900–16,700 calendar years ago)<sup>28</sup>. This age range is based on <sup>14</sup>C-ages for the organic material beneath desert varnish on shoreline cobbles<sup>28</sup>, <sup>14</sup>C-ages for algal carbonate shoreline deposits<sup>28,29</sup> and <sup>230</sup>Th–<sup>234</sup>U isochron ages on the same algal carbonates<sup>29</sup>.



But the binge-and-purge hypothesis suffers from one major drawback—the timing of the surges would be controlled by internal properties of the ice sheet and not by external climatic forcing. If so, it is not so easy to understand what ties the advances of the Chilean mountain glaciers to the surges of the Laurentian ice sheet. As already stated, the two records are as near to synchronous as can be documented by existing radiocarbon measurements (within  $\pm 300$  years). Thus if the binge-and-purge theory is correct, each surge must somehow have sent out a signal to all the world's glaciers telling them to advance. The most likely carrier for this message would be the ocean's thermohaline circulation. Somehow its change would have to have led to a corresponding change in the Earth's temperature (perhaps via a change in the inventory of atmospheric water vapour).

Denton prefers to turn the situation around and invokes global temperature changes to drive both the glacial advances and generation of Heinrich icebergs. He calls on a global cooling to propel the rapid advances which pushed large quantities of Canadian ice into the sea, and allowed the icebergs to be carried far across the sea before melting. A problem with this interpretation is that because the response time of ice sheets to surface forcing is measured in tens of thousands of years<sup>41,42</sup>, a close synchronism with rapidly responding mountain glaciers is not to be expected.

One important testable distinction exists between MacAyeal's and Denton's hypotheses. The former would restrict the release of Heinrich icebergs to the Hudson Straits whereas the latter would have them coming as well from the St Lawrence valley and even New England. There is a possibility that lead isotope measurements on individual feldspar grains, together with K–Ar measurements on individual amphibole grains would allow a determination of the source of Heinrich's icebergs. Their source would be defined unambiguously if they contained exclusively material from the older Precambrian Churchill Province (which underlies Hudson Bay) or if they also contained material from later Precambrian and Palaeozoic igneous bodies located to the south of the Canadian Archaean shield and also in eastern Greenland. Indeed, Gwiazda and Hemming have very recently obtained lead isotope measurements that clearly demonstrate that ice-rafted feldspars from Heinrich layer 2 are exclusively from older Precambrian rocks, whereas their equivalents in ambient glacial age sediment are derived from a wide range of terrains. This finding is consistent with MacAyeal's scenario.

## Summary

So where does this leave us? First, the relationship between Bond cycles and D–O cycles remains a mystery. Bond cycles show up

in the glaciation of mountains in the Southern Hemisphere, pollen records from Florida, and iceberg records from the North Atlantic. Dansgaard–Oeschger cycles are found in global methane levels and in the Greenland ice-core records of temperature and dust. The YD displays characteristics of D–O and Bond cycles. As both phenomena seem to have their roots in the northern Atlantic, it is tempting to suggest the involvement of changes in the mode of thermohaline circulation, driven by the effect of fresh water on the formation of deep water in the northern Atlantic. Perhaps the D–O cycles are caused by a 'salt oscillator' operating in the Atlantic, with Heinrich icebergs disrupting the regular progression of these oscillations. Perhaps the changes in ocean circulation perturb the strength of upwelling in the equatorial zone, affecting the operation of the great tropical convective systems that load the atmosphere with much of its moisture. In this way, the climatic effects of ocean-circulation changes could be carried beyond the northern Atlantic into other parts of the Earth. If so, then one would expect that Denton's mountain glaciation record will eventually reveal lesser advances associated with individual D–O cold phases.

Although this all sounds feasible, I am not convinced. Lurking on the horizon is an as yet untapped source of information—the record of atmospheric <sup>14</sup>C/<sup>12</sup>C ratios. As deep-water formation in the northern Atlantic is presently the dominant conduit for the transfer of <sup>14</sup>C from the upper ocean to the deep sea, any major changes in thermohaline circulation should measurably perturb the distribution of <sup>14</sup>C within the sea and hence <sup>14</sup>C/<sup>12</sup>C ratio in atmospheric CO<sub>2</sub>. So far, we have suitable radiocarbon information (from independently dated tree rings, varved sediments and corals) only for the YD interval. These results clearly eliminate the possibility that thermohaline circulation was shut down during the Younger Dryas. Perhaps instead a shallower conveyor with a source region shifted to lower latitudes operated in its place<sup>43</sup>. In any case the <sup>14</sup>C results loom as dark clouds on the horizon of an ocean-based model. Unfortunately, no atmosphere-based hypothesis has been proposed. Neither models nor theory provide a mechanism whereby the atmosphere on its own could jump from one quasi-stable mode of operation to another.

It must be pointed out that this subject is of more than academic interest. We are currently provoking the Earth's climate with a steady build-up of greenhouse gases. It would be prudent to find out what it might take to kick the system into one of its alternate modes of operation. Could this happen in the absence of a greatly extended ice cover? □

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