Convection with continents-Whitehead

**Laboratory Studies of Mantle Convection with continents and other GFD problems**

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**Abstract**

We review first some laboratory studies that illustrate some of the nonlinearities that arise in geophysical flows. These include convection cells, viscous conduits, waves on the conduits due to changing sources and due to tilt of the conduits. Next, we illustrate fingering structures that result when flows alter their surroundings to retard their continuation. This is typical as hot material from deep in the earth rises to shallow regions where cooling is encountered. We then described recent work concerning the relation between convection cells and material floating at the surface. This is motivated from factors governing convection in the mantle of the earth. Continents are thickened when they collide or when they override ocean trenches. This might retard mantle flow, and it clearly influences the thickness of the continental crust. Other estimates of continental deformation suggest that continents slowly spread out by slow internal deformation and erosion. The link between the surface deflection of a floating layer and the parameters of cellular flow below the layer are quantified. Laboratory experiments show the deflection of a thin horizontal layer of plastic balls (as a model of continental material) floating on viscous silicon oil (as a model of the mantle of the earth) that is forced to adopt a cellular circulation. The oil is a model of a convection cell which laterally sweeps the balls into patches above the descending part of the cell. The balls tend to spread out in opposition to this convergence, resulting in a steady balance that determines the size and thickness of the clump of balls in the downwelling region. The experiment results can be scaled to the mantle convection/continent and they will give measurements of thickness as functions of the mantle speed and spreading speed of the continental material.

1. A review of some past studies in the GFD laboratory

   Laboratory experiments have a long tradition in the Geophysical Fluid Dynamics Laboratory in Woods Hole Oceanographic Institution. Studies in the fifties included many of the first attempts to make a model of ocean circulation, and well-known experiments that illustrated the western intensification of the ocean (Stommel Arons and Faller). In the 1960’s many of the first studies of double diffusion and of the effects of mixing were conducted in this laboratory (Turner 1973).

   This author arrived at Woods Hole Oceanographic Institution in 1972 and was involved in experiments that looked at cellular convection of high Prandtl number fluid at Rayleigh numbers up to about 500,000 (Whitehead and Chan, 1976, Whitehead and Persons, 1978). A summary of this and subsequent experiments is found in Balmforth et al. (2001) A typical flow pattern is shown in figure 1. This is a top view of a shadowgraph of convection cells. The white lines are sheets of cold fluid that gathers near the top of the layer and sinks down to the bottom.
Convection with continents - Whitehead

The dark lines are sheets of hot fluid that gathers near the bottom of the layer and rises up to the top. Many junctions are found where white sheets or black sheets gather as a radial cluster. We call these regions “spoke” flows.

Figure 1. A shadowgraph of spoke flows.

Some laboratory experiments indicated that continents could modulate mantle convection due to their influence on the thermal budget. This modulation takes two forms. First, the continents can act as thermal insulators to the hot mantle. They block direct exposure of the mantle underneath the continent to the surface of the earth and thereby retard cooling of the mantle there. This can produce drift of the floating, insulating continents (Elder 1967, 1968, Zhang and Libchaber 2000). In addition, continental material may have its own heat production that can lead to drift instability with or without mantle convection (Howard et al 1970, Whitehead 1972). Such a drift instability is shown in Figure 2. A float has a heater attached below it. The heater is a fine wire of small diameter that has negligible drag. The hot oil below the floating mass rises and conveys the float in one of the two branches of the convection cell. The heater is thus self-propelled. It can be driven in either direction by the heater as shown in figure 2. In this way, the float with a heater below it acts in a manner similar to the insulators that float on top of convection cells. Theory illustrated the manner in which the phase lag between the temperature field and the heater leads to this lateral drift.
Figure 2. Side view of a heater floating in oil that drifts laterally. The source of heat is located near the bottom of the tank. The triangular float is located on the surface of the oil. The vertical stripes are deflected by the temperature pattern that rises from the heater. The heater is moving toward the right in the upper panel and toward the left in the lower panel.

There was also a large focus on determining the features of buoyantly driven flow when the intruding fluid was injected at the bottom and floated up to the top in the limit of small Reynolds number. Particular interest was paid to the case when the intruding fluid was much lower in viscosity than the other one. In that case, the fluid would start to rise as a sphere followed by a vertical tube which would allow the lower viscosity fluid to rise in a much smaller cross sectional area as shown in figure 3. This has led to many suggestions that the tubes may allow hot material to rise through the earth’s interior in circular conduits, perhaps feeding hotspots and other volcanic centers (Griffiths and Whitehead 2001).
Figure 3. A photograph of a thin layer of lower viscosity fluid rising up through transparent silicon oil by gravitational instability. The bodies become spherical as they detach from the bottom and a conduit continues to drain the remaining material upward. The conduits develop waves if either the source flux is suddenly changed (Scott et al. 1986, Whitehead and Helfrich 1988, Helfrich and Whitehead 1990) or if they are tilted beyond about 60 degrees (Skilbeck and Whitehead 1978, Whitehead 1982). The waves are shown in figure 4. Fluid inside the waves recirculates so that the waves can carry material large vertical distances without substantial mixing. Waves that develop when the conduits are tilted are shown in figure 5.
Figure 4. Side view of a solinoidal wave that propagates up conduits. The camera is moving upward with the wave and time progresses from left to right. Dye inside the wave shows that the material circulates inside the wave and does not leave the wave.

Figure 5. Growing waves that develop in conduits that tilt to more than about 60 degrees lead to instability of the conduit.
Convection with continents - Whitehead

In the 1980’s a number of studies illustrated the manner in which lower viscosty material would develop morphological instabilities if its properties were altered by cooling or deposition as the flow commenced (Whitehead and Helfrich 1990, 1991, Whitehead and Griffiths, 2001). Figure 6 shows the evolution of paraffin that starts flowing radialy from a small source on a level plate that is cold. Above the plate is a transparent lid so that the flow is two dimensional.

![Figure 6](image-url)

**Figure 6.** Left Panel. Paraffin that is developing fingers as it flows radialy outward from a confined source on a cold level plate. Right Panel. Somewhat later, many of the fingers have frozen and stopped. The darkest fluid is flowing out through one channel that continues to stay open. The lighter dyed fluid was flowing out through a second channel that has now stopped.

We also found that water flowing through salt would erode the surface interface of the salt due to a dissolution instability (Whitehead and Kelemen, 1994, Aharanov et al. 1995, Whitehead and Griffiths, 2001). Figure 7 shows dye fingers from a channel within a mixture of salt and glass beads.
Figure 7. The erosion of a mixture of salt and glass balls as water flows down through it from above. This photograph shows dye that was injected into the bottom of the fresh water layer lying above. The fresh water rapidly becomes salty so the dye originally enters the salt layer as a horizontal sheet. The dye breaks up into fingers.

2. Recent studies

Throughout the 20th century, the notion of continental drift has intrigued natural scientists. The concept originally espoused by Wegener, of continents plowing through a solid earth mantle was replaced in the 1960’s by a picture of mantle convection forcing the lateral motion of large, rigid surface plates. In conjunction with all the rich data gathered by earth scientists, the understanding of thermal convection as a likely driving mechanism has become advanced through work in many fields such as geological fluid dynamics, applied physics, engineering and the ocean and atmospheric sciences. Most would agree that large descending slabs of ocean lithosphere are cooling the earth, and that this descent provides much of the traction to move the plates laterally to the trench regions. The plates are cooled by thermal conduction and the hot material is supplied by ascent of mantle under mid-ocean spreading centers.

In spite of these indications that continents can influence mantle motion beneath it, in numerical modeling it is frequently presumed that continents move about passively with the plates. However, continents clearly also undergo internal deformation, particularly after active mountain building episodes (Thatcher, et al 1999). They seem to have undergone a number of episodes of splitting apart and rejoining (Hoffman 1991), and have also experienced collisions with ocean ridges, trenches, and hotspots. Such collisions may extinguish subduction and eventually lead to warming of the mantle under super continents, which leads to their renewed breakup (Gurnis 1988, Zhong and Gurnis 1993, Conrad and Gurnis, 2003. In some cases, the
Convection with continents—Whitehead

collision leads to retreating subduction boundaries (Royden 1993a). In other cases, active collision leads to mountain building (Yin and Harrison 2000) and later extension (Kaufman and Royden 1994) and other alterations of the orogens (Huerta et al. 1998, 1999), including thermal processes (Royden, 1993b, Huerta et al 1996). Recently documented extension has been used along with a hydrostatic approximation to estimate the viscosity of the continental material in the orogens (England and Molnar 1997, Flesch et al 2000). The continents possess deep lithospheric roots that add to their strength, but under some conditions these roots may be dense enough to develop Rayleigh-Taylor instability and sink into the mantle (Conrad and Molnar 1997, Houseman and Molnar 1997).

Although there have been numerous advances in documenting the history and structure of continents, the overall picture of the mechanical interaction of the continents with both plates and mantle convection is even less clear than that of the plates and mantle alone. Although this is mostly due to a lack of the needed data, the unknown laws governing stress, strain, and strain rate throughout the range of physical and compositional parameters for continents also seem formidable.

This reports two studies of the combination between a well-understood feature of mantle convection, namely the cellular motion in the bottom layer and the behavior of a deformable floating body. We seek to determine constraints on the size and thickness of models of continents as a function of overturn speeds of cellular motion below the floating mass.

We hypothesize the following: that the first order effect of mantle convection on continents is to attract them to subduction zones, especially on average over long geological times. Once the continent material arrives at a subduction zone, the continental crust is thickened from the strong local convergence by mountain building. If the continental material is not swept into the mantle, this increases the average thickness of the continents, causing the continent surface to be elevated on average and the crust to be thicker and thus extend to greater depth, and the area of the continents to be decreased. The newly built mountains are then eroded very actively and they also spread apart naturally. The attraction to subduction zones is not documented in the geological record to my knowledge, but we see good examples of continent-subduction interaction. Such has happened both in North America and in the Himalayas within the past 40-80 my and it is presently seen next to the Andes and in the Alps. Moreover, the continents are thick, cold, and strong and their presence may extinguish subduction zones. This has also been seen in both these regions. Thus we have good examples of continent-subduction collision over the past few million years. We note that it would be difficult to reconstruct all historical collisions of continents and subduction zones over the history of the Earth, so that any kind of formulas or overarching principles that might express the rates of the above processes might be helpful in speculating about the past earth.

2. Modulation of a layer by cellular motion

The formation of the surface continental material into lumps was probably produced by mantle convection. This process is sensitive to the mechanical properties of both the mantle and the continental crust. The idealized model developed here is the very simplest example of the formation of lumps we can find. It illustrates the sensitivity to the parameters of the driving and the physical properties of the liquids in the two layers. Consider a layer of proto-continent material with viscosity, \( \mu_1 \), density \( \rho_1 \), and thickness \( l \) above a second mantle material with viscosity, \( \mu_2 \), density \( \rho_2 \), and depth \( L \) in a field of gravity \( g \). The top material lies initially at uniform depth everywhere on the top of the mantle. We ask “what is the effect of mantle motion from a lateral velocity \( U \sin k x \) in the x-direction that is imposed along the bottom of the mantle
Convection with continents—Whitehead

at \( z = -L \)"? The elevation of the top of the continent \( \eta_1 \), and the interface between continent and mantle \( \eta_2 \) are to be determined. These calculations assume a steady-state. The equations of viscous flow are used

\[
0 = -\nabla p_i + \mu_i \nabla^2 u_i
\]

along with the continuity equation

\[
\frac{\partial u_i}{\partial x} + \frac{\partial w_i}{\partial z} = 0
\]

where \( i = 1, 2 \) denote the top and bottom layer, respectively. The boundary conditions are linearized at the interface between the two layers (which is taken to be at \( z=0 \)). They are the matching of lateral velocity \( u_t = u_2 \), vertical velocity, and lateral stress

\[
\mu_1 \left( \frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x} \right) = \mu_2 \left[ \frac{\partial u_2}{\partial z} + \frac{\partial w_2}{\partial x} \right].
\]

In addition, the condition of vertical stress determines a value for \( \eta_2 \).

\[
2\mu_1 \frac{\partial u_1}{\partial x} + P_1 - 2\mu_2 \frac{\partial u_2}{\partial x} - P_2 = g(\rho_2 - \rho_1)\eta_2
\]

The boundary conditions on the top of the upper layer are applied at \( z = l \). These conditions are zero vertical velocity and zero lateral stress. If the velocities are known, the condition of zero vertical stress produces a value for \( \eta_1 \). The final two boundary conditions are \( w_2 = 0 \) and \( u_2 = U \sin kx \) at \( z = -L \). The solutions are of the form \( \sinh kz \)

\[
w_i = A_i \sinh kz + B_i \sinh kz + C_i \cosh kz.
\]

The six constants \( A_i \), \( B_i \) and \( C_i \) are found by inverting the matrix equation that comes from the matching and boundary conditions

\[
\sum a_{in} A_i + b_{in} B_i + c_{in} C_i = F_n
\]

where \( n=1, 6 \). The matrix and the solutions are shown listed in Appendix 1.

The solution for \( \eta_2 \) gives the depth of the interface between the two fluids, which corresponds to “continental roots”, from stress developed by circulation in the mantle. It is found by inverting the matrix equation and by using the solutions in equation 3. It is

\[
\eta_2 = \frac{\eta_{02}}{l} \left[ \frac{1}{kL \cosh kL - \sinh kL} + \frac{\lambda \sinh kL \sinh^2 kL + \mu_0 (\lambda \cosh kL \sinh kL \cosh kL - kL)}{\sinh^2 kL - k^2 L^2 \sinh^2 kL + \mu_0 [k] \sinh kL \cosh kL \cosh kL - kL \sinh kL \cosh kL - kL} \right]
\]

where \( \eta_{02} = \frac{2U k^2 \mu_2}{g(\rho_2 - \rho_1)} \lambda = L / \ell \), and \( \mu_0 = \mu_2 / \mu_1 \).

The deflection is linearly proportioned to the parameter group \( \eta_{02} \), times layer depth \( l \), and the rest of the function depends on the ratio of mantle depth to layer thickness \( \lambda \), the viscosity ratio \( \mu_0 \), and each layer depth times the wave number. The solution for \( \eta_1 \) gives values of the elevation of the continent. It is

\[
\eta_1 = \frac{\eta_{01}}{l} \left[ \frac{kL \cosh kL - \sinh kL}{\sinh^2 kL - k^2 L^2} \sinh^2 kL + \mu_0 [k] \sinh kL \cosh kL \cosh kL - kL \sinh kL \cosh kL - kL} \right]
\]

where \( \eta_{01} = \frac{2U k^2 \mu_2}{g \rho_1} \). Let us determine an estimate for root deflection that would be produced by forcing with typical values of mantle convection as presently understood. We take \( U = 3 \times 10^{-9} \text{ms}^{-1} \), which is the spreading rate of the Pacific plate. This assumes that the flow speed in the deep return flow region is about the same size as the surface plate speed. Consider first a series of relatively small
Convection with continents-Whitehead

convection cells with wavelength of $6 \times 10^3$ km. Acceleration of gravity is $g = 9.8$ m/s$^2$, density difference between mantle and continent $\rho_2 - \rho_1$, is typically 600 kg/m$^3$, and we take upper layer thickness $l$ of the continental crust to be 15 km. This is the value if all continental crust were spread evenly over the globe. Take that the primary mantle flow is shallow — of the order of $L = 300$ km. We pick mantle viscosity of $\mu_2 = 4 \times 10^{21}$ pa s, a value used for an analysis of asthenospheric counterflow (Turcotte and Schubert 2002 page 250) and we specify that continent viscosity is one order of magnitude greater, so that $\mu_0 = 0.1$. This gives from equation 6 a value $\eta_2 / l$ of about 0.6.

Since this value is beyond the magnitude permitted by this linearized theory, the large size means that the top layer is expected to greatly thin over regions of mantle divergence and greatly increase its thickness over the mantle convergent regions. If such a layer were to break apart, this would lead to continents in the form we see them today. Instead, if we used a mantle viscosity of $\mu_2 = 10^{21}$ pa s, and a depth of 2800 km which are typical for whole mantle convection, the value $\eta_2 / l$ is about 1.0. Doubling the wavelength increases $\eta_2 / l$ up to 2.9.

Thus we see that the top layer is distorted by stresses from circulation within the bottom layer. A usual opinion is that continents are not disturbed by stress from the mantle because they are mechanically so strong that they behave as rigid material that does not flow, but only breaks like a brittle solid. In that limit, the solid can be taken as a fluid of very large viscosity. This calculation shows that the upper layer viscosity, which only arises in the parameter $\mu_0$, only weakly affects the size of the results. For example, if $\mu_0$ is decreased by two orders of magnitude, so that continent viscosity is increased by two orders of magnitude, the above numbers are changed by less than 10%. The role of the viscosity of the top layer is to retard the flow at the interface, and thus slow the rate of flow in the lower layer. In steady-state, the continent density is rearranged so that the pressure gradients from the elevation of the surface layer match the stress exerted on the top layer by the bottom layer, rather than alter the pressure distribution in that layer. Thus, the continent elevation reflects the stress that the mantle flow exerts on the top layer from below.

The above calculation is deliberately idealized and artificial to emphasize the predominant interaction between floating continental crust and mantle motions. This is best pictured as a crude model of the evolution of the continental crust over the past two or three billion years of the earth. The surface layer experiences two driving forces, a stress from the mantle motion that tends to drive the surface layer into clumps, and a divergence from gravitational spreading that tends to smooth out the clumps.

In the actual Earth, the lateral compression of continents is found predominantly in regions where mountains are being built. This had long been inferred from innumerable geological studies. Recently, it is also directly measured as compression between plate motions in the mountain building regions of the Americas and Asia. The compressive mountain building arises from collision of continents and subduction zones in the Rockies and Andies, and with collision between two continents in combination with subduction in the Himalayas. Let us estimate how much elevation has been added to the continents for the past 20 million years. Assume a doubling of continental crust thickness, from 40 km to 80 km, over three regions. First, a surface area in South America of about 120 km wide and 5000 km long with a total area of $6 \times 10^5$ km$^2$. In North America we assign a width of 600 km and a length of 3000 km for a total area of $1.8 \times 10^6$ km$^2$. For the Himalayas we assign an area of 1000 km and a width of 2000 km to give $2 \times 10^6$ km$^2$. All three give a total of $4.4 \times 10^5$ km$^2$ over the globe that have doubled
the thickness of the continental crust over the past 20 million years. At present, the total area of
continents on the earth is roughly 30% of the surface of the earth or 120 x 10^6 km^2. Thus,
without erosion or the gravitational divergence of mountain regions we could extrapolate
backward in time that the earth would have a decreased continent area from over 40% of the
globe to 30% over roughly the past 180 million years. This is an enormous amount of areal
contraction and an equally great amount of thickening of the crust. Naturally, the present epoch
might easily be unusually active tectonically. However, if the continents have been assembled in
supercontinents twice in the past billion years and have undergone a large amount of
fragmentation and drift in between, then there could have been many such active periods in the
past two billion years.

There are two well-documented processes that argue that the above did not happen. First,
the mountain regions are laterally extending even while they are being compressed. England and
Molnar (1997) use the extension rates to estimate an effective viscosity of the earth that is
roughly ten times the commonly estimated mantle viscosity.

The second is the well-known freeboard argument. We present a constraint on continent
freeboard here assuming conservation of continent material and conservation of water. The
conservation of continental crust is equivalent to assuming that over a geological time of a
billion years or so, any continental material that has been eroded is deposited on the sea floor
either as sediment or as ocean sediment. It is then subducted at trenches and returned to the
surface by island arc volcanism where it joins with the sides of the contents. The conservation of
water assumes that subducted water is largely returned to the surface by volcanoes and that
whatever water is liberated from the mantle or returned to the mantle is small over the past
couple of billion years or so. Using conservation of continental mass, water, and using the
isostatic approximation for continents floating in a mantle, figure 8 shows an estimate of the
mean elevation of continents and ocean water surface as a function of the area of the continents
on the surface of the earth.
Figure 8. Elevation of the ocean surface and the shelf break with different areas of continents on the surface of the earth.

To make this figure, we started the calculations by taking estimates from Sverdrup et al (1970) tables 3 and 4 for the present ocean area of $3.6 \times 10^8$ km$^2$ and mean depth of 3.8 km. The ocean area includes almost 10% submerged continent, but for these crude calculations, we prefer to use the figures from the tables directly without alteration. Land area from table 3 is $1.5 \times 10^8$ km$^2$, and ocean and land add up to the surface area of the earth with land covering 29% and the ocean the rest. We also take the average continent crust thickness to be 40 km. Equating land to continent, this makes the present volume of continental crust equal to $6 \times 10^9$ km$^3$. Using a value of density of continent material of 2900 kg m$^{-3}$ and of mantle material of 3300 kg m$^{-3}$ ext, we can estimate that the continent surface floats at an elevation above the ocean floor of $(3300 - 2900)/3300 = .12$ times the thickness of the continental crust, which is 4.8 km. Thus the continent floats on average 1.0 km above the ocean surface. Since at the present time the edge of the continent (shelf break) is at 100 m depth on average, we take the present shelf break to be 1.1 km below the average continent depth. Corrections are not included for numerous layers (layers
Convection with continents—Whitehead

of ocean crust, ocean sediment, the water itself, mid-ocean ridges or the thickness of the thermal boundary layer of the ocean floor) to keep the problem very simple.

If we then assume that at some past time the area of the continents was different, we calculate the thickness of continental crust and the depth of the ocean water with the new areas, and plot them. This is done in figure 8. As continent area increases, the average continent crust thickness decreases so that the continent floats at a lower elevation with respect to the sea floor.

If at times continents were perhaps spread over a larger area of the earth’s surface, they would be more submerged and mostly under water. In that case, we might assume that continent erosion would not be active. Plate collisions might then gradually compress the edges of the continents and make them thicker.

In contrast, if there were times when the continents occupied a smaller area than now, then their average elevation would be high above the ocean surface. Figure 8 shows that if continents covered only 20% of the earth surface, then the shelf break would 2.4 km above sea level, and average continent elevation would be 3.4 km above sea level. We could expect much stronger erosion during such times.

I suggest that the average elevation of continents above sea level be called. If erosion is the primary cause of “global freeboard”, we conclude that the eroded material is completely removed from continent surfaces and deposited on the ocean floor. Over long geologic time such as a billion years, the material from continents that is placed on the ocean floor is removed, since ocean floor is young. At the present time, most material on the ocean floor above the crust is ocean sediment. Such material, although greatly variable in its composition and thickness, must have continent origins even though it is highly changed chemically from the original continental material. The present ocean sediments are about one km in thickness and cover an area 3.6 × 10⁸ km² so their volume is 3.6 × 10¹⁰ km³. This is about 5% of the present volume of continents. Taking the average age of sea floor to be 100 my, then, if we assume that the present rate of sedimentation were to apply for a billion years, half the continent crust volume of 6 × 10⁹ km³ is deposited on the ocean floor. If this is true, either

a. The eroded material is returned to continents, possibly through a number of pathways such as by being glued onto the sides during collisions of a continent with trenches, island arcs or another continent, by being returned to the surface along with island arc volcanism, and by being emplaced under the continents by the subducting slabs.

or

b. The eroded material is swept into the mantle and the volume of continental crust is decreasing. In this case, the continent area on the surface of the earth is decreasing with time.

If erosion is not the primary cause of “global freeboard”, then sea level plays a part in the viscous spread of the continents.

Laboratory studies of floating bodies in cellular motion

Experiments with a rotating cylinder illuminate some of the features of material that floats on the top of a flowing viscous material. We filled a cylinder of 13.3 cm diameter and 9.6 cm along axis length with 6 cm of 100 cs silicon oil. Approximately 600 small (0.64 cm diameter) polypropylene balls were then added. One hundred balls with half the diameter were also added to allow a little more mobility to the sphere matrix through mobility of dislocations, and to enable the circulation to be more clearly seen. The cylinder was covered with a rigid lid and turned so that the axis was horizontal and connected to a variable speed motor so the cylinder could rotate about its axis. Thus, gravity was at right angles to the axis. Upon rotation, the
Convection with continents-Whitehead

silicon oil developed a circulation cell driven by the walls of the cylinder. This circulation swept
the floating balls toward the descending region. Figure 9 shows the result for one rate of rotation

Figure 9  Photograph of a cluster of floating spheres in a counterclockwise rotating cylinder. The camera looks along both the axis of the cylinder and the rotation axis. Gravity is directed downward. Some of the smaller spheres are detached from the clump by saltation and are returning to the surface in a curved trajectory.

Two aspects are notable. First, a clump of spheres is drawn downward to provide a basement root to the cluster on the descending side. Second. The spheres are elevated above the oil surface over the descending regions. This elevation is a natural consequence of the mechanical laws. Even for a purely viscous fluid, uplift of the top surface is always present if the fluid is floating above another fluid that is also forced downward. The size of the basement root is proportional to rotation rate. More importantly, the clump size follows the overturning velocity divided by the ascent velocity of a sphere. This is shown in figure 10.
Figure 10. Results of the experiment shown in figure 9. Depth of the root $D-D_c$ divided by layer ("continent") thickness $D_c$ as a function of the rotation velocity divided by sphere ascent velocity.

3. Summary
Work in the GFD laboratory has illustrated a number of features of flow in Earth Science problems. The nonlinearities inherent in fluid problems require experiments to help us understand what structures and instabilities are selected by the fluids as they convey heat and momentum.

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Appendix 1
The boundary and interface conditions can be expressed as a matrix equation (note ignore the underlines in the matrix, they are proofing marks)
Convection with continents-Whitehead

\[
\begin{vmatrix}
  kcL & (-sL - kLcL) & cL + kLsL & 0 & 0 & 0 \\
  -sL & LsL & -LcL & 0 & 0 & 0 \\
  k & 0 & 1 & -k & 0 & \frac{1}{\sinh kl} \\
  0 & \mu_2 & -k & 0 & \mu_1 & 0 \\
  0 & 0 & 0 & sl & lsl & lcl \\
  0 & 0 & 0 & 2k^2sl(2kcl + 2k^2lsl) & 2ksl + 2k^2lcl & 0 \\
\end{vmatrix}
\]

\[
\begin{bmatrix}
  A_2 \\
  B_2 \\
  C_2 \\
  A_1 \\
  B_1 \\
  C_1 \\
\end{bmatrix} = \begin{bmatrix}
  U_k \\
  0 \\
  0 \\
  0 \\
  0 \\
  0 \\
\end{bmatrix}
\]

where \( cL = \cosh kl \), \( cl = \cosh kl \), \( sL = \sinh kl \), and \( sl = \sinh kl \).

The solutions for the coefficients are

\[
A_2 = 2U_k^2 [\mu_1 L \sinh kl \sinh^2 kl + \mu_2 L \cosh kl(-kl + \sinh kl \cosh kl)] / \text{Det}
\]

\[
B_2 = 2U_k^2 [\mu_1 (\sinh kl - k \cosh kl) \sinh^2 kl] / \text{Det}
\]

\[
C_2 = 2U_k^2 [\mu_1 \sinh kl \sinh^2 kl + \mu_2 \sinh kl \sinh kl - \sinh kl \cosh kl] / \text{Det}
\]

\[
A_1 = 2U_k^2 [\mu_2 (\sinh kl - k \cosh kl)] / \text{Det}
\]

\[
B_1 = 2U_k^2 [\mu_2 \sinh kl - k \cosh kl] \sinh^2 kl / \text{Det}
\]

\[
C_1 = -2U_k^2 [\mu_2 \sinh kl - k \cosh kl] \sinh kl \cosh kl / \text{Det}
\]

where

\[
\text{Det} = -2\mu_1 [\sinh^2 kl - k^2 L^2] \sinh^2 kl + 2k \mu_2 [\sinh kl \cosh kl - kL][kl - \sinh kl \cosh kl]
\]

References


Convection with continents-Whitehead


Convection with continents—Whitehead


