Mesoscale Eddy - Internal Wave Coupling. Symmetry, Wave Capture and Results from the Mid-Ocean Dynamics Experiment

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Abstract

Vertical profiles of horizontal velocity obtained during the Mid-Ocean Dynamics Experiment (MODE) provided the first published estimates of the high vertical wavenumber structure of horizontal velocity. The data were interpreted as representative of the background internal wavefield and thus, despite some evidence of excess downward energy propagation associated with coherent near-inertial features that was interpreted in terms of atmospheric generation, these data provided the basis for a revision to the Garrett and Munk spectral model.

These data are reinterpreted through the lens of 30 years research. Rather than representing the background wavefield, atmospheric generation, or even near-inertial wave trapping, the coherent high wavenumber features are characteristic of internal wave capture in a mesoscale strain field. Wave capture represents a generalization of critical layer events for flows lacking the spatial symmetry inherent in a parallel shear flow or isolated vortex.
1. Introduction

Conducted during March-July of 1973, the Mid-Ocean Dynamics Experiment (MODE) was one of the first concentrated studies of mesoscale ocean variability. The experiment featured arrays of moored current meters, neutrally buoyant floats, standard hydrographic station techniques and the use of novel vertically profiling instrumentation.

Vertical profiles of horizontal velocity obtained with a free-falling instrument using an electric field sensing technique (Sanford 1975) provided the first published estimates of the high vertical wavenumber structure of horizontal velocity. The data were assumed to be representative of the background internal wavefield and thus, despite some evidence of excess downward energy propagation (Leaman and Sanford 1975) that was interpreted in terms of atmospheric generation (Leaman 1976), the data provided the basis for a revision to the isotropic Garrett and Munk spectral model (Garrett and Munk 1975). These data and their analysis are seminal in their influence of how we think about the oceanic internal wavefield.

The point of this paper is to suggest an alternate interpretation of those data, that the high wavenumber contributions are dominated by coherent features characteristic of a ‘shrinking catastrophe’ (Jones 1969) or ‘wave capture’ (Bühler and McIntyre 2005) scenario of internal wave – mesoscale eddy interaction. Wave capture is phenomenologically distinct from a parallel shear flow critical layer [e.g. Jones (1967)] or near-inertial internal wave trapping (Kunze 1985).

This interpretation is motivated by recent developments featuring the following results:

- a regional characterization of the background internal wavefield rather than the previous universal model (Polzin et al. 2006),
- revised estimates of net energy transfers between the internal wavefield and the mesoscale eddy field that indicate wave-eddy coupling is a significant, if not the primary, regional source of internal wave energy (Polzin 2006), and
- the development of a heuristic local characterization of internal wave-wave interactions (Polzin 2004) and use of that characterization in a radiation balance scheme (Müller 1976) to quantify the magnitude of the wave-eddy coupling process (Polzin 2006), with the further suggestion that wave-eddy coupling might explain some of the regional variability alluded to above (Polzin et al. 2006).

The MODE velocity profile data do not fit easily into the regional characterization promoted by Polzin et al. (2006). In that study, Eulerian frequency ($\sigma$) spectra whiter (less steep) than $\sigma^{-2}$ typically accompany vertical wavenumber ($m$) spectra redder (steeper) than $m^{-2}$. Rather than exhibiting a power law fit, the MODE gradient spectra are peaked at vertical wavelengths smaller than 100 m. This oddball warranted further investigation.

The interpretive context here is that of the WKB approximation and ray-tracing within a 3-d quasigeostrophic flow field, [Bretherton (1966); Jones (1969)]. Pertinent results are summarized in the Appendix. Internal waves are assumed to be of small amplitude and have small spatial scales relative to a geostrophically balanced background $\overline{u}(x, y, z)$ that evolves over a much longer time scale. Spatial gradients in the background are assumed to be sufficiently small that wave-mean interactions affect wave propagation only through an advective Doppler shift, $k \cdot \overline{u}$. Ray tracing features an action ($A = E/\omega$) conservation statement with variations in intrinsic frequency $\omega = \sigma - k \cdot \overline{u}$ being offset by variations in energy ($E$) following ray paths. It is crucial that the reader be cognizant that wave-mean interactions can be qualitatively different for 2- and 3-dimensional background flows.
Despite the complexity of a three dimensional background state, a simple characterization of the interaction is possible. Bühler and McIntyre (2005) point to an analogy between internal wave propagation and the problem of particle pair separation in incompressible 2-D turbulence. In this relative dispersion problem, particle pairs undergo exponential separation when the rate of strain exceeds relative vorticity:

\[ S_s^2 + S_n^2 > \zeta^2 \]  

with \( S_s \equiv \tau_x + \tau_y \) the shear component of strain, \( S_n \equiv \tau_x - \tau_y \) the normal component and \( \zeta \equiv \tau_x - \tau_y \) relative vorticity. Equation (1) is simply the Okubo-Weiss criterion [e.g., Provenzale (1999)]. Bühler and McIntyre argue that the problem of small amplitude waves in a larger scale flow field is kinematically similar to particle pair separation. If strain dominates vorticity, the ray equations lead to exponential increase/decrease in the density of phase lines, i.e., an exponential increase/decrease in horizontal \( k_h \):

\[ k_h = (k^2 + l^2)^{1/2} \]  

(Fig. 1). Vorticity simply tends to rotate the horizontal \( \text{wavevector} \) in physical space. The collapsing of phase lines in an eddy strain field provides a simple picture of how an internal wave packet interacts with an eddy strain field.

Jones (1969) and Bühler and McIntyre (2005) further argue for a ‘shrinking catastrophe’ or ‘wave capture’ scenario. Simply put, the vertical wavenumber is slaved to the horizontal, so that exponential growth of the horizontal wavenumber implies either exponential growth or decay of the vertical wavenumber in the presence of thermal wind shear. Whether the vertical wavenumber grows or decays depends upon the sign of the horizontal wavenumber relative to the thermal wind shear. Those waves with growing horizontal and vertical wavenumber magnitude will tend to be captured within the extensive regions of the eddy strain field and eventually dissipate: strain acts as a funnel to collect high wavenumber low intrinsic frequency waves. Asymptotically, a captured wave will tend to an aspect ratio of

\[ \frac{f m}{N k_h} \sim \frac{k \cdot \bar{\mathbf{u}}_z / N}{k_h \sqrt{D / f}}, \]  

with \( D \equiv (S_n^2 + S_s^2 - \zeta^2)/4 \), vertical wavenumber \( m \), buoyancy frequency \( N \) and Coriolis frequency \( f \). This finite aspect ratio implies the captured wave tends to an intrinsic frequency larger than \( f \). Bühler and McIntyre (2005) refer to wave capture as a non-trivial variant of a critical layer in a two dimensional flow, which in turn is characterized by linear growth of the cross-stream and vertical wavenumbers. The intrinsic frequency consequently tends to \( f \) in the 2-d problem. See the Appendix for details.

The situation envisioned by Jones (1969) and Bühler and McIntyre (2005) is an idealized representation of the oceanic mesoscale for several reasons. First, although there is arguably a scale separation between internal waves and mesoscale eddies, it is not obvious that this scale separation is sufficient for the asymptotic results quoted above to be applicable: wave propagation may result in the termination of a capture event. A second complication is one of time dependence in the slowly evolving mesoscale field. Müller (1976) points out that a resonance condition is possible if the progression of the ray trajectory matches the phase velocity of the mesoscale. Within the thermocline, mesoscale features are observed to migrate westward at a speed of approximately 2 km day\(^{-1}\), or about 2 cm s\(^{-1}\), and somewhat faster (5 cm s\(^{-1}\)) at depth (Freeland and Gould 1976). These westward drifts are also typical group velocities for high vertical wavenumber, near-inertial internal waves. Finally, waves with increasing horizontal wavenumber magnitude and decreasing vertical wavenumber magnitude are undergoing transport to higher intrinsic frequency. With a buoyancy profile that decreases with depth, such
transport can lead to trapping in the upper ocean as the wave reflects from the ocean surface and lower turning points.

Previous analyses of oceanic observations have described the essential behavior of internal wave propagation in a 3-dimensional background flow. Mied et al. (1987) and Mied et al. (1990) use ray-tracing to depict the rotation and change in magnitude of the horizontal wavevector and evolution of vertical wavenumber for a coherent near-inertial wave packet propagating in a mesoscale eddy field. Joyce and Stalcup (1984) document a high frequency wave embedded in an upper ocean front. All these studies discuss the departures of the background flow from a state of symmetry, i.e. a planar shear flow (jet) or azimuthal vortex (ring). None provide the succinct summary inherent in the Okubo-Weiss criterion (1) nor make a distinction between relative vorticity and rate of strain.

In the absence of such a succinct summary it is easy to appreciate that near-inertial internal wave trapping (Kunze 1985) has provided the conceptual paradigm for internal wave - mean flow interactions. Kunze (1985) argues that the effective lower bound of the internal waveband is not the Coriolis frequency \( f \), but rather \( f + \zeta / 2 \), so that near inertial waves (where near inertial is taken to be near the effective Coriolis frequency) located in regions of negative relative vorticity will have frequencies below \( f \) and so will be unable to propagate outside that region.

Near-inertial internal wave trapping was originally motivated by observations (Kunze and Sanford 1984) of large amplitude high wavenumber near-inertial waves in an upper ocean frontal regime. Such waves were found to the west of a southward velocity maximum, i.e. in a region of negative relative vorticity. Kunze (1985) argues that the wave-mean scale separation requirement of the WKB approximation can be relaxed and derives a dispersion relation that, in the limit of small Doppler shifting (wave propagation across the frontal jet), can produce the phenomenology of wave trapping. Subsequent detailed studies (Kunze et al. 1995; Kunze and Toole 1997) justifying this characterization have been in symmetric (2-d) background flows (a Gulf Stream Warm Core Ring and vortex cap atop a sea mount, respectively) in which the observed wavefields had negligible Doppler shifting. In the sea mount example, the generation of a subinertial diurnal internal tide was documented. In the warm core ring, the horizontal structure of the wavefield was nearly identical to that of the ring and the presumed source of the observed wavefield was inertial pumping associated with mixed layer motions oscillating at the effective Coriolis frequency, Weller (1982).

In a more generic 3-d frontal regime or mesoscale eddy field for which (1) applies, it is not clear that near-inertial internal wave trapping is a relevant conceptual paradigm for at least two reasons. First, Doppler shifting will typically make an \( O(1) \) contribution to the dispersion relation (Polzin et al. 1996), significantly larger than the \( O(R_o = \zeta / f) \) correction that is responsible for trapping nominally subinertial \( (f + \zeta / 2 \leq \omega \leq f) \) internal waves in regions of negative relative vorticity, (Kunze 1985). See the Appendix for further detail. Second, a notable loose end is that Kunze’s near-inertial wave trapping scenario requires trapped waves to originate inside regions of negative relative vorticity. D’Asaro (1995) finds no evidence that near-inertial mixed layer oscillations are refracted by the relative vorticity structure of a weak 3-d eddy field. In retrospect, it is obvious that the original interpretation of observations reported in Kunze and Sanford (1984) as an example of near-inertial internal wave trapping should be regarded with caution. That data set consisted of cross-frontal surveys with no ability to constrain the along front phase variations associated with Doppler shifting and Kunze (1985) develops the idea of near inertial wave trapping by taking the expedient limit of setting the Doppler shift to zero.

In the following, the MODE data are examined for characteristics of wave propagation
relative to the the mean flow gradients and an interpretation is forwarded that they are consistent with the wave capture scenario rather than representing the background internal wave state or the direct products of atmospheric forcing.

2. The MODE Data Set, Revisited

The data examined here were obtained as a times series of 28 deployments of an electromagnetic velocity profiler within 20 km of the MODE 1 central mooring (28° N, 69° 40' W) during June 11-15. Leaman and Sanford (1975) and Leaman (1976) report on a subset (20) of these profiles that were obtained directly on top of the central mooring. Sanford (1975) uses these profiles (profiles 219-247) to examine the spatial and temporal variability over small horizontal and temporal lags. The following analysis focuses upon 37 profiles (of 56 possible up and down traces) having data gaps no larger than three contiguous points. Data gaps have been filled through linear interpolation. The conductivity sensor used in this field program is noisy and consequently buoyancy perturbations $b' = g(\rho - \overline{\rho})$ have been estimated with potential temperature ($\theta$) alone. This implicitly assumes a constant temperature-salinity ($\theta - S$) relation. The analysis is consequently restricted to main thermocline (470-1100 m) and abyssal (2100-2730 m) depths. The $\theta - S$ relation at these depths is climatologically tight. Uncertainties attributable to this assumption are $O(10\%)$ within the main thermocline and do not affect the interpretation of the results presented here. At intermediate levels influenced by Mediterranean Water characteristics, water mass variability precludes buoyancy estimates. Temperature gradient estimates at depths greater than 3000 m are increasingly contaminated by instrumental noise. Diagnostics for this study are provided by relations based upon linear internal wave kinematics, Table 1. In the following $E = E(m)$ is the vertical wavenumber ($m$) energy spectrum, with subscripts denoting various decompositions.

Prior analysis of these data points out that velocity profiles separated by half an inertial period tend to mirror image each other, Fig. 2. Thus much of the finescale velocity structure is near-inertial. In contouring the 5-day time series of velocity profiles, upward phase propagation is evident, Fig. 3. This implies an excess of downward energy propagation and is consistent with the observed clockwise (cw) rotation with depth of the velocity vector, Leaman and Sanford (1975).

But with the hindsight of 30 years and the personal experience of analyzing thousands of similar profiles, my claim is that data from the thermocline region (500-1000 m) are subtly odd. First, the velocity profile data are not self-similar with depth. The thermocline region appears to have an excess of small scale energy, which can be quantified with the gradient spectra, Fig 4. In particular, clockwise ($2m^2E_{cw}$) shear is generally enhanced over counter-clockwise ($2m^2E_{ccw}$) and is peaked about wavelengths ($\lambda_v$) of 60 m. The excess energy is near-inertial in character, and this excess can be quantified through the ratio of horizontal kinetic ($E_h$) to potential energy ($E_p$), Fig. 5. This ratio exhibits a peak coincident with the peak in the cw spectrum. Abyssal spectra (Fig. 4) are also peaked, but at smaller wavenumbers ($\lambda_v = 320$ m) which can’t be explained simply through buoyancy scaling: the WKB approximation implies $\lambda_v \propto 1/N$ and the ratio between thermocline and abyssal stratification is a factor of 3.3, smaller than the ratio of the wavenumber peaks. Secondly, the time-depth series (Fig. 3) reveals the presence of coherent wave packets. The presence of coherent features is not representative of the ‘background’ wavefield. A vertical wavenumber domain coherence analysis between velocity and buoyancy perturbations (Fig. 6) and the two horizontal velocity components (Fig. 7) returns
large estimates of coherence at the peak in the shear spectrum, $\lambda_v \approx 60$ m. Coherence estimates are uniformly largest at 64 and 58.2 m vertical wavelengths. In the following an attempt is made to diagnose wavepacket characteristics using amplitude, coherence and phase estimates at these peak wavenumbers.

a. Main Thermocline Wave Estimates [$\lambda_v = 64$ m and $\lambda_v = 58.2$ m]

1) Estimates of Wave Frequency

One estimate of wave frequency can be obtained from the estimates of velocity - buoyancy phase, Fig. 6 and Table 1. The phase estimates exhibit a trend over the range of wavenumbers comprising the packet. Noting, however, that the product of the two phase estimates $[\tan(\varphi_{v'v'}) \tan(\varphi_{u'v'}) = -\omega^2/\omega^2$ for a single wave] is much more constant than their ratio, the phase estimates at the packet peak are combined to estimate

$$\omega = 1.0 \pm 0.1 f$$

with indicated error representing one standard deviation, Bendat and Piersol (1986)$^1$.

This frequency estimate is compatible with that obtained by interpreting the observed energy ratio estimates over the same bandwidth as the phase estimates,

$$R_{\text{energy}} \equiv \frac{E_k}{E_p} = 12.6,$$

in terms of a single wave:

$$\frac{\omega}{f} = \sqrt{\frac{E_k + E_p}{E_k - E_p}} = 1.08.$$

This second frequency estimate is arguably biased by contributions associated with the background wavefield and vortical motions (Polzin et al. 2003). Taking this contribution to be given by the incoherent power, and attributing this entirely to $E_p$, an alternate estimate of the wave packet energy ratio is:

$$R_{\text{energy}} \equiv \frac{E_k}{E_p} = 18 - 24,$$

with wave frequency

$$\frac{\omega}{f} = \sqrt{\frac{E_k + E_p}{E_k - E_p}} = 1.04 - 1.06.$$

A third frequency estimate is possible using the rotary estimates:

$$R_{\text{rotary}} \equiv \frac{E_{cw}}{E_{ccw}} = 8.3,$$

for which

$$\frac{\omega}{f} = \frac{\sqrt{R_{\text{rotary}} + 1}}{\sqrt{R_{\text{rotary}} - 1}} = 2.1.$$

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$^1$Uncertainty estimates presented in this work represent random errors assuming Gaussian statistics. Bias errors are possible, for example, in interpreting the energy ratio $R_{\text{energy}}$ as a frequency or aspect ratio characterizing a broadband spectrum, e.g. Polzin et al. (1995). Bias errors are not, in general, reported here as such estimates require making unverifiable assumptions about the statistics of the contaminating field.
The implication here is that there is a significant contamination to the single wave interpretation of the rotary diagnostic. If $\omega = 1.08 f$, the single wave rotary ratio is 676. Nonlinearity can provide a rational for reduced rotary ratios at near-inertial frequencies.

In the cascade representation of nonlinearity described in (Polzin 2004), momentum conservation is obtained by backscattering into an oppositely signed wavevector at a rate proportional to the downscale energy cascade. For a boundary source, this closure scheme returns a peak in the ratio of upward and downward propagating wave spectra at wavenumbers a factor of two to three smaller than that defined by $2 \int_0^{m_c} m^2 E_k dm = 0.7 N^2$. In the background wavefield, $m_c = 0.1$ cpm. The peak in $cw/ccw$ spectral ratio occurs at a wavenumber $m_p = 58.2$ m for which the shear variance $2 \int_0^{m_p} m^2 E_k dm = 0.32 N^2$. Interpreting rotary ratio as being significantly contaminated and rejecting the rotary estimate of wave frequency is consistent with that closure scheme.

A fourth estimate is that of Eulerian frequency. Assuming a single plane wave solution, wave phase ($\varphi$) is given by the linear relation:

$$\varphi = mz - \sigma t, \quad (3)$$

so that with positive $\sigma$, wave crests (lines of constant phase) propagate upward if $m > 0$. Here the Fourier transform is used to isolate the packet wavelength over the transform interval so that the term $mz$ in (3) is effectively constant. The rotary decomposition effectively isolates upward and downward phase propagation for near-inertial waves, so that the time rate of change of observed $cw$ Fourier coefficient phase, Fig. (10), provides an estimate of the wave packet’s Eulerian frequency. Linear regression of the phase against time returns:

$$\sigma = 4.4 \pm 0.15 \times 10^{-5} \text{s}^{-1},$$

with the quoted uncertainty representing one standard error. The estimate is subinertial ($f = 6.8 \times 10^{-5} \text{s}^{-1}$) and linear internal wave kinematics (i.e. a super-inertial intrinsic frequency) requires significant Doppler shifting of a northward propagating wave.

2) Horizontal Wavenumber Azimuth Estimates

Using the linear diagnostic $\tan(\varphi^v^u) / \tan(\varphi^v^u) = k^2 / -l^2$, estimates of velocity-buoyancy phase return, with one standard deviation uncertainty:

$$\frac{|k|}{|l|} [\lambda_v = 64m, \lambda_v = 58.2m] = [1.09(0.95 - 1.25), 1.99(1.69 - 2.34)].$$

Phase propagation of this wave is to the north and west, in addition to being upward. The linear diagnostic for the orientation of the major axis of the current ellipse (Table 1) returns:

$$\theta_n[\lambda_v = 64m, \lambda_v = 58.2m] = [0.97, 0.64] \text{ radians.}$$

With $(k, l) = k_h[\cos(\theta_n), \sin(\theta_n)]$

$$\frac{|k|}{|l|} [\lambda_v = 64m, \lambda_v = 58.2m] = [0.69, 1.34].$$

The trend of the wavevector’s horizontal azimuth with increasing vertical wavenumber estimated from the orientation of the major axis of the horizontal current ellipse is consistent
with the horizontal azimuth estimated from the velocity-buoyancy phase. The mean, however, differs. The major axis estimate of the horizontal azimuth ($\theta_n$) is regarded as being less reliable than that associated with the velocity-buoyancy phase because of possible subtractive cancellation in the denominator of the normal coordinate definition (Table 1). Consequently the velocity-buoyancy phase estimates of horizontal azimuth are used below.

The dispersion relation provides an estimate of horizontal wavenumber magnitude $[k_h = \pm m(\omega^2 - f^2)^{1/2}/N]$, and thus with $|k|/|l| = 1.5$ and $\omega = 1.08 f$:

$$(k, l) = (-2\pi/11, 2\pi/17) \text{ km}^{-1}.$$ 

3) Wave Mean Interactions

Further insight can be gained by interpreting the data in the context of the geostrophic flow field. The mean baroclinic shear profile (Fig. 11) indicates southward, surface intensified flow. The MODE Central current meter at nominally 3000 m depth indicates a mean current of $(u, v) = (5.8, 2.8) \text{ cm s}^{-1}$ over the time period of the profiler data set, so that only minor adjustments to the baroclinic profile from the electromagnetic velocity profiler (Fig. 11) are required to render the velocity estimates absolute. The Doppler shift associated with advection by the mean field is significant, $-\vec{v} \cdot \vec{n} \approx \frac{2\pi}{17} \text{ km} \cdot 0.07 \text{ m s}^{-1} = 2.6 \times 10^{-5} \text{ s}^{-1}$, relative to $f = 6.8 \times 10^{-5} \text{ s}^{-1}$ and makes an $O(1)$ contribution in the dispersion relation. Advection by the mean flow at the level of the transform interval, $(\vec{u}, \vec{v}) = (0, -7) \text{ cm s}^{-1}$, dominates the rate at which the coherent features can propagate through: $C_g = (-1.3, 0.8) \text{ cm s}^{-1}$. The Eulerian frequency and Doppler shift estimates, $\sigma - \vec{v} \cdot \vec{n} = 4.4 \pm 0.15 \times 10^{-5} \text{ s}^{-1} + 2.6 \pm 0.4 \times 10^{-5} \text{ s}^{-1}$ combine for a consistent estimate of the intrinsic frequency with $\omega = 1.08 f = 7.3 \times 10^{-5} \text{ s}^{-1}$. With horizontal phase propagation to the NW and downward energy propagation in a southward jet, the coherent features are propagating in thermal wind shear with increasing vertical wavenumber, $dm/dt = -\vec{v} \cdot \vec{z} > 0$.

Having ascertained the importance of Doppler shifting and thermal wind shear for the vertical evolution of the thermocline wavepacket, the remaining question is the role of the horizontal gradients. Maps of the geostrophic stream function (McWilliams (1976); The Mode-I Atlas Group (1977)) indicate a straining feature immediately to the north of the experimental site, Fig. 12. In the analysis of Bühler and McIntyre (2005) and Jones (1969), this strain will preferentially orient wave fronts, cascade wave phase to higher horizontal wavenumber and act as a funnel in advecting high wavenumber low intrinsic frequency waves southward. On the other hand, the same map could be cited to support the significance of relative vorticity at the time and location of the observations.

One proposed role of relative vorticity is near-inertial wave trapping. Kunze (1985) advocates the use of

$$\omega \cong f + \zeta/2 + \frac{N^2 k_h^2}{2 f m^2} + \frac{1}{m} [\vec{u} z - \vec{k} \vec{v} z]$$

rather than the more conventional

$$\omega = \left(\frac{N^2 k_h^2 + f^2 m^2}{m^2}\right)^{1/2}.$$
Estimating relative vorticity \( \zeta = \nabla_x - \nabla_y \) magnitude as \( \zeta \cong \nabla_x = \nabla/L \), with \( \nabla = -0.07 \text{ m s}^{-1} \) taken as the average velocity in the absolute profile (Fig. 11) over the transform interval and length scale \( L = 100 \text{ km} \) taken from the geostrophic streamfunction map, Fig. (12), the contribution of relative vorticity to the proposed dispersion relation is nearly two orders of magnitude smaller than either intrinsic, Eulerian, or Coriolis frequencies. The term representing buoyancy forces is an order of magnitude larger. Consequently I am led to the inference that the dynamics in this three dimensional system is dictated by variations in the Doppler shift rather than variations in relative vorticity.

A second proposed role is through (1). Averaged over daily intervals, southward thermocline shear exhibits neither a decreasing nor increasing trend and suggests that, with the observed westward drift of the southward jet (The Mode-I Atlas Group 1977), the observations were obtained near the velocity maximum of the southward jet. The velocity maximum represents a region of sign transition for the relative vorticity and implies strain locally dominates vorticity. However, it is not simply the local conditions that dictate the wave packet’s characteristics.

In the context of a ray tracing analysis the observed packet characteristics represent the time history of wave-mean interactions along the ray path. For near-inertial waves with horizontal group velocity small relative to the background velocity, the ray trajectories are clearly dominated by horizontal advection. It is a straightforward matter to summarize the stream function in Fig. (12) with an analytic representation and back trace internal waves through it\(^2\). It is more involved, but still possible, to estimate the geostrophic stream function with a time dependent objective mapping procedure and back trace rays through that field. Experience with the analytic representations suggest that, since the system is dominated by horizontal advection, it is a trivial matter to select initial conditions that are not statistically different from the observed wave characteristics that have ray trajectories leading back to where the rate of strain variance exceeds the vorticity variance. But without observations of the spatial/temporal evolution of the wave packet, neither exercise proves anything other than that the observations are plausibly consistent with the dynamics being dominated by variations in Doppler shifting.

Asymptotically, the combination of horizontal straining and propagation in vertical shear will produce a wave aspect ratio of (Bühler and McIntyre 2005):

\[
\frac{f m}{N k_h} \sim \frac{\nabla_z / N}{k_h \sqrt{D}/f} = 3.8
\]

with \( \nabla_z = -2.4 \times 10^{-4} \text{ s}^{-1} \) and \( \sqrt{D} \cong \nabla_y = \nabla/L \), with \( \nabla = -0.07 \text{ m s}^{-1} \) taken as averages from the absolute velocity profile (Fig. 11) over the transform interval and length scale \( L = 100 \text{ km} \) taken from the geostrophic streamfunction map, Fig. (12). Predicted energy ratios are

\[
\frac{E_k}{E_p} = 2 \times \left( \frac{f m}{N k_h} \right)^2 + 1 = 30,
\]

which is significantly larger than either the observations (12.6) or that associated with the GM model (3). Note that maxima in vertical shear and horizontal strain are likely not collocated so that \( \nabla_z / \sqrt{D} \) varies over the advective time scale \( \nabla/L \cong \sqrt{D} \) and that the estimated zonal

\(^2\)Try, for example, \( \psi(x, y, z, t) = \psi_o \sin(K x - \Omega t) \sin(L y) \cos(M z) \) with \( (\nabla, \nabla) = (\psi_y, -\psi_x) \), \( \Omega = -\beta K / [K^2 + L^2 + (f M / N)^2] \), \( \beta = 2 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1} \), \( M \) representing the first baroclinic mode in a 2000 m deep ocean, \( f = 6.8 \times 10^{-5} \text{ s}^{-1} \), \( N = 4.2 \times 10^{-3} \text{ s}^{-1} \), \( K = L = 2\pi / 400 \text{ km}^{-1} \) and \( \psi_o \) taken to give maximum velocities of 0.25 m s\(^{-1}\), \( \psi_o = 0.25 / \sqrt{K^2 + L^2} \text{ m}^2 \text{ s}^{-1} \) and initial position \( x(t = 0) = [x = 0, y = -100 \text{ km}, z = 1100 \text{ m} \text{(above bottom)}] \).
group velocity is similar to the observed westward drift of the mesoscale eddy field. These are conditions not considered in the asymptotic result (2).

I simply conclude that the MODE thermocline data are consistent with the dynamics being dominated by variations in Doppler shifting, of which wave capture is an asymptotic expression.

b. Abyssal Wave Estimates \([\lambda_v = 320 \text{ m}]\)

The abyssal wave signatures are more difficult to characterize, in part because the wave packet at \(\lambda_v = 320 \text{ m}\) does not stand out as strongly from the background, but also because of lower signal to instrumental noise ratios. Whatever the cause, velocity-buoyancy coherence is not statistically different from zero, Fig. 8. The restricted set of diagnostics are:

\[
R_{\text{energy}} \equiv \frac{E_k}{E_p} = 5.0, \\
R_{\text{rotary}} \equiv \frac{E_{cw}}{E_{ccw}} = 5.2 \quad \theta_n = 1.20 \text{ radians} \\
\sigma = 7.70 \pm 0.08 \times 10^{-5} \text{ s}^{-1}.
\]

The major axis lies in the NNW/SSE direction.

Taking these estimates at face value, then, and interpreting them in terms of a single wave:

\[
\frac{\omega}{f} = \sqrt{\frac{E_k + E_p}{E_k - E_p}} = 1.22.
\]

\[
\frac{|k|}{|l|} [\lambda_v = 320 \text{ m}] = 0.39.
\]

Assuming propagation to the northwest,

\[(k, l) = (-2\pi/25, 2\pi/9.8) \text{ km}^{-1}.
\]

Advection by the mean flow \((\overline{u}, \overline{v})=(4.7, 2.1) \text{ cm s}^{-1}\) is difficult to distinguish in magnitude from the nominal group velocity \(C_g = (-1.5, 3.7) \text{ cm s}^{-1}\) associated with the coherent features. Doppler shifting,

\[-k\overline{u} - l\overline{v} \simeq \frac{2\pi}{25 \text{ km}} 0.047 \text{ m s}^{-1} - \frac{2\pi}{9.8 \text{ km}} 0.021 \text{ m s}^{-1} = 1.2 \times 10^{-5} \text{ s}^{-1} - 1.3 \times 10^{-5} \text{ s}^{-1},
\]

is not significantly different from zero, indicating that wave crests are aligned with the streamlines. The flow field at these depths is directed slightly north of east, The Mode-I Atlas Group (1977), rather than southward as noted in the thermocline.

3. Summary and Discussion

The data set of velocity and temperature profiles obtained by Tom Sanford during MODE (Sanford 1975; Leaman and Sanford 1975; Leaman 1976) contains a near-inertial wave packet with vertical wavelength of 60 m and wavevector directed upward and to the northwest. Ratios of horizontal kinetic to potential energy are significantly larger in the main thermocline than at mid-depth or specified through the GM models. The large ratios indicate greater near-inertial
energy and are dominated by contributions from the wave packet. Despite these observations providing the basis for a revision to the Garrett and Munk empirical description of the background wavefield (Garrett and Munk 1975), the presence of the wave packet and anomalous characteristics at high vertical wavenumber are not consistent with the data set representing the background wavefield. Moreover, the wave packet is not likely to be the direct by-product of atmospheric generation (Leaman 1976) as its vertical group velocity places it some 80 days away from the surface, comparable to the dissipation time scale for the entire background internal wavefield. A near-inertial wave having the observed vertical wavelength and amplitude would likely dissipate over a much shorter time scale. The MODE thermocline data are consistent with the dynamics being dominated by variations in Doppler shifting. Near-inertial wave trapping (Kunze 1985), for which Doppler shifting is assumed to be negligible, is not an appropriate characterization of the wave-mean interaction in this 3-d background field.

Doppler shifting of the wave packet is significant and the vertical wavenumber magnitude is increasing as the wave propagates in thermal wind shear at the base of the thermocline. Bühler and McIntyre (2005) argue that the problem of small amplitude waves interacting with a larger scale 3-d flow field only through Doppler shifting is kinematically similar to the issue of particle pair separation. If the background strain variance dominates the background vorticity variance, the ray equations lead to an exponential increase/decrease in horizontal wavenumber. Vorticity simply tends to rotate the horizontal wavevector in physical space. In this 3-d problem, the vertical wavenumber is slaved to the horizontal, so that exponential growth of the horizontal wavenumber implies either exponential growth or decay of the vertical wavenumber in the presence of thermal wind shear. Those waves with growing horizontal and vertical wavenumber magnitude will tend to be captured within the extensive elements of the eddy strain field and eventually dissipate: an extensive element acts as a funnel to collect high wavenumber low intrinsic frequency waves. Estimates of the geostrophic streamfunction (McWilliams 1976) place the observations within an extensive element of the mesoscale strain field.

This mechanism is essentially that invoked by Müller (1976) in a much more sophisticated treatment of a radiation balance equation. That approach enables calculation of the rate at which energy and momentum are transferred from the mesoscale eddy field to the internal wavefield. That model treats mesoscale eddies as a 3 dimensional field and permanent transfer is associated with wave dissipation, but one’s notion of “dissipation” needs to include the irreversible effects of weak nonlinearity acting on the internal wavefield. With this expanded notion and, if the ability of internal wave packets to escape capture are accounted for, Müller’s model can be manipulated to make sensible predictions for the observed transfer rates (Polzin 2006).

The bulk of the literature regarding internal wave-mean flow interactions focusses upon symmetric flows, but the consequences of assuming symmetric background states are virtually unappreciated. The introduction of symmetry in the case of parallel shear flows (Jones 1967) or circular vortices (Ooyama 1967) greatly simplifies the analysis. But it comes at a cost. In a zonal flow, the flux of zonal angular pseudomomentum \( C_g k \hat{A} \) is nondivergent, for a circular vortex the flux of azimuthal pseudomomentum is nondivergent. This constraint is usually referred to as Andrews and McIntyre’s generalized Eliassen-Palm flux theorem:

\[
\frac{d}{dt} k \hat{A} + \nabla \cdot F = D + O(\alpha^3); \tag{4}
\]

stating that in the absence of dissipation \( D \) and nonlinearity (small wave amplitude \( \alpha \) limit), and for steady conditions, the pseudomomentum flux \( F \) is spatially nondivergent. In the ray
tracing limit, $F = kC_gA$ and the zonal wavenumber $k$ is constant, so that the Eliassen-Palm flux theorem is nothing more than an action flux conservation statement.

What is not immediately obvious is that with action conservation, the pseudomomentum flux is not conserved for asymmetric (3-d) flows. A simple rationalization of the difference in behavior between 2-d and 3-d systems comes from theoretical physics: Each symmetry exhibited by a Hamiltonian system corresponds to a conservation principle [Nöther’s theorem, e.g. Shephard (1990)]. For spatial symmetries the conservation principle concerns momentum: axisymmetric flows preserve the flux of pseudomomentum in the symmetric coordinate. The straining of waves provides the essential mechanism through which streamwise pseudomomentum is not conserved.

There are profound consequences for pseudomomentum and vorticity that are addressed in Bühler and McIntyre (2005) and followed up in a companion manuscript (Polzin 2006). The momentum flux associated with a plane internal wave of infinite spatial extent is nondivergent and that plane wave has no potential vorticity signature. However, a wavepacket of finite extent in both horizontal directions will have a momentum flux divergence associated with its envelope structure, which in turn induces accelerations on the packet scale resulting in a dipole vortex structure. In a 3-d system, pseudomomentum is not conserved following wave-mean interactions and Bühler and McIntyre (2005) argue that with modulation of the wave pseudomomentum are modulations in the potential vorticity signature of the wavepacket. The modulation of wave pseudomomentum by mesoscale eddy strain is especially significant as it provides a mechanism for damping potential vorticity anomalies associated with the mesoscale eddy field: The wave packet undergoing capture has a vorticity signature that opposes the vorticity distribution in the eddy field, Fig. 12. Dissipation of the captured wave implies a permanent mixing of potential vorticity.

This transfer of wave pseudomomentum for eddy potential vorticity is a characteristic of waves interacting with a 3-d background. It does not occur within a 2-d (symmetric) background. Thus for the purpose of this paper, a mesoscale eddy is not an axisymmetric vortex or jet. The pertinent quantity is a nonzero rate of strain, and on this ground, a linear Rossby wave field qualifies.

Finally, the Okubo-Weiss criterion (1) is a generic description of wave-mean interactions when the mean is an asymmetric nondivergent flow. This characterization (1) comes about simply through the Doppler shift and is independent of the character of the dispersion relation. In particular, it carries over to the interaction of Rossby Waves with the General Circulation and may be a key feature of upgradient potential vorticity transfers and/or Recirculation Gyre dynamics.

In summary, my hypothesis is that the bandwidth limited, coherent features in this data set are a product of the horizontal strain and vertical shear within the geostrophic flow field.
Acknowledgments.

I am indebted to Tom Sanford for his assistance in making this investigation possible. In forwarding this interpretation, I am cognizant of being afforded the luxury of hindsight. The motivation for pursuing reanalysis of the MODE data is not to refute the prior analysis. Thirty years worth of communal research and personal experience analyzing many similar data sets provide a much more substantial backdrop against which to appreciate the idiosyncrasies of this particular data set. The motivation is the historical context of this data set and an appreciation that answers to some very profound and fundamental questions about rotating stratified fluids have been in sight for more than three decades.

I thank M. McIntyre for forwarding a preprint of Bühler and McIntyre (2005) and subsequent discussions about the topic. Salary support for this analysis was provided by Woods Hole Oceanographic Institution bridge support funds.
APPENDIX

Wave-Mean Interactions in the Ray Tracing Paradigm

Consider trying to describe the interaction of internal waves and mesoscale eddies through a self consistent expansion of the equations of motion. Such a derivation starts by invoking a decomposition of velocity \[ \mathbf{u} = (u, v, w) \], buoyancy \[ b = -g \rho / \rho_0 \] with gravitational constant \( g \) and density \( \rho \) and pressure \( \Pi \) fields into ‘mean’ (\( \bar{\cdot} \)) and small amplitude internal wave (\( \cdot' \)) perturbations on the basis of a time scale separation: \( \phi = \bar{\phi} + \phi' \) with \( \bar{\phi} = \tau^{-1} \int_0^\tau \phi \, dt \) in which \( \tau \) is much longer than the internal wave time scale but smaller than the eddy time scale. Progress depends upon deriving a wave equation and seeking approximate solutions to that equation.

a. asymmetric (3d) background fields

For 3d \( \bar{\phi} = \bar{\phi}(x, y, z) \) background fields, derivation of a wave equation is limited to backgrounds having small lateral and vertical gradients \( \text{with Rossby number} \ R_o = \zeta / f = (\nabla_x - \nabla_y) / f \ll 1 \) and Froude number \( F_r = \left( \frac{u^2 + v^2}{N} \right)^{1/2} / N \ll 1 \) and requires that the spatial scales of the wavefield be much smaller than the background. This is the ray tracing paradigm with locally valid plane wave solutions (i.e. all wave variables are assumed to be proportional to \( \exp(i[\mathbf{k} \cdot \mathbf{x} - \sigma t]) \) with wavenumber \( \mathbf{k} = (k, l, m) \), horizontal wavenumber magnitude \( k_h = (k^2 + l^2)^{1/2} \) and Eulerian frequency \( \sigma \).

Ray tracing implies continuity of a real valued phase function \( \Omega = \omega + \mathbf{k} \cdot \mathbf{u} \), with intrinsic frequency given by a dispersion relation. Here the dispersion relation is \( \omega^2 = \left( \frac{N^2 k_h^2 + f^2 m^2}{m^2} \right) \). Ray tracing also implies conservation of wave action, \( A = E / \omega \), so that for steady conditions in the absence of nonlinearity and dissipation the action flux is non-divergent:

\[
\nabla \cdot (\nabla + C_k) A = 0. \tag{A1}
\]

The evolution of the wavevector following the wavepacket is given by the spatial gradient (denoted by \( \nabla \)) of the phase function:

\[
\frac{d\mathbf{k}}{dt} = -\nabla \Omega \tag{A2}
\]

Component-wise for the wavevector evolution:

\[
\frac{dk}{dt} = -k \bar{\nu}_x - l \bar{\nu}_y - m \bar{\nu}_z \tag{A3}
\]
\[
\frac{dl}{dt} = -k \bar{\nu}_y - l \bar{\nu}_x - m \bar{\nu}_z
\]
\[
\frac{dm}{dt} = -k \bar{\nu}_z - l \bar{\nu}_x - m \bar{\nu}_y.
\]

Consistent with the slowly varying approximation, the velocity gradients are viewed as constant along ray paths and (A3) represents a system of linear first order constant coefficient differential equations. Assuming that \( \mathbf{k} \propto e^{\alpha t} \), the system (A3) can be reduced to a cubic equation for \( \alpha \):

\[
\alpha^3 + \alpha \left[ \bar{\nu}_x \bar{\nu}_y - \bar{\nu}_x \bar{\nu}_y + \bar{\nu}_z (\bar{\nu}_x + \bar{\nu}_y) - (\bar{\nu}_y \bar{\nu}_z + \bar{\nu}_x \bar{\nu}_z) \right] \\
+ \bar{\nu}_z (\bar{\nu}_x \bar{\nu}_y - \bar{\nu}_y \bar{\nu}_x) + \bar{\nu}_y (\bar{\nu}_x \bar{\nu}_z - \bar{\nu}_x \bar{\nu}_z) + \bar{\nu}_x (\bar{\nu}_y \bar{\nu}_z - \bar{\nu}_y \bar{\nu}_z) = 0. \tag{A4}
\]
In the quasigeostrophic approximation, for which $\zeta / f \equiv R_o \ll 1$ and the characteristic 'x' and 'y' length scales of the geostrophic flow field are nearly equal, $\nabla z \sim O(R_o^2)$. At lowest order (A4) reduces to:

$$\alpha = \pm [S_n^2 + S_s^2 - \zeta^2]^{1/2}/2.$$  \hspace{1cm} (A5)

in which

$$\zeta \equiv \nabla_x - \nabla_y$$  

(A6)

$$S_n \equiv \nabla_x - \nabla_y$$

$$S_s \equiv \nabla_x + \nabla_y$$

$$\Delta \equiv \nabla_x + \nabla_y$$

the variables $S_n$ and $S_s$ represent normal and shear components of the rate of strain tensor, $\zeta$ is relative vorticity and $\Delta$ is the horizontal divergence.

The phenomenology of (A5) is simple. Solutions to (A5) represent exponential growth and decay if $S_n^2 + S_s^2 > \zeta^2$ and have an oscillatory component otherwise.

**Wave Capture**

Jones (1969) and Bühler and McIntyre (2005) further argue for a ‘shrinking catastrophe’ or ‘wave capture’ scenario. Since

$$\frac{dm}{dt} = -k \nabla_z - \nabla_z,$$

exponential increase of $k_h$ corresponds to exponential increase/decrease of $m$, depending upon the sign of $(\nabla_z, \nabla_z)$ relative to the horizontal wavevector $(k, l)$. Those waves with growing vertical wavenumber will tend to be captured within the extensive regions of the eddy strain field and eventually dissipate, leading Bühler and McIntyre (2005) to characterize the interaction as a nontrivial variant of critical layer behavior. A captured wave with exponentially growing horizontal and vertical wavenumber will asymptotically tend to a super-inertial intrinsic frequency that depends upon the aspect ratio of the background flow, (2).

b. symmetric (2d) backgrounds

Internal wave - mean flow interactions are qualitatively different for symmetric flows such as a zonal jet with $[\nabla, \nabla] = [\nabla(y, z), 0]$ or an axisymmetric vortex. While one can derive wave equations for such flows without going into the low $R_o, F_r$ parameter regime and without the requirement of a scale separation, the impact of symmetry on behavior is best appreciated in those limits. Component-wise for the wavevector $(k, l, m)$ in a zonal $[\nabla(y, z)]$ flow:

$$\frac{dk}{dt} = 0$$

$$\frac{dl}{dt} = -k \nabla_y$$

$$\frac{dm}{dt} = -k \nabla_z$$  \hspace{1cm} (A7)

Differentiating with respect to time:

$$\frac{d^2k}{dt^2} = 0$$
\[
\frac{d^2 l}{dt^2} = 0 \\
\frac{d^2 m}{dt^2} = 0 
\] 

(A8)

The growth of meridional and vertical wavenumbers are linear in time and the zonal wavenumber is constant.

Critical Layers

Critical layer behavior is exhibited as a wave packet, having constant zonal wavenumber (k) and Eulerian frequency \( \sigma \), evolves through vertical propagation in a sheared flow \( \overline{u}(z) \) such that the intrinsic frequency

\[
\omega = \sigma - k\overline{u} \to f
\]

tends to \( f \).

c. near-inertial internal wave trapping

The interaction between internal waves and geostrophically balanced background flows discussed so far has only been through the Doppler shift. An added layer of complexity happens when one relaxes the scale separation requirement and introduces mean flow gradients directly into the dispersion relation. Kunze (1985) produces a dispersion relation by taking the determinant of the linearized equations of motion. This procedure systematically neglects gradients of Doppler shifting that are the same order of magnitude as the background gradients Kunze (1985) is attempting to introduce into the dispersion relation, Polzin et al. (1996). Kunze (1985)’s dispersion relation,

\[
\omega \approx f + \frac{\zeta}{2} + \frac{N^2 k^2 h^2}{2f m^2} + \frac{1}{m}[l\overline{u}_z - k\overline{v}_z] 
\]

(A9)
in not correct at \( O(F_r) \). The expression (A9) was used in conjunction with ray-tracing arguments to explain an observed correlation between energy and relative vorticity in an upper ocean frontal regime, (Kunze and Sanford 1984). Kunze et al. 1995 and Kunze and Toole 1997 have used this approach to explain observations of nominally subinertial \( (f + \zeta/2 < \omega < f) \) internal waves having negligible Doppler shift in symmetric background flows (a Warm Core Ring and the vortex cap atop Fieberling Guyot). These examples can be seen as special cases of wave-mean interactions for which the proposed dispersion relation (A9) reduces to that consistent with solutions to a wave equation, Kunze and Bos (1998). The critical layer in this problem is provided by vertical propagation at constant intrinsic frequency \( \omega \).

d. Spontaneous Imbalance

Ray tracing is based upon the continuity of a real valued phase function \( \Omega \). As one reaches the \((R_o, F_r) \sim O(1)\) region of parameter space, a geostrophically balanced flow field can spontaneously degenerate into internal waves through ‘ageostrophic’ instabilities (Molemaker et al. 2005), implying a complex frequency, or otherwise be considered to ‘force’ the internal wave-field (Ford et al. 2000). In either case, the transfer of energy and momentum from balanced to unbalanced motions is believed to be weak in the small Rossby number, Froude number limit.
Those analyses, however, do not consider the effects of Doppler shifting and it is worth considering what happens to (A4) as the $[R_o, F_r \sim O(1)]$ parameter space is approached. In this instance, the factors in (A3) involving the vertical velocity gradients are not small. The prefactor multiplying $\alpha$ becomes:

$$S_n^2 + S_s^2 - \zeta^2 \rightarrow \sum_{i=1}^{3} S_{ni}^2 + S_{si}^2 - \zeta_i^2 - \Delta_i^2$$

in which the subscript $i = 1 \rightarrow 3$ denotes the $x - z$, $y - z$ and $x - y$ planes. Beyond noting that $w_z(\bar{u}_x \bar{v}_y - \bar{v}_x \bar{u}_y) = \Delta (S_n^2 + S_s^2 - \zeta^2 - \Delta^2)$, (A3) appears not to admit to a simple characterization: strain, vorticity and divergence in all three planes can be significant.
References


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A map of the 500 m geostrophic stream function (McWilliams 1976; The Mode-I Atlas Group 1977) from MODE, day 165, 1973 [June 14± 3 days] with a schematic depiction of wave crests (thick solid lines at approximately one horizontal wavelength separation) and wavevector (the arrow) associated with the coherent features noted in data reported by Sanford (1975) and Leaman and Sanford (1975) and reanalyzed here. The streamfunction map indicates a straining feature immediately to the north of the experimental site (28° N, 69° 40' W with data taken over June 11-15, a time frame that includes Day 165). In the ‘shrinking catastrophe’ and ‘wave capture’ scenarios, this strain will preferentially orient wave fronts and cascade wave phase to higher horizontal wavenumber. The orientation of the wave phase in the horizontal and vertical is consistent with the (Bühler and McIntyre 2005) wave capture scenario for a wave propagating downward in the southward, surface intensified jet. Also depicted is the dipole structure (Bühler and McIntyre 2005) associated with the wavepacket and it’s circulation. Note that this vorticity signature opposes that associated with the mesoscale field: potential vorticity contours differ from the geostrophic stream function contours only in the advection of relative vorticity, McWilliams (1976). High and low pressure centers are associated with negative and positive relative vorticity, respectively. Bühler and McIntyre (2005) forward the hypothesis that the dipole becomes locked into the mean flow as the internal wave dissipates. Thus wave capture provides a mechanism for mixing potential vorticity.
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