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An abyssal recipe

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ABSTRACT

Fine- and microstructure observations indicate bottom-intensified turbulent dissipation above rough bathymetry associated with internal wave breaking. Simple analytic representations for the depth profile of turbulent dissipation are proposed here under the assumption that the near bottom wavefield is dominated by a baroclinic tide. This scheme is intended for use in numerical models and thus captures only the gross features of detailed solutions to the energy balance of the internal wavefield. The possible sensitivity of the magnitude and vertical variability of the dissipation rate profile to various environmental parameters is discussed. An expression for the diapycnal buoyancy flux is presented that explicitly treats the difference between the height of an isopycnal above the mean bottom and the actual bottom. This returns a diapycnal velocity estimate that is consistent with both tracer observations of downwelling and a basin scale mass budget that requires upwelling.

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1. Introduction

The rate of diapycnal mixing relates through the buoyancy equation (McDougall, 1991) and vorticity dynamics (Stommel and Aarons, 1960) to the intensity of upwelling and horizontal circulation in the abyssal ocean. Similarly, the intensity of diapycnal mixing relates to the ability of the abyssal ocean to store heat and carbon. Mixing associated with internal wave breaking is an important, and perhaps even dominant, part of the diapycnal transformation for deep and bottom waters (Polzin et al., 1997; Ledwell et al., 2000). Parameterization of this mixing is therefore a key ingredient to understanding the centennial to millennial time scale variability of the oceans and may play a role on shorter times scales as well.

The observed dramatic enhancement with depth/abrupt decay of turbulent dissipation with height above bottom in vertical profile data obtained during the Brazil Basin Tracer Release Experiment (BBTRE) is linked to a similar enhancement/decay of a bandwidth limited shear spectrum (Polzin, 2004b). The observed near boundary shear spectrum is peaked at vertical wavelengths of about 100 m. Internal waves of this scale do not propagate quickly (the internal wave group velocity is roughly a wavelength in a wave period). Thus the energy of these waves is dissipated near the boundary, reducing the amplitude of the shear spectrum and thereby resulting in a spatial decay of the turbulent dissipation. The abrupt decay of dissipation is simply a signature of the spatial scale of the peak in the shear spectrum. The intent of this work is to turn that insight into a dynamically based parameterization, in contrast to ad hoc parameterizations used in Simmons et al. (2004), Saenko and Merryfield (2005) and Jayne (2009), for example. The key issue is treatment of nonlinearity in the internal wavefield.

There are three distinct regimes in which nonlinearity plays a role in the energy balance of the internal tide. The first dynamical regime is a boundary layer of O(10)'s of meters high in which the nonlinear response includes nonhydrostatic effects (Gemmrich and van Haren, 2001; Aucan et al., 2006). The characterization of this boundary layer as an "internal swash zone" may be an apt metaphor and nonhydrostatic effects could be significant above this boundary layer (Legg and Klymak, 2008). The internal swash zone is not well sampled in the Brazil Basin data set as the free-fall instrumentation was unable to reliably get closer than 50 m from the bottom. The second dynamical regime is a near boundary region of O(500) m extent characterized by overturning directly associated with velocity and density gradients in the semidiurnal tide and hence relatively strong wave-wave interactions. In Polzin (2004b) this near boundary regime was addressed using a local (in vertical wavenumber) flux characterization for nonlinear transfers in a quadratically nonlinear system. Finally, the farfield will be characterized by weak wave-wave interactions. The parametric subharmonic instability is especially pertinent to far field

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dynamics as the semidiurnal lunar (M_2) tide can decay into nearinertial products equatorward of 28.9° latitude (MacKinnon and Winters, 2005; Hibiya et al., 2002). Near-inertial shear will impact the dissipation profile directly through increased shear variance leading to shear instability and indirectly as increased shear variance leading to changes in the decay scale within the strongly nonlinear near boundary region.

Here I follow the lead of Polzin (2004b) and interpret the abrupt decay of the dissipation profile as a strongly nonlinear process in the near-boundary regime. Internal swash zone and weakly nonlinear far field dynamics are both neglected. The rational for doing so is that the strongly nonlinear closure formulated in Polzin (2004a) permits analytic solutions characterizing the interplay of wave propagation and wave dissipation. The resulting solutions are discussed below (Sections 2.1 and 2.2), extended to include effects associated with wave propagation in non-uniform buoyancy profiles (Section 2.3) and related to external environmental parameters such as the barotopic tidal velocity and topographic characteristics (Section 2.4). A full depth specification for the buoyancy flux associated with wave breaking is presented in Section 3.2. Transferring this profile to a model representation that does not fully resolve the bottom topography is addressed in Sections 3.4 and 3.5. A Discussion focuses upon issues of buoyancy forcing and the abyssal Brazil Basin mass budget (Section 4.1) and a possible climate change scenario (Section 4.2). A summary concludes.

2. Ingredients

The key to producing a dynamically based parameterization of the dissipation profile associated with internal wavebreaking is to link the dissipation profile to the finescale internal wave shear producing that dissipation. This is done in Polzin (2004b) by identifying analytic solutions to a radiation balance equation. Those solutions relate a dissipation profile $\epsilon(z)$ given by

$$\epsilon(z) = \frac{\epsilon_0}{\left(1 + z/z_0\right)^2},\tag{1}$$

having magnitude ϵ_0 and scale height z_0 (Fig. 1) to a bandwidth limited vertical wavenumber (m) energy spectrum [E(m)] given by

$$E(m) = \frac{bm_0^2}{m^2} \left(1 - \frac{m_0^2}{m^2} \right),\tag{2}$$

Fig. 2. The analytic expression (1) depicts the turbulent decay of a bandwidth limited finescale internal wavefield (2) propagating away from the bottom boundary. The decrease in dissipation with height above bottom is directly related to the decrease in wave amplitude associated with the dissipation. Given this formulation, a parameterization can be formulated by relating the spectral amplitude *b* and bandwidth m_0 to variability in stratification, topography and tide through models of internal tide generation and wave scattering. A list of these ingredients is provided as Table 1.

2.1. The basic dissipation profile

Despite the idealized nature of the analysis presented in Polzin (2004b), quantitative agreement can be found between a theoretical prediction for the near-bottom profile of turbulent dissipation and dissipation data presented in Polzin et al. (1997) (Fig. 1). A reasonable fit of (1) to the dissipation data can be obtained for $\epsilon_0 = 1 \times 10^{-8}$ W/kg and $z_0 = 150$ m, Fig. 1. The variable z in (1) represents a height above boundary (*hab*) coordinate system of a single profile and Fig. 1 presents an average dissipation profile in this coordinate system. This differs from averaging in a depth or isopycnal coordinate system and this distinction is crucial when



Fig. 1. Dissipation vs. height-above-bottom over rough bathymetry in the Brazil Basin. The dissipation data (thick line) represent an average over the 30 stations which appear in Fig. 3 and east of 18° W in Fig. 2 of Polzin et al. (1997). The thin line represents a fit to the data. See also Figs. 3 and 5.



Fig. 2. Bell's model (the fundamental tone and harmonics are shown as thin lines, their sum is denoted by the thick solid line) and the idealized solution spectra. Note the dominance of the fundamental tide (M_2) in Bell's model. Both sum $[s(m) = E^+(m) + E^-(m)]$ and difference $[d(m)E^+(m) - E^-(m)]$ idealized solution spectra are represented as thick dashed lines. The difference spectrum serves as the bottom boundary condition and thus is to be directly compared to Bell's model, which serves as the source. The sum spectrum (2) is used to set the dissipation profile. The amplitude of the sum spectrum and low wavenumber limit m_o of the idealized solution are indicated in the figure.

attempting a closure for general circulation models. The distinction will be addressed in Section 3.4.

Eqs. (1) and (2) represent a nonlinear propagation model and the effort presented here differs substantially from other use of these data. St. Laurent et al. (2002) fit exponentials to turbulent dissipation data from the BBTRE. They then attempt to extrapolate

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Table 1	
List of ingredients.	

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	П	$(1 - R_f)\epsilon_{ij-\text{total}}$	Total buoyancy flux	(27)
	$\epsilon_{ij ext{-total}}$	$\epsilon_{ii} + \frac{\Delta N_{ij}^2}{\frac{\Delta H_{ij}}{\Delta H_{ij}}}$	Total dissipation	(26)
	P.,	$\int_{0}^{0} N_{ij}^{2} dz$	Total nower input	(13)
	i ij	$\frac{1-P_{ij}}{1-R_{j}}\int_{0}^{y}\epsilon_{ij-total}dz$		(15)
	Δ	$\frac{1-\kappa_{f}}{\rho}P_{ij}-\epsilon_{oij}z_{oij}$	Residual dissipation	(25)
	$\epsilon(z)$	$\frac{\epsilon_o}{(1+z/z_o)^2}$	Basic ϵ profile	(1)
	$\epsilon(z^*)$	$\frac{\epsilon_o N^2(z)/N_o}{(1+z^2/z_o)^2}$	<i>N</i> -scaled ϵ profile	(6) and (11)
	ϵ (Hab)	$\frac{\epsilon_0 z_0^2}{(z_0 + Hab)^2 - 3h^2} Hab > \sqrt{3}h_0$	ϵ_{ij} depth profile	(30)
	ϵ (Hab)	$\frac{\epsilon_o z_o (Hab + \sqrt{3}h_o)}{2\sqrt{3}h_c (z_c + Hab + \sqrt{3}h_c)} 0 < Hab < \sqrt{3}h_o$	ϵ_{ij} depth profile	(30)
	Z	2 (510 (20 + 110 + (510)	Depth	
	Ζ*	$\int_{0}^{z} \frac{N_{ij}^{2}(z')}{N_{ij}^{2}} dz'$	Buoyancy scaled depth	(5) and (12)
	h	h = Hab - hab	Topographic height	-
	hab	h = Hab - hab	Height above local bottom	-
	Hab	h = Hab - hab	Height above mean bottom	-
	ϵ_{oij}	$\epsilon_o \left[\frac{N_{oij}}{N_o}\right]^4 \left[\frac{V_{oij}}{V_o}\right]^{4\nu-2} \left[\frac{l_{oij}h_{oij}}{l_oh_o}\right]^4$	ijth bottom dissipation	(14)
	Z _{oij}	$Z_o \left[\frac{N_{oij}}{N_o} \right]^{-3} \left[\frac{V_{oij}}{V_o} \right]^{3-2\nu} \left[\frac{I_{oij}h_{oij}}{I_o h_o} \right]^{-2}$	ijth dissipation scale height	(15)
	P _{ij}	$Po\left[\frac{V_{oij}}{N_o}\right] \left[\frac{V_{oij}}{V_o}\right]^2 \left[\frac{h_{oij}}{h_o}\right]^2 \left[\frac{l_{oij}}{l_o}\right]$	<i>ij</i> th power input	(16)
	Po	3.7 mW/m ²	Reference power input	(17)
	ϵ_o	$1.0 imes 10^{-8} \text{ W/kg}$	Reference dissipation	(18)
	Z ₀	150 m	Reference dissipation scale height	(3) and (19)
	No	$9.6 \times 10^{-4} \ s^{-1}$	Reference stratification	(20)
	(U_o, V_o)	(0.021,0.025) m/s	Reference barotropic tidal velocities	(21)
	H(k, l)	$H(h_o, k_o, l_o, v)$	Topographic spectrum	(8)
	(k_o, l_o)	$(2.2\times 10^{-4}, 1.0\times 10^{-3})\ m^{-1}$	Reference topographic scales	(23)
	h _o	110 m	Reference rms topographic height	(22)
	v	0.9	Topographic power law	(8)
	mo	$\simeq 2\pi N_o/V_o$	Characteristic vertical wavenumber	(3), (4) and (10)
	ω	$1.4025 \times 10^{-4} \ s^{-1}$	Wave frequency	
	bm_{o}^{2}		Amplitude of gradient spectrum	(3) and (4)
	~			

those curve fits using ideas grounded in linear wave dynamics with no consideration of the nonlinear aspects of the problem. A second point of departure from St. Laurent et al. (2002) is the interpretation of the Brazil Basin data set itself. The average profile defined in Fig. 1 is intended to characterize dissipation above abyssal hills external to deep canyons. This profile differs significantly from the corresponding profile in St. Laurent et al. (2001), their Fig. 3, left most panel. In short, the difference results from St. Laurent et al.'s (2001) use of the Smith–Sandwell bathymetry to characterize the geographic location of the vertical profile (dissipation) data. The Smith–Sandwell data were not adequate for this application and the results were not ground truthed with center-beam data. A detailed discussion is provided in Polzin (2008).

2.2. The relation of the dissipation profile to the finescale internal wavefield

The turbulent dissipation profile (1), Fig. 1, represents the spatial decay of wave energy in a bandwidth-limited finescale internal wavefield (2), Fig. 2, propagating away from the bottom boundary. The analysis presented in Polzin (2004b) relates the spectrum and the dissipation profile as:

$$z_0^{-1} = 2A\alpha\beta(\omega)N^{-2}bm_0^4,\tag{3}$$

$$\epsilon_0 = (1 - R_f) A \phi(\omega) N^{-1} b^2 m_0^4, \tag{4}$$

where the flux Richardson number ($R_f \cong 0.2$) expresses the partitioning of turbulent production into potential energy fluxes and dissipation; α is a nondimensional, O(1) constant which is estimated

numerically to be α = 2.31; *A* = 0.20 is a nondimensional constant expressing the strength of the wave–wave interactions¹ and the factors $\beta(\omega)$ and $\phi(\omega)$ are spectrally weighted versions of

$$\begin{split} \phi(\omega) &= (\omega^2 + f^2) \Big[(\omega^2 - f^2) (N^2 - \omega^2) \Big]^{1/2} \Big/ \Big[\omega^2 (N^2 - f^2) \Big] \quad \text{and} \\ \beta(\omega) &= (\omega^2 + f^2) N \Big/ \Big[\omega \Big[(\omega^2 - f^2) \Big(N^2 - \omega^2 \Big) \Big]^{1/2} \Big]. \end{split}$$

See Polzin (2004b) for further discussion of the frequency domain.

The fit parameters in (1) correspond to a spectral level of $bm_0^2 = 2.1 \times 10^{-5} \text{ s}^{-2}/\text{m}^{-1}$ and vertical wavelength $2\pi/m_0=375\text{m}$ for $\omega = 1.4025 \times 10^{-4} \text{ s}^{-1}$ (an M_2 semi-diurnal internal tide), $R_f = 0.2$, A = 0.20, $N = 1 \times 10^{-3} \text{ s}^{-1}$ and $f = 0.53 \times 10^{-4} \text{ s}^{-1}$. Observed shear spectra are in reasonable agreement with these spectral parameters (Polzin, 2004b). The issue of specifying the vertical profile of turbulent dissipation has thus been cast into a problem of specifying the internal wave spectral parameters at the bottom boundary using models of wave generation and scattering. Before doing so, the issue of wave propagation in non-uniform stratification is addressed.

2.3. Buoyancy scaling of the dissipation profile

Variable stratification adds an additional complication. Buoyancy scaling under the Wentzel–Kramers–Brillouin (WKB) approximation returns the result that the vertical wavenumber of a wave packet varies in proportion to *N*, which in turn implies an $^{-1}$ There is a typographical error in Polzin et al. (1995) that leads to *A* being quoted as 0.1 in Polzin (2004a).

additional transport of energy to smaller scales, Appendix B. Such effects can be described by buoyancy scaling the vertical coordinate in (1) as:

$$z^* = \int_0^z \frac{N^2(z')}{N^2(z'=0)} dz'$$
(5)

and invoking the hydrostatic versions of $\phi(\omega)$ and $\beta(\omega)$ to obtain

$$\epsilon = \frac{\epsilon_0 N^2(z) / N^2(z=0)}{\left(1 + z^*/z_0\right)^2}.$$
(6)

The variability of N(z) is sufficiently small in the bottom most 1500 m that the fit parameters are not affected ($z_o = 150 \text{ m}, \epsilon_o = 1 \times 10^{-8} \text{ W/kg}$). At shallower depths, however, increasing stratification coupled to nonlinearity serves to transport the remaining energy efficiently to small scales, Appendix B.

2.4. A generation model and topography

A linear model of internal tide generation using continuous topography will return the result that the internal tide energy density is proportional to the topographic slope variance. This is problematic given the topological character of mid-ocean ridge bathymetry. It can be described as fractal² (Goff and Jordan, 1988), which implies the topographic slope variance is unbounded as smaller and smaller scales are included in the slope estimate. Using a linear model and a continuous representation of mid-ocean ridge bathymetry, the predicted internal wavefield will have infinite energy density and infinite shear. This is aphysical as either adiabatic or diabatic nonlinearity will serve to damp the smallest-scale response. This issue, and its resolution, are discussed in greater detail in Polzin (2004b).

The resolution defined in Polzin (2004b) is to use a quasi-linear spectral model of internal tide generation that incorporates horizontal advection of the barotropic tide into the momentum equations (Bell, 1975):

$$E_{\text{flux}}(k,l,\omega_n,z=0,t) = \frac{n\omega_1}{2\pi^2} \Big[\Big(N^2 - n^2 \omega_1^2 \Big) \big(n^2 \omega_1^2 - f^2 \big) \Big]^{1/2} [k^2 + l^2]^{-1/2} \\ \times H(k,l) J_n^2 \Big(\Big[\Big(k^2 U_o^2 + l^2 V_o^2 \Big) / \omega_1^2 \Big]^{1/2} \Big)$$
(7)

Here $E_{\text{flux}}(k, l, \omega)$ is the horizontal wavenumber-frequency spectrum for the vertical energy flux, ω_1 is the fundamental frequency of the barotropic tide (M_2) , n an integer such that $n\omega_1 < N$ and $\omega_n = n\omega_1$ represents the nth harmonic. The function J_n is a Bessel function of order n and the factors U_o and V_o in its argument represent the amplitude of the barotropic tide. The function J_n represents the effects of horizontal advection by the barotropic tide. It serves as a smoothing function at high wavenumber and thereby avoids the problems of infinite energy and shear. The topographic spectrum H(k, l) can expressed in terms of Goff and Jordan's (1988) anisotropic parametric representation:

$$H(k,l) = \frac{4\pi v h_o^2}{l_o k_o \left(\frac{k^2}{k_o^2} + \frac{l^2}{l_o^2} + 1.0\right)^{(\nu+1)}},$$
(8)

where k_o and l_o are roll-off wavenumbers, v prescribes a high wavenumber power law, and h_o is the rms height. This parametric representation seeks to capture variability associated with abyssal hills created by faulting and volcanism at mid-ocean ridge spreading centers. It does *not* seek to describe larger scale offset fractures such as the canyon in the BBTRE or isolated seamounts. Abyssal hill morphology is believed to exhibit a regional statistical homogeneity related to spreading rates, the visco-elastic properties of magma at the spreading center, etc. The parameter v exhibits little variability in comparison to h_o and (k_o, l_o) . The method used to estimate v employs an objective estimate of noise. See Goff (1991) for further discussion.

In previous work (Polzin, 2004b) I used values for the Mid-Atlantic Ridge at 26°S from tables in Goff (1991). Use of those parameters with TPXO (Egbert et al., 1994) derived estimates of the barotropic tide $[(U_o, V_o) = (2.1, 2.5) \text{ cm/s}]$ returned a 7.6 mW m⁻² estimate of the total energy flux using (7). Swath bathymetry was obtained on the last of four BBTRE cruises in April–May of 2000. Analysis of these multibeam data returns parameter estimates of $(k_o = 2.2 \times$ $10^{-4} \text{ m}^{-1}, l_o = 1.0 \times 10^{-3} \text{ m}^{-1}, h_o = 110 \text{ m}, \text{ and } v = 0.90$; John Goff, personal communication, 2002; see also Appendix A). These values return an energy flux estimate of 3.8 mW m⁻², indistinguishable from the depth integrated dissipation data, 3.7 mW m⁻².

Finally, (7) can be converted to a 1-D horizontal spectrum by integrating over the orientation of the horizontal wavevector and then converted to a vertical wavenumber frequency spectrum by invoking a linear dispersion relation, Fig. 2. The shear spectrum rolls off at a vertical wavelength of about 100 m and it is this high wavenumber peak that is to be associated with the parameters ϵ_{0} and m_{0} in (3) and (4).

Identification of the advective roll-off in Bell's model with the idealized solution is the crux of this parameterization scheme.

3. The recipe

The recipe presented below documents a dynamically consistent extrapolation of the BBTRE data. There are three basic parameters in specifying the dissipation profile: (i) the bottom dissipation ϵ_o , (ii) the scale height z_o and (iii) the depth integrated dissipation rate. That process is conducted in five stages: (i) mapping the generation model (7) onto the finescale radiation balance equation solutions (2), (ii) transferring those scalings onto the dissipation profile (1), (iii) accounting for an energy flux residual, (iv) transferring from an observational coordinate system into a model coordinate system to obtain a mean buoyancy flux profile, and (v) treating the residual flows implied by the buoyancy flux divergence.

3.1. Mapping the generation model onto the nonlinear propagation model

Identification of m_0 as the roll-off N/V_o in Bell's model and bm_0^2 as the shear spectral density at that roll-off, the following functional dependencies result:

$$bm_0^2 \propto V_o^{2\nu-1} l_o^2 h_o^2 N^3,$$
 (9)

$$m_0 \propto N/V_o,$$
 (10)

where *v* is the high wavenumber power law of the topographic spectrum and l_oh_o is proportional to the rms topographic slope at the energy-containing scales of the topography. The factor k_oU_o does not appear in the parameterization scheme. The energy flux is dominated by larger scales than either the energy or shear, but the energy flux in the minor axis coordinate still dominates the major axis coordinate unless $U_o \gg V_o$. The roll-off m_0 results from the advective smoothing in the generation model (7) that decreases the high wavenumber internal wave shear in the topographic minor axis coordinate. The use of V_o in the numerator of (10) appears robust apart from the perverse instance in which $V_o = 0$.

3.2. Transfer the spectral domain scalings onto the dissipation profile

Let subscripts of *ij* denote values at a particular x-y grid point. The near-boundary dissipation profile is specified as,

² For this discussion the fractal designation implies a high wavenumber power-law of k_h^{-2} to k_h^{-3} for the 1-D bathymetric spectrum.

$$\epsilon_{ij}(z) = \frac{\epsilon_{oij} N_{ij}^2(z) / N_{oij}^2}{\left(1 + z^* / z_{oij}\right)^2},\tag{11}$$

in which z^* is a scaled height coordinate,

$$z^* = \int_0^z \frac{N_{ij}^2(z')}{N_{oij}^2} dz'$$
(12)

and z' increases from the bottom. The z_{oij} parameter is a scale height, ϵ_{oij} is the dissipation rate at the bottom and N_{oij} is the stratification at the bottom for the *ij*th grid point.

Following Polzin (2004b), the total dissipation $(\epsilon_{ij-\text{total}})$ is assumed to be locally in balance with the rate energy is converted from the barotropic tide to internal waves, P_{ij} :

$$P_{ij} = \frac{\rho}{1 - R_f} \int_0^{H_{ij}} \epsilon_{ij\text{-total}} \, dz,\tag{13}$$

where R_f is the flux Richardson and I have assumed $R_f = 0.20$. The assumption of a vertically 1-D balance is supported by scaling arguments (Polzin, 2004b) and the O(1) efficiency of the wave scattering process above the Mid-Atlantic Ridge (Polzin, manuscript in preparation-b). This assumption will need to be reassessed for faster spreading mid-ocean ridges that are not as rough as the Mid-Atlantic Ridge.

The parameterization proceeds as follows. Let P_o , ϵ_o , N_o , (U_o, V_o) and (k_o, l_o) represent reference values for the various parameters. The dissipation profile (11) can be expressed in terms of:

$$\epsilon_{oij} = \epsilon_o \left[\frac{N_{oij}}{N_o} \right]^4 \left[\frac{V_{oij}}{V_o} \right]^{4\nu-2} \left[\frac{l_{oij}h_{oij}}{l_o h_o} \right]^4, \tag{14}$$

$$Z_{oij} = Z_o \left[\frac{N_{oij}}{N_o} \right]^{-3} \left[\frac{V_{oij}}{V_o} \right]^{3-2\nu} \left[\frac{l_{oij}h_{oij}}{l_o h_o} \right]^{-2},$$
(15)

$$P_{ij} = P_o \left[\frac{N_{oij}}{N_o} \right] \left[\frac{V_{oij}}{V_o} \right]^2 \left[\frac{h_{oij}}{h_o} \right]^2 \left[\frac{l_{oij}}{l_o} \right], \tag{16}$$

where the parameterization is normalized to the Brazil Basin observations:

 $P_o = 3.7 \text{ mW/m}^2,$ (17)

$$\epsilon_o = 1.0 \times 10^{-8} \,\mathrm{W/kg},\tag{18}$$

 $z_{\rm o} = 150 \, {\rm m},$ (19)

$$N_0 = 9.6 \times 10^{-1} \, \mathrm{s}^{-1}, \tag{20}$$

$$(U_o, V_o) = (0.021, 0.025) \text{ m/s}, \tag{21}$$

$$h_{\rm o} = 110 \, {\rm m},$$
 (22)

$$(k_o, l_o) = (2.2 \times 10^{-4}, 1.0 \times 10^{-5}) \text{ m}^{-1},$$
 (23)
 $v = 0.9.$ (24)

The functional dependence of ϵ_o and z_o upon $N, h_o, (k_o, l_o)$ and (U_o, V_o) in (14) and (15) comes directly from (9) and (10), and (16) results from (7).

3.3. Account for a residual dissipation

The difference between the power input and the depth integrated dissipation in the profile (11) is given by a residual Δ :

$$\Delta = \frac{1 - R_f}{\rho} P_{ij} - \int_0^\infty \frac{\epsilon_{oij} \, dz}{\left(1 + z/z_{oij}\right)^2} \cong \frac{1 - R_f}{\rho} P_{ij} - \epsilon_{oij} z_{oij}.$$
(25)

I simply assume that the residual is distributed with depth as a constant diffusivity:

$$\epsilon_{ij\text{-total}} = \epsilon_{ij} + \frac{\Delta N_{ij}^2}{\int_0^{H_{ij}} N_{ij}^2 dz}.$$
(26)

This gives a reasonable approximation to the observed dissipation profile over the entire water column, Fig. 3. The total turbulent buoyancy flux profile Π is thus:

$$\Pi = -\frac{g}{\rho_o} \frac{R_f}{1 - R_f} \epsilon_{ij\text{-total}}.$$
(27)

The dissipation profile resulting from (26) is quite similar to that resulting from much more complicated numerical solutions (Polzin, manuscript in preparation-a) to the nonlinear propagation model defined in (34) and (35). The standard invocation of a flux-gradient relation in (27) leads to $\Pi = -K_{\rho}N^2$. The closure (27) is intended to be implemented as a flux Π , rather than an eddy diffusivity K_{ρ} .

An insight is that weakly nonlinear processes will tend to result in $\epsilon \propto N^2$ and near-boundary dissipation will be dominated by tidal and Lee wave processes rather than the scattering of the background wavefield. Thus (26) hides many sins.

3.4. Transfer the recipe to a mean coordinate system

The preceding analysis assumes a height above bottom coordinate system (z = hab, say), in which the bottom is the actual bottom, rather than a highly smoothed version used by general circulation models (to be denoted by z = Hab) (Fig. 4). The issue is that $\epsilon(z = \overline{hab} = Hab)$ can be a poor approximation of $\overline{\epsilon(z = hab)}$, in which a horizontal or isopycnal average is denoted with the overbar. Let p(hab|Hab) represent the probability that the actual bottom is at z = hab given that the height above the mean bottom is Hab. Then the average buoyancy flux across the mean surface (horizontal or isopycnal) is $-\frac{g}{\rho_o} \frac{R_f}{1-R_f} \epsilon(Hab)$ with:

$$\epsilon(Hab) = \int_0^\infty \epsilon(hab)p(hab|Hab)\,dhab.$$
(28)



Fig. 3. Dissipation vs. height-above-bottom over rough bathymetry in the Brazil Basin. These dissipation data appear in Fig. 1. Here the full-depth profiles are displayed as the thick lines. Data from the bottom most 1500 m have been averaged in a height above bottom (*hab*) coordinate system. Data from shallower depths are plotted in a depth coordinate system. The thin line closely resembling the dissipation profile represents (26) using (6). The average observed buoyancy frequency and diffusivity profiles are plotted as dashed and dotted lines. The average water depth of these 30 profiles is 4400 m.



Fig. 4. Diagram illustrating the difference between the height above bottom coordinates assumed in the analytic analysis (*hab*, dashed arrow) and that assumed in a numerical model (*Hab*, solid arrow). Grey shading represents bathymetric data obtained with a center beam system along a ridge to the northeast of the injection site. See also Fig. 9 for a multi-beam estimate. The variable *hab* is the height above the actual bottom whereas *Hab* is the height of an isopycnal surface or model level above the mean bottom (4200 m here). The indicated isopycnal happens to be a fit to the isopycnal at the injection surface.

Implicit in this expression is that $\epsilon(hab) = 0$ if the topography intrudes across the mean surface. An analytic estimate of $\epsilon(Hab)$ is given below so that the method can be copied.

A convenient representation is to assume the bathymetry to be a succession of triangular planforms (a sawtooth profile) with height h = Hab - hab probability distribution:

$$p(h) = \frac{1}{2\sqrt{3}h_o} |h| \leqslant \sqrt{3}h_o,$$

$$p(h) = 0 |h| \geqslant \sqrt{3}h_o,$$
(29)

in which *h* is the topographic height about *Hab* having a variance of h_o^2 . This can easily be translated into an expression for p(hab|Hab) given p(hab|Hab)dhab = p(h)dh with the restriction that hab > 0. Given (29), the depth profile of dissipation (28) becomes:

$$\begin{aligned} \epsilon(Hab) &= \frac{\epsilon_o z_o^2}{(z_o + Hab)^2 - 3h_o^2} Hab > \sqrt{3}h_o, \\ \epsilon(Hab) &= \frac{\epsilon_o z_o \left(Hab + \sqrt{3}h_o\right)}{2\sqrt{3}h_o \left(z_o + Hab + \sqrt{3}h_o\right)} \ 0 < Hab < \sqrt{3}h_o. \end{aligned}$$
(30)

The resulting dissipation profile is nearly uniform near the boundary $(Hab < \sqrt{3}h_o)$. Values of (30) larger than (1) are noted at greater distances. The two solutions become similar in the limit that $Hab \gg h_o$, Fig. 5. The resulting $\epsilon(Hab)$ profile is sensitive to the height variance h_o^2 and care must be exercised to use fully resolved estimates of the topographic variance.

Despite the idealized representation of the statistical distribution of topographic heights, the gross features of the resulting dissipation profile (30) appear robust. For example, the assumption of Gaussian statistics for h does not substantively alter the near-bottom *Hab* decrease in dissipation, Figs. 5 and 6.

3.5. Diapycnal advection

Complete the transfer from a rough basin to a smooth dish with a porous bottom by specifying a diapycnal velocity at the bottom boundary.



Fig. 5. Dissipation profiles in both local (hab – dashed line) and mean (Hab) coordinates. The bold solid line was estimated with Gaussian h probability distribution, the thin solid line represents (30).

There is a very serious consideration which, at this point, does not have a good resolution. If implemented as a buoyancy flux, the parameterization culminating in (26) will produce downwelling over the tops of the topographic roughness. This is easily seen from the diapycnal advection-diffusion balance in the density equation:

$$w^* N^2 = \partial(\Pi) / \partial z_*. \tag{31}$$

Here w^* is the diapycnal velocity and z_* is the diapycnal coordinate. This equation, apart from nonlinearities in the equation of state (McDougall, 1991), is exact. Thus the sign of w^* is determined by the sign of $\partial(\Pi)/\partial z_*$, which is in the downwelling sense in the hab coordinate, Fig. 6 (Π is monotonically decreasing). This implies the production of dense water. Analysis of tracer observations imply a diapycnal velocity of about $-3 \times 10^{-7} > w^* > -5 \times 10^{-7}$ m s⁻¹ below the tracer injection level (Ledwell et al., 2000), which was estimated to be 500 m above abyssal hill summits immediately to the north and south of the injection site [See Ledwell et al. (2000) for caveats about particulate scavenging of the tracer. Note also that the tracer estimate of diapycnal velocity uses the function form (1) and so is not entirely independent of the analysis presented here.]. The diapycnal velocity profile resulting from (31) using either the Gaussian or sawtooth topographic probability distribution returns estimates of w^{*} in a Hab coordinate that are consistent with the tracer estimate. Both analytic estimates feature deep upwelling near the mean depth Hab = 0.

Specification of this deep diapycnal velocity needs to be consistent with mass conservation. This requires either explicit



Fig. 6. Diapycnal velocity profiles in both local (*hab*) and mean (*Hab*) coordinates. The top of the box at *Hab* = 560 m represents the height of the injection surface above mean depth of sample boxes 2.1-5.3 (Fig. 9). The width of the box represents the range of the tracer based estimate of diapycnal velocity below the injection surface. The line coding is as in Fig. 5.

representation of horizontal conduits (canyons) and horizontal fluxes or specification of the diapycnal velocity at the bottom face of a grid cell and nonlocal accounting of the mass budget. Guidance is provided in Section 4.1.

3.6. Further caveats

- The prescription is for the buoyancy flux, not the mass diffusivity: The equations of motion are forced by the divergence of momentum and bouyancy fluxes. This work represents a parameterization of the buoyancy flux. It is not necessary, and quite insensible, to invoke a flux gradient relationship and implement the buoyancy flux in terms of a diffusivity.
- The prescription is only for the vertical faces of a grid cell: Before implementing this scheme, the entire buoyancy budget of a grid cell requires further consideration. The answer is likely to depend quite significantly upon model resolution.
- *Bathymetry:* Goff's parametric representation (8) does not capture low wavenumber variance associated with offset fractures. This can lead to an underestimate of the corresponding low vertical wavenumber internal wavefield and associated energy flux. Applying a two dimensional Fourier transform to the local swath bathymetry returns estimates of the energy flux that are 20–40% larger than using (8). This "missing" energy flux is accounted for in this local balance by normalizing the parameterization to the BBTRE observations.

Given that mid-ocean ridge roughness exhibits at least a regional statistical homogeneity (Goff, 1991) and the information required to account for variability of topographic roughness in (14)–(16) and transformation to a *Hab* coordinate system requires fully resolved (e.g. center- or multi-beam) bathymetry which is generally not available, I suggest accounting for variability in $h_o(k_o, l_o)$ on a regional or ridge-wise basis. See Goff and Arbic (submitted for publication) for a description of such variability.

When the topography is smooth, the choice of a constant background diffusivity is not unreasonable in light of the observations (Polzin et al., 1997).

• The energy spectrum of the nonlinear propagation model (2) has been identified with one aspect of the generation model (7). The reader should be cognizant of the following:

First, the idealized solutions are not a complete description of the generation and scattering processes: they are too bandwidth-limited and, as a consequence, do not capture the entire energy flux associated with the generation process. The observed spectra, as well, exhibit significantly more energy at low wavenumber (Polzin, 2004b). The neglected energy is likely to be dissipated in or near the thermocline where the observations indicate $K \cong (0.1-0.2) \times 10^{-4} \text{ m}^2 \text{ s}^{-1}$. The missing energy flux is added to the dissipation profile as a constant diffusivity.

Second, (7) describes only the generation process, not the scattering process. Arguably (Polzin, 2004b, manuscript in preparationb) the roll-off in the spectrum at $m_0 \propto N/V_o$ might be better described as a smoothing operation with vertical scale given by $m_0 \propto N/U_{\rm rms}$, in which $U_{\rm rms}$ represents the root-mean-square tidal velocity along the topographic minor axis and the spectral level *b* interpreted as being the sum of both generation and scattering processes. This roll-off should be a generic feature of both barotropic and baroclinic flow over topographic roughness for baroclinic waves of large vertical wavelength, $(m^{-1} \gg h_o)$ (Polzin, 2004b, manuscript in preparation-b).

Third, and finally, Bell's (1975) model has an infinitesimal amplitude bottom boundary condition which assumes ray trajectories are more steeply sloped than the topography, $l_0 h_0 \ll \omega_1 / N$. The use of such a bottom boundary condition requires further justification that is beyond the scope of this paper. The infinitesimal amplitude bottom boundary condition assumes that internal wave ray trajectories are more steeply sloped than the bottom topography. This is clearly not true here, see Polzin (2004b), Fig. 8. However, as noted in Polzin (2004b), enhanced shear anticipated with finite amplitude effects is not apparent in the observations. A possible resolution is that the topography is two dimensional and anisotropic, so that rather than being constrained to go up and over supercritical topography in one dimension, water parcels may be 'blocked' in that direction and forced across topography in the subcritical direction. A suitable finite amplitude bottom boundary condition for anisotropic, twodimensional bathymetry has not been defined. The energy flux associated with finite amplitude bathymetry tends to saturate at a constant value when the slope of internal wave ray trajectories equals that of the topographic slope (Garrett and Kunze, 2007) and this is approximately the case for the abyssal hills here as $l_0 h_0 \sim \omega_1 / N$.

4. Discussion

4.1. The Mid-Atlantic Ridge as a lung

The initial (Polzin et al., 1997) BBTRE investigation raised a very interesting question about the mass budget of the abyssal Brazil Basin. In a diapycnal advection–diffusion balance (31) the sign of the diapycnal velocity w^* is given by the sign of $\partial_{z_*} \epsilon$ if the mixing efficiency $\frac{R_f}{1-R_f}$ is assumed to be constant. The dramatic enhancement of ϵ above rough topography implies downwelling and we



Fig. 7. Depth-longitude section of diapycnal diffusivity in the Brazil Basin. The figure appears in Mauritzen et al. (2002), with data appearing originally in Polzin et al. (1997) and Ledwell et al. (2000). Note the nonuniform color map. The thin white lines mark the observed depth of the 0.8 °C and 1.8 °C potential isotherms. The thicker white lines with arrows are a schematic representation of the streamfunction estimated from an inverse calculation (St. Laurent et al., 2001) and are intended to portray the zonal overturning circulation and modification of Bottom Water in the Brazil Basin. Each horizontal bin in the diffusivity map represents an individual profile.



Fig. 8. A schematic depiction of the proposed secondary circulation along the canyon axis. Lines without arrows represent isopycnals. Lines with arrows represent particle trajectories with net upwelling across isopycnals. Upwelling in this schematic is associated with the near boundary decay of the internal tide generated/scattered in association with abyssal hills atop the ridge, along with (28). The black foreground is the bathymetry profile along the canyon axis. The peaks atop the ridge are abyssal hills. The 3-D image of topography extends from approximately 18°1′W 21°40′S to 17°15′W 21°33′S, Fig. 9, with a perspective oriented toward 345 °T and 1° from the horizontal. The canyon axis is significantly smoother than the adjacent ridge, but is occasionally cut by abyssal hills extending from the ridge. These features form sills that block deep flow up the canyon axis.

hypothesized that the mass budget would be closed by upwelling within the canyons rather than above rough topography or over the much smoother western half of the basin. We forwarded the conjecture that, on a broad scale, the Mid-Atlantic Ridge could be viewed as a permeable, sloping boundary with sinks for the densest waters in the Brazil Basin at the depths of the canyon mouths and sources of water at depths about the canyon heads. Unstated was a back of the envelope calculation implying substantial flow up the offset fractures: the requirement of laundering 2×10^6 m³ s⁻¹ of dense Antarctic Bottom Water through some 30 canyons in the abyssal Brazil Basin having a characteristic cross-sectional area $\frac{1}{2}HW$ with height H = 700 m and width W = 20,000 m returns an

average velocity of 0.02 m s^{-1} . Additional funding was obtained to deploy a mooring in the offset fracture with the primary goal of documenting such a mean flow and the secondary goal of documenting temporal characteristics of the internal wavefield. The back-of-the-envelope calculation is in remarkable agreement with two-year averaged estimates of flow in the canyon (Thurnherr et al., 2005).

This scheme is quantified in the streamfunction determined from an inverse model [St. Laurent et al., 2001, see also Fig. 7]. In the St. Laurent et al. (2001) work, the Smith–Sandwell bathymetry is used to characterize the geographic location of vertical profile (dissipation) data. Their binning structure emphasized canyon sidewalls as the locus of maximum dissipation (Thurnherr et al., 2005). This is an artifact of using the Smith–Sandwell bathymetry: the loci of maximum dissipation are abyssal hill regions external to the canyons, Polzin (2008). In St. Laurent et al. (2001), upwelling is regarded as occurring within canyons in concert with small mixing efficiencies $\binom{R_f}{1-R_f} \ll 0.2$ over the bottom most 300 m, in which the small mixing efficiencies are optimally determined from an inverse model. The resulting overturning streamfunction represents only 20% of the anticipated basin average. Here upwelling is diagnosed from (31) in a *Hab* coordinate system and may be significantly larger. Accurate estimates of the height variance are required to obtain a representative upwelling profile: The Smith–Sandwell bathymetry product is not sufficient.

The buoyancy budget within the fracture zone valleys of some 0.02 m s⁻¹ up-canyon flow demands significant diapycnal transformations in combination with net upwelling. This constraint leads (Thurnherr et al., 2005) to conjecture that the diapycnal transformations may very well be associated with sill processes rather than the tidal generation scheme proposed here. In a related work (Polzin, 2008) I examine the circumstantial evidence presented by Thurnherr et al. (2005) in support of the sill process interpretation and find that, apart from isopycnals dipping at sills cutting across the fracture zone valleys, the evidence does not support that interpretation.

The conundrum I posed by the inference of downwelling above rough topography, averaging dissipation profiles in a heightabove-bottom (*hab*) coordinate system and equating that coordinate in the context of a diapycnal advection–diffusion balance (6) is probably misleading. Averaging in a height above the *mean* boundary (*Hab*) coordinate system returns a positive diapycnal velocity estimate as an isopycnal encounters the tops of the abyssal hills. The resolution with St. Laurent et al. (2001) is the recognition that the Smith–Sandwell product is not adequate to provide a binning structure for vertical profile data and it does not resolve the height variance required to infer upwelling rates from (31).

A schematic of a circulation scheme that does not require significant upwelling in association with sills is rendered in Fig. 8. Here a sill is simply depicted as blocking the deepest flow and the deep up canyon flow is balanced by diapycnal upwelling off the canyon axis. This contrasts with inferences one might be tempted to draw about deep up canyon flow and the nearly vertical 0.8 °C isotherm in Fig. 7.

The total upwelling in this regime can figure significantly in the abyssal Brazil Basin mass budget. Taking a characteristic upwelling rate of 1.5×10^{-6} m s⁻¹ over the bottom most 300 *Hab* (Note that Fig. 6 does not extend to negative *Habs* included in this 300 m), the zonal extent of the upwelling regime can be estimated as this height scale divided by the mean slope of the Mid-Atlantic Ridge ($\cong 7 \times 10^{-4}$) and the meridional extent estimated as the length of the Brazil Basin, 3×10^{6} m. The back-of-the-envelope budget returns some 2 Sv of upwelling. This is the correct order of magnitude to balance the input of dense water through the Vema Channel and additional inputs associated with downwelling.

In this scenario, canyons are simply conduits for deep flow rather than the locus of upwelling. These canyon flows supply water to be upwelled in the smaller spaces in between individual abyssal hlls, Fig. 8. Net upwelling appears only in the *Hab* coordinate and after due consideration of fully resolved bathymetry. Abyssal hills play a critical role in the ventilation (aspiration) of the abyss. An analogy to the functioning of a lung may be useful.

4.2. Climate implications

It is fairly well established that the meridional overturning circulation is particularly sensitive to diapycnal mixing and freshwater forcing in the North Atlantic and wind stress in the Southern Ocean (Bugnion et al., 2006). Of particular concern here is the potential shutdown of Deep Water production in the North Atlantic with either increasing hydrologic forcing or decreasing diapycnal mixing, e.g. Zhang et al. (1999), which has been investigated under both 'constant diffusivity' and 'constant available energy' scenarios (Nilsson and Walin, 2001). A significant caveat is that the upper limb of the meridional overturning circulation responds most directly to mixing in the thermocline, Jayne (2009).

The analysis presented here returns a dramatic dependence of $\epsilon_{oij} \propto N^4$ and $z_{oij} \propto N^{-3}$ on buoyancy frequency, though as a measure of the depth integrated dissipation, their product is sensibly $\epsilon_{oij} z_{oij} \propto N$. The proposed parameterization scheme does not fit nicely into either the 'constant diffusivity' or 'constant available energy' scenarios: The major feature of abyssal stratification in the Atlantic Ocean is the Antarctic Bottom Water/North Atlantic Deep Water interface. If an increase in freshwater forcing were to shut down the production of North Atlantic Deep Water, that interface may intrude much further into the North Atlantic and potentially result in a dramatic change in the distribution of diapycnal mixing. I do not believe the linkages and consequences under the proposed parameterization can be understood without further investigation.

5. A summary of half-baked ideas

A dynamically based parameterization for the dissipation profile has been presented and appropriately modified for inclusion in general circulation models. There are a great number of caveats associated with this process and so substantive improvements to the parameterization scheme over time are likely. Thus the ideas presented here should be considered as half-baked. Being dynamically based, though, the scheme can be tested and modifications pursued in a rigorous fashion.

That parameterization identifies a functional form for the nearboundary dissipation profile (11), dynamically extrapolates that profile to account for geographic variability in tides, topography and stratification (14)–(16), closes the internal wave energy budget by assuming generation balances dissipation as a local process (26), and then modifies that result for inclusion in general circulation models that do not fully resolve topographic variability (28) and (31).

The premise of this paper is that the energetics of the finescale internal wavefield above rough topography is dominated by tidal frequencies and that this is a local process. These assumptions are open to question and documenting the oceanic wavefield to test these hypotheses is a matter of ongoing research. My perspective is that topographic roughness renders the internal wave energy balance to be essentially one-dimensional. This is a fundamental departure from the established nonlocal paradigm. A significant part of past research on internal waves involves the search for energy sources in the context of the Garrett and Munk spectrum. These sources have been extremely difficult to identify, in part because simple model estimates tend to indicate most sources to be of similar order of magnitude. There are many candidates for sources of energy for the oceanic internal wavefield. The problem is discriminating between them. Sources have not been obvious, in part because the energy input at each event tends to be a relatively small fraction of the total energy flux resident in the background internal wavefield. Several analogies likening this process to small impulses acting on a massive flywheel, random sources of heat within a efficiently conducting thermal block, or the scattering of light by a fog bank have been invoked. My personal experience is that it is easy to over- or misrepresent the strength of individual sources on the basis of simple model calculations which are unconstrained by sufficient observations. Here,

however, I through caution to the wind and include only tides in this recipe.

Altering this recipe to account for different sources through ϵ_o , z_o and P_{ij} could be an easy matter if those other sources are identified. The most pressing issues are the generation/decay of quasi-stationary Lee waves (Nikurashin and Ferrari, submitted for publication) and subsequent momentum transfer.

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Appendix A. Multibeam statistics

Multibeam bathymetry data were obtained on the fourth and final Brazil Basin Tracer Release Experiment cruise in April–May of 2000 from the R/V Knorr, Fig. 9. The data were processed by Peter Lemmond (Woods Hole Oceanographic Institution) and John Goff (personal communication, 2002) provided estimates for his parametric spectral representation (8) based upon those data, Table 2. The data were divided into an 8 row by 3 column matrix of sample boxes for this analysis. Parameter values quoted in Section 2.4 represent averages over rows 2–5, i.e. sample boxes 2.1–5.3.

Appendix B. Idealized solutions to a radiation balance equation

The analytic expression (1) and the parametric specifications of ϵ_0 and z_0 in terms of m_0 and b result from a coupled system of partial differential equations which govern the vertical evolution of the vertical wavenumber spectrum (Polzin, 2004a,b). These equations are of the form:

$$\pm \frac{\partial}{\partial z} C_{gz} E^{\pm} + \frac{\partial F^{\pm}}{\partial m} = S_o^{\pm} - S_i^{\pm}$$
(32)

in which $\pm C_{gz}E^{\pm}$ represents the vertical flux of energy (C_{gz} is taken to be positive definite) and the energy spectra of the upward and downward propagating wavefield are denoted as $E^+(m)$ and $E^-(m)$, respectively. The factor F^{\pm} similarly represents the transport of energy through the vertical wavenumber domain for the upward + and downward – propagating wavefields. The transport F^{\pm} associated with internal wave–wave interactions is a quadratic function of the spectral level, and so (32) represents a nonlinear propagation model. The dissipation rate is estimated as:

$$\epsilon = (1 - R_f)[F^+ + F^-] \tag{33}$$

in the limit as $m \to \infty$. The right-hand-side of (32) represents explicit sources and sinks in the spectrum, e.g. transfers between discrete wavenumbers and frequency associated with resonant interactions. The representation forwarded in Polzin (2004b) is:

$$\frac{\partial E^{+}(m)}{\partial z} + \frac{Am}{N^{2}} \frac{\partial [\beta(\omega)m^{4}E^{+}(m)(E^{+}(m) + E^{-}(m))]}{\partial m}$$
$$= \frac{A\beta(\omega)m^{4}}{2N^{2}} [E^{-}(m) - E^{+}(m)][E^{+}(m) + E^{-}(m)]$$
(34)

and

$$\frac{\partial E^{-}(m)}{\partial z} + \frac{Am}{N^{2}} \frac{\partial [\beta(\omega)m^{4}E^{-}(m)(E^{+}(m) + E^{-}(m))]}{\partial m}$$
$$= \frac{A\beta(\omega)m^{4}}{2N^{2}}[E^{+}(m) - E^{-}(m)][E^{+}(m) + E^{-}(m)].$$
(35)

The stratification has explicitly been assumed constant here. These equations represent a balance between the vertical propagation of wave energy $\partial(\pm C_{gz}E^{\pm})/\partial z$ and the downscale transport of energy $\partial F^{\pm}/\partial m$ in which the right-hand side serves to conserve wave momentum. The right-hand side will tend to make the wavefield vertically isotropic at small scales, thereby creating a downward propagating wavefield from a source at the bottom boundary. Thus the upward and downward propagating wavefields are explicitly coupled by nonlinearity. The bottom-boundary condition is a unidirectional source at the bottom with planar reflection from a flat bottom for the downward propagating wavefield. An approximate solution to (34) and (35) is

$$E^{+} + E^{-} = \frac{1}{1 + 2A\alpha\beta(\omega)N_{0}^{-2}bm_{0}^{4}z} \frac{bm_{0}^{2}}{m^{2}} \left(1 - \frac{m_{0}^{2}}{m^{2}}\right), \tag{36}$$

with corresponding dissipation rate

$$\epsilon(z) = \frac{(1 - R_f) A\phi(\omega) N^{-1} b^2 m_0^4}{\left[1 + 2A\alpha\beta(\omega) N_0^{-2} b m_0^4 z\right]^2}.$$
(37)

The solution is approximate because the spectral shape needs to be determined numerically. It happens that this shape is quite well described by $\frac{bm_0^2}{m^2} \left(1 - \frac{m_0^2}{m^2}\right)$ with $\alpha = 2.31$ (Polzin, 2004b). Buoyancy scaling under the WKB approximation gives the

Buoyancy scaling under the WKB approximation gives the change with N of vertical wavenumber for a single internal wave as (Leaman and Sanford, 1975):

$$m \cong \hat{m}N/N,\tag{38}$$

where \hat{m} and \hat{N} are reference values of m and N(z). The relation (38) assumes the hydrostatic approximation, in which case the group velocity (C_{gz}) is independent of N(z) at constant m. The change of vertical wavenumber implies a transport of energy through the vertical wavenumber spectrum which needs to be accounted for in (34) and (35). The transport rate is:

$$F = E\frac{dm}{dz}\frac{dz}{dt} = E\hat{m}\frac{dN}{dz}\hat{N}^{-1}C_{gz} = \frac{\omega^2 - f^2}{\omega}E\frac{dN}{dz}N^{-1},$$
(39)

where the direction of upward energy propagation has been chosen. With this flux law, the hydrostatic versions of (34) and (35) become:

$$\frac{\partial E^{+}(m)}{\partial z} + m \frac{\partial [E^{+}(m)N_{z}N^{-1} + A\beta(\omega)N^{-2}m^{4}E^{+}(m)(E^{+}(m) + E^{-}(m))]}{\partial m}$$

= $\frac{A\beta(\omega)m^{4}}{2N^{2}}[E^{-}(m) - E^{+}(m)][E^{+}(m) + E^{-}(m)]$ (40)

and

$$-\frac{\partial E^{-}(m)}{\partial z} + m \frac{\partial [-E^{-}(m)N_{z}N^{-1} + A\beta(\omega)N^{-2}m^{4}E^{-}(m)(E^{+}(m) + E^{-}(m))]}{\partial m}$$
$$= \frac{A\beta(\omega)m^{4}}{2N^{2}}[E^{+}(m) - E^{-}(m)][E^{+}(m) + E^{-}(m)].$$
(41)

Analytic progress can be made by forsaking momentum conservation, in which case the explicit coupling on the right-hand side of (40) and (41) vanishes and, for a unidirectional source, (40) becomes

$$\frac{\partial E^{+}(m)}{\partial z} + m \frac{\partial (E^{+}(m)N_{z}N^{-1} + A\beta(\omega)m^{4}N^{-2}E^{+}(m)^{2})}{\partial m} = 0.$$
(42)



Fig. 9. Multi-beam bathymetric map with HRP station positions posted as white filled circles for profiles contributing to the average profile in Fig. 1. Another four stations in this average are located atop the ridge flank to the northeast. Black filled circles represent the mooring position to the east and the injection site to the west. The data were divided into 24 separate sample boxes for analysis. See Table 2 for results of the parametric fits. The sample box key in column one of Table 2 is the row.column of the grid overlaid on the bathymetry.

Table 2

Fits to multibeam data for Goffian statistics (Goff, 1991). The estimates presented here were provided by John Goff (personal communication, 2002). Values quoted in Section 2 are averages over sample boxes 2.1–5.3. The tracer was injected on an isopynal at a mean depth of 4010 m. The mean depth of sample boxes 2.1–5.3 is 4570 m. The power law ν was fixed as $\nu = 0.90$ in this analysis. The parameters λ_n and λ_s correspond approximately half a horizontal wavelength in the along strike (λ_s) and across strike (λ_n) directions.

Sample box #	Depth	h_o (m)	Azimuth	λ_n (km)	λ_s (km)	Anisotropy
1.1	4567	112 ± 11	-7.8 ± 3.1	2.6 ± 0.6	11.3 ± 3.9	4.3 ± 1.4
1.2	4568	94 ± 21	-8.4 ± 4.3	6.1 ± 2.3	31.2 ± 27.5	5.1 ± 4.2
1.3	4635	115 ± 15	-10.8 ± 3.0	3.1 ± 0.8	15.4 ± 6.8	5.0 ± 2.1
2.1	4699	128 ± 10	1.7 ± 4.0	2.7 ± 0.5	8.6 ± 2.4	3.2 ± 0.9
2.2	4401	114 ± 15	-7.2 ± 3.4	$\textbf{3.8} \pm \textbf{1.1}$	18.2 ± 9.5	4.7 ± 2.3
2.3	4509	89 ± 11	-10.1 ± 2.8	3.2 ± 0.9	20.0 ± 10.1	6.2 ± 2.9
3.1	4708	103 ± 9.6	3.4 ± 4.2	3.1 ± 0.7	10.3 ± 3.3	3.3 ± 1.0
3.2	4485	98 ± 16	-1.6 ± 5.9	5.2 ± 1.6	17.2 ± 8.9	3.3 ± 1.6
3.3	4274	145 ± 19	-2.2 ± 3.1	3.7 ± 1.0	20.1 ± 10.4	5.5 ± 2.7
4.1	4504	86 ± 10	-10.1 ± 3.0	3.3 ± 0.9	17.7 ± 8.3	5.4 ± 2.4
4.2	4574	147 ± 25	0.9 ± 4.1	5.3 ± 1.7	24.7 ± 15.7	4.7 ± 2.8
4.3	4539	100 ± 19	-14.9 ± 3.6	5.5 ± 1.9	32.1 ± 26.0	5.8 ± 4.4
5.1	4719	87 ± 7	-2.9 ± 1.7	1.9 ± 0.4	14.0 ± 5.0	7.3 ± 2.5
5.2	4732	112 ± 9	-5.1 ± 3.2	2.3 ± 0.5	9.1 ± 2.7	3.9 ± 1.2
5.3	4690	92 ± 8	-8.6 ± 1.8	2.0 ± 0.4	15.0 ± 9.8	7.7 ± 2.8
6.1	4870	153 ± 24	-8.8 ± 2.6	4.2 ± 1.3	29.2 ± 20.1	7.0 ± 4.5
6.2	4603	142 ± 19	-5.4 ± 2.2	3.3 ± 1.0	25.7 ± 19.6	7.7 ± 4.4
6.3	4593	150 ± 22	-13.3 ± 4.2	4.4 ± 1.3	18.9 ± 9.9	4.3 ± 2.1
7.1	4669	150 ± 31	-6.4 ± 2.7	5.6 ± 2.0	47 ± 92	8.4 ± 8.6
7.2	4613	148 ± 22	-4.3 ± 5.9	4.9 ± 1.5	16.1 ± 7.9	3.3 ± 1.6
7.3	4489	149 ± 22	-13.1 ± 6.0	4.6 ± 1.4	15.1 ± 7.1	3.2 ± 1.9
8.1	4706	111 ± 21	-6.0 ± 2.5	4.7 ± 1.6	43 ± 46	9.0 ± 9.1
8.2	4699	175 ± 24	-24 ± 6.8	4.1 ± 1.2	11.9 ± 9.4	2.9 ± 1.3
8.3	4686	153 ± 24	-10.8 ± 3.7	4.1 ± 1.3	22.2 ± 14.2	5.4 ± 3.2

Solutions to (40) are no longer separable. A solution to (40) is:

$$E^{+} = \frac{1}{1 + 2A\alpha\beta(\omega)N_{0}^{-4}bm_{0}^{4}\int_{0}^{z}N^{2}(z')\,dz'}\frac{bm_{0}^{2}N^{2}}{m^{2}N_{0}^{2}}\left(1 - \frac{m_{0}^{2}N^{2}}{m^{2}N_{0}^{2}}\right), \quad (43)$$
 with $\alpha = 2.0$ and dissipation rate

$$\epsilon(z) = \frac{(1 - R_f)A\phi(\omega)N^{-1}b^2m_0^4N^4N_0^{-4}}{\left[1 + 2A\alpha\beta(\omega)N_0^{-4}bm_0^4\int_0^z N^2(z')\,dz'\right]^2},\tag{44}$$

which is similar to the solution for the case of constant stratification (36) and (37), but with *N*-scaled vertical wavenumber and vertical

coordinate. The effect of scaling the vertical wavenumber in proportion to *N* amounts to an increase of high wavenumber spectral density (which depends on vertical wavenumber as m^{-2}) in proportion to N^2 . This implies a simple shift of the abscissa in Fig. 2. In contrast, energy transport to higher wavenumber associated with wavewave interactions affects a decrease in the amplitude of the spectrum, or shift in the ordinate. The increase in the high wavenumber portion of the spectrum associated with buoyancy scaling implies an increase in the energy transport to smaller scales via wave-wave interactions. The influence of buoyancy scaling is to increase spectral levels, which in turn makes wave-wave interactions more efficient and implies an increase in turbulent dissipation. Internal wave energy is thereby dissipated closer to the bottom boundary by wave-wave interactions in the presence of vertical stratification which increases with height above the bottom.

Implications of increasing stratification for the vertical diffusivity can be succinctly summarized. In the limit of large wavenumber, F/N^2 depends on N only through the vertical coordinate, i.e.

$$\kappa_{\rho} \cong \frac{F}{N^2} \cong \left(1 + 2A\alpha\beta(\omega)bN_0^{-4}m_0^4 \int_0^z N^2(z')\,dz'\right)^{-2}.$$
(45)

The effect of increasing stratification is to decrease the scale height over which the diffusivity decays. The coupling of increasing stratification and wave-wave interactions inhibits the flux of energy from the abyss to the main thermocline. Energy generation into small vertical scales at the bottom boundary has little effect on main thermocline spectral levels and thus the diffusivity there.

Given that there is little difference between the two solutions at constant *N*, it seems to make sense to implement the buoyancy scaling and accounting for the effects of momentum conservation in an ad hoc manner by retaining $\alpha = 2.31$ rather than $\alpha = 2.0$. This ad hoc approach adds little complexity to the closures (1)–(4).

Appendix C. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ocemod.2009.07.006.

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