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Detrital Remanent Magnetization: Viscosity Theory of the Lock-in Zone

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Post-depositional remanent magnetization (PDRM) becomes permanent within a lock-in zone, whose apparent time delay (depth) and time span (thickness) are related to the characteristic time constants of compaction and magnetic grain rotation. Two simple models of the PDRM lock-in zone are described, using Yaskawa's idea of effective sedimentary viscosity, and Mooney's empirical relation for the viscosity of a concentrated suspension of spheres. The impulse response of the lock-in zone determines the fidelity of the paleomagnetic recording.

INTRODUCTION

Detrital remanent magnetization (DRM) is mainly a post-depositional process (PDRM) in which the mechanical alignment of magnetic grains becomes progressively secured by the consolidation of the sediment [*Irving and Major*, 1964; *Kent*, 1973]. Until the magnetic grains become locked into place, they are free to follow the directional fluctuations of the ambient magnetic field, behaving as miniature compasses (see *Opdyke* [1971] and *Verosub* [1977] for useful background on remanent magnetism in soft sediments.)

Yaskawa [1974] considered the sediment itself to be a viscous medium in which the magnetic particles are immersed (see also *Y. Otofuji* and *S. Sasajima* [1980] and *Hamano* [1980]). Expulsion of the interstitial water during sediment compaction causes the effective viscosity to rise due to particle crowding and friction. This increasingly retards the efficiency of the magnetic alignment process. The rate of alignment is characterized by a time constant that varies with the effective viscosity.

Experimental DRM/PDRM gives the appearance of locking into place quickly to produce a sharp recording of the ambient magnetic field. In particle settling, the recording is established at the water/sediment interface, or very closely beneath it [e.g., *Lövlie*, 1974; *Barton and McElhinny*, *Barton et al.*, 1980]. Typically, the DRM/PDRM appears to respond with time constants of minutes to days, comparable to the length of the experiment itself. During longer term experiments, still brief by geological standards, the magnetization undergoes little or no apparent change after the first several days.

In nature, the time that is available for completing the PDRM process is many orders-of-magnitude longer than the time available in the laboratory. If PDRM were a continually ongoing process, merely becoming slower as time passed, then the eventual outcome could depend more heavily on the longer time scales than on the shorter ones. Thus, the laboratory experiments could be giving incorrect information about the natural PDRM process.

Our purpose in this paper is to describe a simple theory that leads to a PDRM lock-in zone [*Niitsuma*, 1977], where a smooth representation of the magnetic field fluctuations is recorded and made permanent. The key element of the DRM/PDRM model is its time constant (τ) profile, where τ is dependent on time or depth, which must have very small values at the top of the sediment column, relative to the geomagnetic periods of interest, and very large values in the long term, relative to the age of the material. The time constant profile gives rise to a lock-in zone if these conditions are met, regardless of whether or not the magnetic particles are all identical. The time constant profile determines the impulse response of the PDRM system, including the mismatch between the age of the sediment and the age of its constituent magnetization. (We will now drop the distinction between DRM and PDRM, since the DRM/PDRM system can be considered continuous.)

MAGNETIC ALIGNMENT OF DETRITAL COMPASSES

For a spherical magnetic particle immersed in a Newtonian fluid, the balance between magnetic and viscous torques is described by

$$m\,B\sin\Phi - 8\pi\eta r^3\frac{d\Phi}{dt} = \frac{2}{5}\rho\, r^5\frac{d^2\Phi}{dt^2} \qquad (1)$$

where

m = grain magnetic moment;
B = applied magnetic induction
Φ = angle between m and B;
η = viscosity of the medium;
r = grain radius;
ρ = grain density;
t = time.

and *MKS* units are used.

For grain sizes encountered in DRM, the inertial torque is negligible in comparison with the viscous drag for media at least as viscous as water. The equation resulting from discarding the inertial term is easily solved under the small angle approximation $\sin\Phi \approx \Phi$, yielding

$$\ln\frac{\Phi(t)}{\Phi_o} = -\int_{t_o}^{t}\frac{dt}{\tau(t)} \qquad (2)$$

where

$$\tau(t) = \frac{8\pi r^3}{m\,B}\eta(t) \qquad (3)$$

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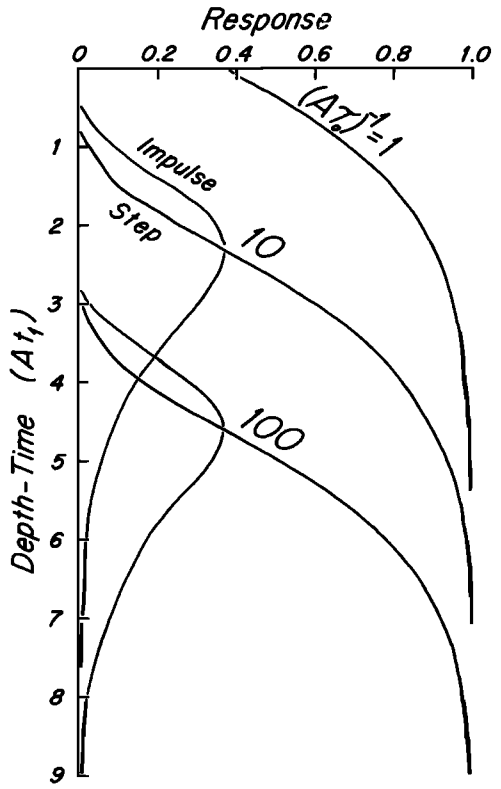

Fig. 1. Step response and impulse response curves of the PDRM lock-in zone for the exponentially increasing time constant mode. A is the characteristic growth rate of the magnetic alignment time constant, τ_o is the initial time constant, and t_1 is the age of the sediment (or depth if the accumulation rate and the compaction together yield a linear age versus depth curve).

θ_0 is the initial angle between m and B, and t_o is the initial time. The alignment time constant is represented by $\tau(t)$, which in water varies from 10^{-2} s to 1 s for typical magnetite grains in the earth's field [*King and Rees*, 1966; Stacey, 1972]. The time constant is proportional to the time varying (or depth varying) effective viscosity $\eta(t)$, which forms the basis of the viscosity theory of PDRM.

Equation (2) gives the log PDRM step response R, which can be differentiated to obtain the impulse response S. If a range of initial time constants $\tau(t_o)$ were involved, then this impulse response would have to be convolved with the appropriate time constant density distribution function $F(\tau(t_o))$ to obtain the system impulse response.

EXPONENTIALLY INCREASING TIME CONSTANT MODEL

Imagine a magnetic alignment time constant profile $\tau(t)$ that increases exponentially as a function of age t

$$\tau(t) = \tau_o e^{At} \quad (t \geq 0) \tag{4}$$

where A is the characteristic growth rate of the alignment time constant.

If a step change Φ were to occur in the ambient field direction when the sediment was age t_1, then the log of the step response R of the system after infinite time would be given by (2) as

$$\ln R = \ln \frac{\Phi}{\Phi_o} = \frac{1}{A\tau_o} e^{-At_1} \tag{5}$$

The impulse response obtained by differentiating (5) is

$$S = -R \ln R \tag{6}$$

The responses R and S (Figure 1) define the lock-in zone. In the present formulation, they are constant in shape and in thickness, as parameterized by $A\tau_o$. At the sediment surface R undergoes a step, because of the perfect alignment of the just settled particles.

The lock-in time delay [e.g., *Dymond*, 1969], the time equivalent of the lock-in depth, lies at the peak of S, at $-\log(A\tau_o)$ where $R = e^{-1}$. The lock-in time span, a time equivalent thickness, is taken to be the $R = 0.1 - 0.9$ response interval, the value being $3.08A^{-1}$. Thus, for example, grains with $\tau_o = 10^{-2}$ s $-$ 1s in a medium consolidating at $A^{-1} = 1$y (3.16×10^7 s) would display a step response delayed by only 17 $-$ 20 s, but smeared over an approximately 3y range. Figure 2 shows the lock-in depth and thickness for various $A\tau_o$.

The reason for choosing an exponential time constant profile is that the consolidation of continuously-accumulated sediments often proceeds in an exponential fashion in its early stages [e.g., *Lambe and Whitman*, 1969]. If the viscosity does vary linearly with the particle concentration of the suspension [*Einstein, 1906; Happel and Brenner*, 1965, Chapter 9], then (4)—(6) are an appropriate description of the response of an idealized sediment.

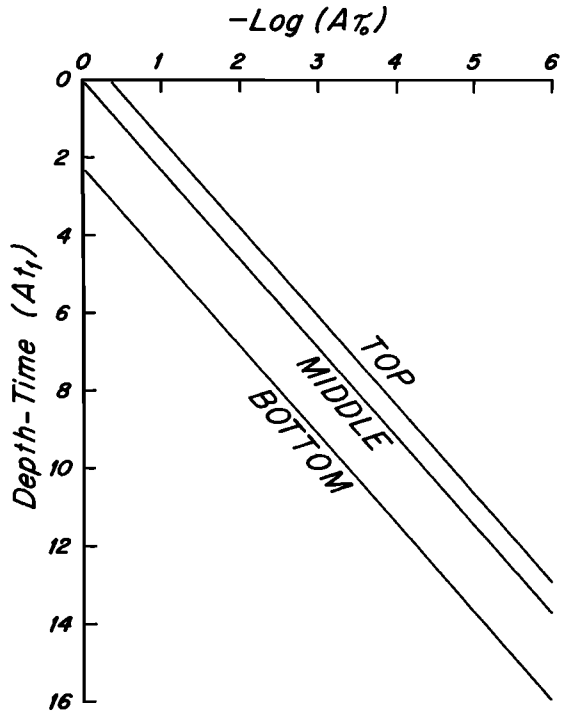

Fig. 2. Position of the PDRM lock-in zone in the sediment column. 'Middle' is the lock-in depth, located at the peak of the impulse response curve. 'Top' and 'bottom' refer to the 0.1 and 0.9 positions of the step response curve, taken to define the width of the lock-in zone. The position of the lock-in zone (At_1) depends on the PDRM system's characteristic parameters (A and τ_o).

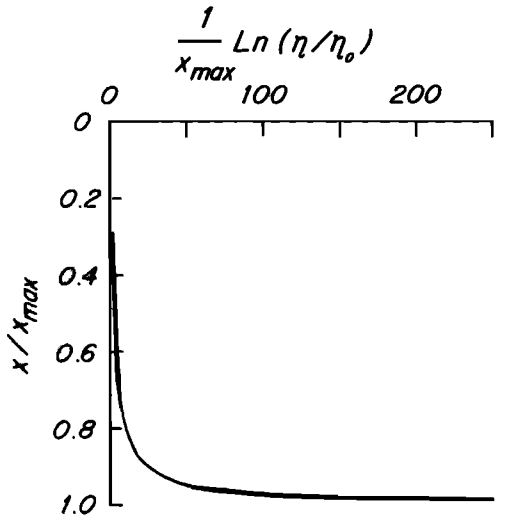

Fig. 3. Graph of the Mooney [1951] empirical formula for the viscosity of a concentrated suspension of spheres (equation (7) in the text).

MOONEY VISCOSITY LAW WITH EXPONENTIAL CONSOLIDATION PROFILE

The viscosity of a suspension of constant-size spheres is a function of the particle concentration x, and of the maximum permitted concentration x_{max}. Relative to the fluid viscosity η_o, the suspension viscosity η is [*Mooney*, 1951; *Happel and Brenner*, 1965, Chapter 9]

$$\frac{\eta}{\eta_o} = \exp \frac{2.5x}{1 - x/x_{max}} \tag{7}$$

The expression is shown in Figure 3, where careful note should be made of the axis labels.

The critical concentration x_{max} ranges from 0.52 for cubic closest packing to 0.74 for hexagonal closest packing of constant size spheres. Values of $x_{max} = 0.52 - 0.62$ have been observed in random packing experiments, and

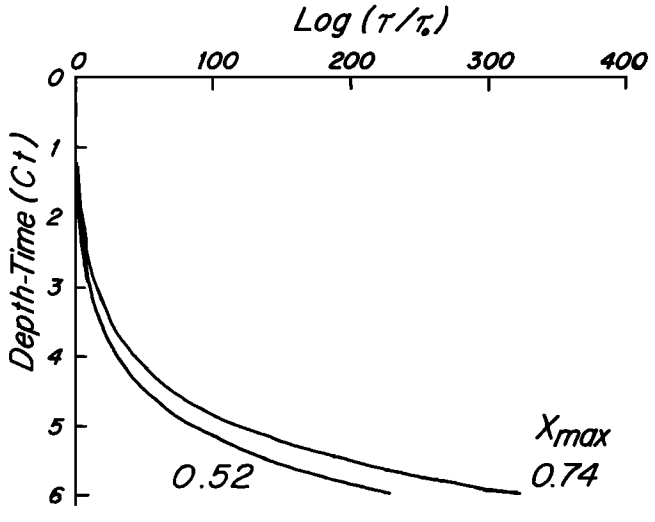

Fig. 4. Magnetic alignment time constant as a function of age in the sediment column, for cubic closest (0.52) and hexagonal closest (0.74) packing of spheres. The curve illustrates text equation (9), the Mooney adaptation.

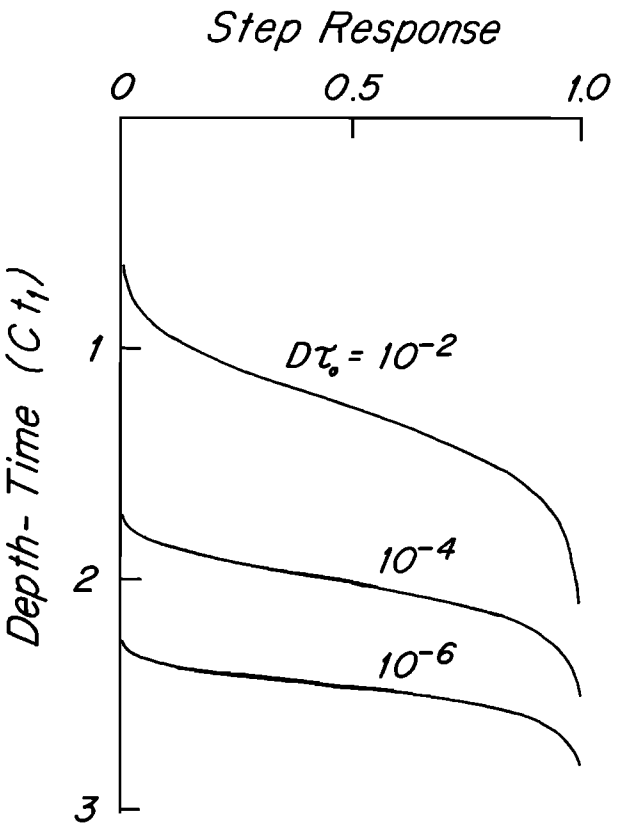

Fig. 5. Step response for the Mooney PDRM model described by text equation (10), for various values of the system parameters.

stable loose-packings packings down to $x_{max} = 0.125$ are known. The form of the Mooney law has proven to be very successful experimentally, although the apparent value of x_{max} is sometimes smaller than the actual concentration measured near the critical point. This is attributed to the accumulation of a fluid layer around each particle, such as what occurs with clays, which increases its effective volume and hence the effective concentration of the suspension.

The density of the sediment is a measure of its particulate concentration. The density increases from the value of water near the sediment surface to some nominal maximum value at depth, excluding the effects of chemical diagenesis. Density profiles vary widely, depending on the sedimentation rate and on the nature of the sediment itself. An upper mixed-layer exists in most sediments because of bioturbation. For the purpose of our model, we will assume an exponential density profile to express a consolidation process that is rapid at the outset and becomes very slow in the long term. The formula for this relative particulate concentration is

$$\frac{x}{x_{max}} = 1 - e^{Ct} \tag{8}$$

where C is the characteristic rate of consolidation.

From (3), (7) and (8), the time constant profile is

$$ln \frac{\tau}{\tau_o} = 2.5 x_{max} (e^{Ct} - 1) \tag{9}$$

Figure 4 shows (9) in terms of Ct for the x_{max} range of 0.52 – 0.74.

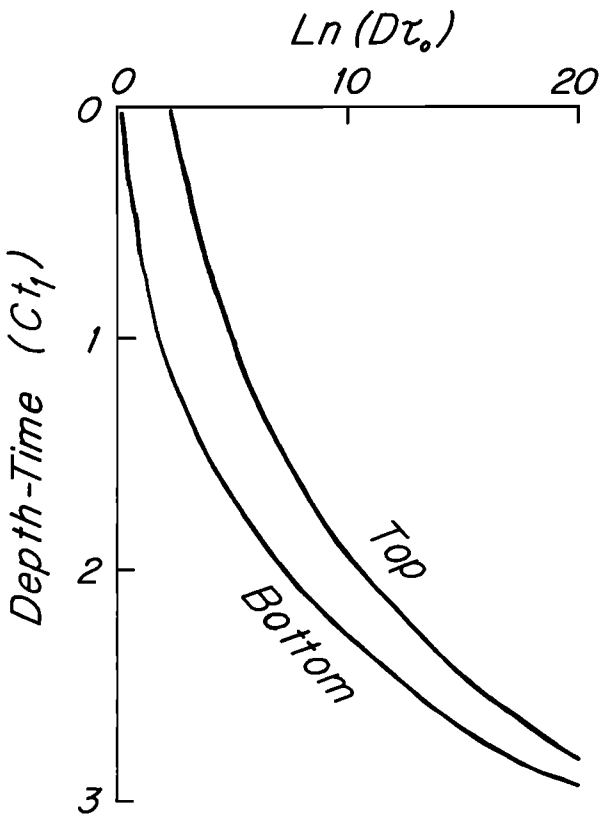

Fig. 6. Position of the PDRM lock-in zone for the Mooney model. The curves illustrate the 0.1—0.9 step response interval, using text equation (10).

The step response, from equations (2 and 9), is:

$$ln\ R = ln\ \frac{\Phi(t)}{\Phi_1} = -\frac{1}{D\tau_o} \int_{Ct_1}^{Ct\to\infty} e^{-e^u}\, du \qquad (10)$$

where

$$D = 2.5\, x_{max}\, C\, e \qquad (11)$$

Graphs of the step response are shown in Figure 5 for various values of the scale factor $D\tau_o$. This scale factor alone characterizes the lock-in zone for this particular model, as did $A\tau_o$ in the model discussed previously.

The position of the lock-in zone is shown in Figure 6, where once again it is seen that the relative thickness of the lock-in zone diminishes with depth. This is intuitively correct, because the closer to the x_{max} singularity the lock-in process occurs, the sharper the response will be.

Discussion

The first of the two PDRM models is purely descriptive. It assumes that the time constant of detrital grain rotation increases exponentially with age after deposition. The resulting impulse response, i.e., the expression of the lock-in zone, (1) is constant in shape, (2) has an approximate width of $3A^{-1}$, and (3) appears at an age offset of $A^{-1}\tau_o$. The width depends solely on the compactive time constant profile, while the depth depends also on the initial rotational time constant of the grain. The advantage of this model is that it is a simple one, requiring only the lock-in depth and width to specify it completely.

The second model injects an empirical theory of viscosity into the hypothesized PDRM mechanism. One outcome of this model is that it may stimulate some experimentation with magnetized microspheres of uniform size. Even though they may be a naive representation of actual sediments, the fundamental understanding of PDRM may be significantly accelerated by such work. The advantage of microspheres is that their behavior is tractable theoretically. By dealing first with materials whose theories are already well developed, insight may be gained about much more complex systems such as natural sediments.

The unifying ideas of post-depositional magnetic alignment and sedimentary viscosity have gained widespread popularity amongst workers in this field. The task before us at present is to discover the actual physical principles that underlie those labels. Laboratory DRM/PDRM experiments are growing in sophistication and are aimed at identifying the most important physical parameters, with the ultimate purpose of devising efficient techniques for measuring them. One very promising method is centrifuging [*Otofuji and Sasajima*, 1977], which makes the depositional process uniform and accelerates the compaction process that appears to control PDRM. It is hoped that bulk physical measurements and remagnetization experiments will prove useful for characterizing the directional and intensity recording capabilities of magnetic detritus, in a fashion analogous to the methods used for studying thermal remanences. The likelihood that this will occur cannot be foreseen. Our purpose has been to stimulate further thinking about DRM/PDRM that will be useful in this quest.

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