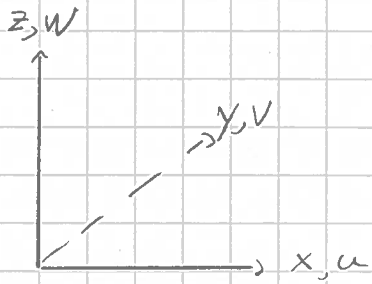
A FEW WORDS ABOUT OCEAN DYNAMICS



x : longitude coordinate

y : latitude coordinate

z : vertical coordinate

u, v, w : velocity components

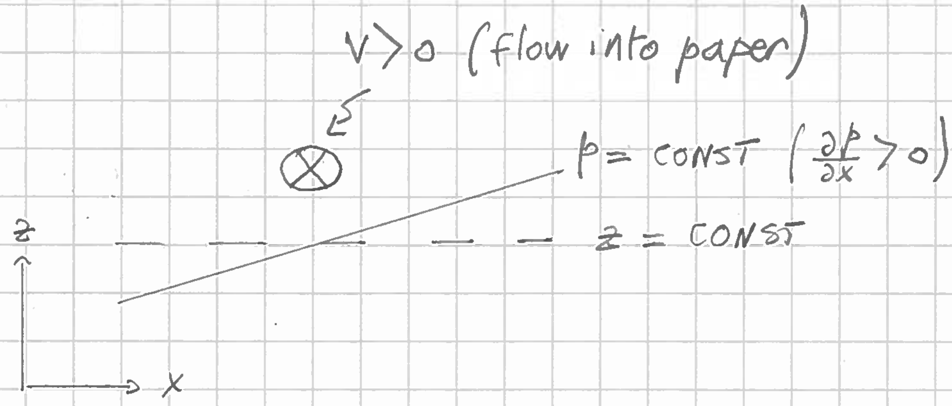
Geostrophic Equations

$$\boxed{\begin{aligned} -fv &= -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \qquad (1a) \\ fu &= -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \qquad (1b) \end{aligned}}$$

where

- u : zonal velocity
- v : meridional velocity
- f : Coriolis parameter : $f = 2\Omega \sin\phi$, where
  - Ω = Angular velocity of Earth's rotation
    $\left(\Omega = \frac{2\pi}{T_{Rot}},\ T_{Rot} \simeq 86400 \text{ sec}\right)$
  - ϕ = latitude
- ρ_0 : reference density (e.g., $\rho_0 = 1025 \text{ kg m}^{-3}$)
- $\frac{\partial p}{\partial x}$: zonal gradient of pressure
- $\frac{\partial p}{\partial y}$: meridional

Example



$$\left.\begin{array}{l} \frac{\partial p}{\partial x} > 0 \\ f > 0 \text{ (Northern Hemisphere)} \end{array}\right\} \text{ so } \quad v = \frac{1}{f\rho_0} \frac{\partial p}{\partial x} > 0$$

Hydrostatic Equation

$$\boxed{\frac{\partial p}{\partial z} = -\rho g} \qquad (2)$$

where

- $\partial p/\partial z$: vertical gradient of pressure
- g : acceleration due to gravity ($g \simeq 9.81 \text{ m s}^{-2}$)
- ρ : density of seawater. For a linear equation of state:

$$\rho = \rho_0 - \rho_0 \alpha (T - T_0) + \rho_0 \beta (S - S_0)$$

where T : temperature

S : salinity

ρ_0, T_0, S_0 : reference values (CONSTANTS)

- $\alpha > 0$: thermal expansion coefficient
- $\beta > 0$: haline contraction coefficient

"Thermal Wind" Equations

$\frac{\partial}{\partial z}$ (1a) yields: $\quad -f\frac{\partial v}{\partial z} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial x}\right) = \frac{-1}{\rho_0}\frac{\partial}{\partial x}\left(\frac{\partial p}{\partial z}\right)$ (3a)

$\frac{\partial}{\partial z}$ (1b) yields: $\quad +f\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\frac{\partial p}{\partial y}\right) = \frac{-1}{\rho_0}\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial z}\right)$ (3b)

But since $\frac{\partial p}{\partial z} = -\rho g$ (eq. 2), (3a) & (3b) yield

$$-f\frac{\partial v}{\partial z} = \frac{-1}{\rho_0}\frac{\partial}{\partial x}\left(-\rho g\right) = +\frac{g}{\rho_0}\frac{\partial \rho}{\partial x}$$

$$f\frac{\partial u}{\partial z} = \frac{-1}{\rho_0}\frac{\partial}{\partial y}\left(-\rho g\right) = +\frac{g}{\rho_0}\frac{\partial \rho}{\partial y}$$

So:

$$f\frac{\partial v}{\partial z} = \frac{-g}{\rho_0}\frac{\partial \rho}{\partial x} \qquad (4a)$$

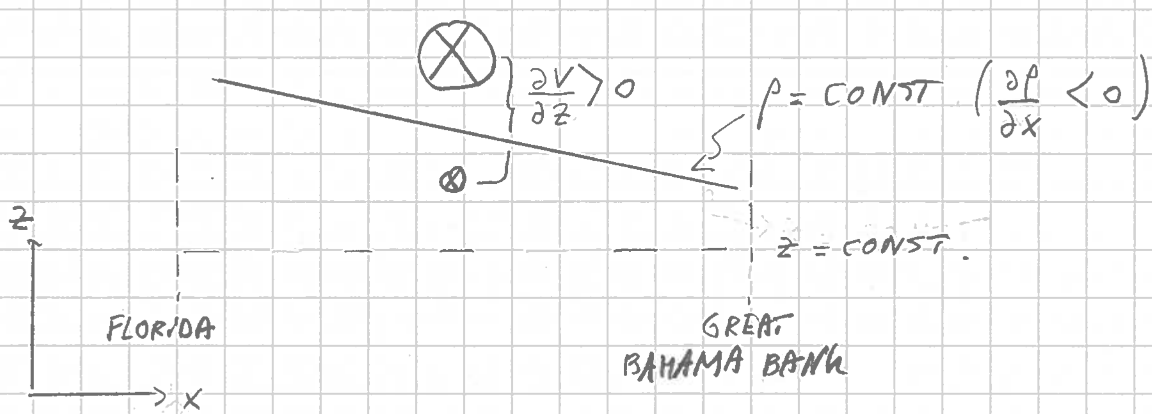
$$f\frac{\partial u}{\partial z} = \frac{g}{\rho_0}\frac{\partial \rho}{\partial y} \qquad (4b)$$

Eqs. (4a) – (4b) are the "thermal wind" equations.

Example

$f > 0$

$\frac{\partial \rho}{\partial x} < 0$



Eq. (4a) shows that $\frac{\partial v}{\partial z} > 0$ $\left(\frac{\partial v}{\partial z} = \text{vertical shear in current velocity}\right)$