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Parameterizing Tidal Dissipation over Rough Topography

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Abstract. The traditional model of tidal dissipation is based on a frictional bottom boundary layer, in which the work done by bottom drag is proportional to a drag coefficient and the velocity cubed. However, away from shallow, coastal regions the tidal velocities are small, and the work done by the bottom boundary layer can account for only weak levels of dissipation. In the deep ocean, the energy flux carried by internal waves generated over rough topography dominates the energy transfer away from barotropic flow. A parameterization for the internal wave drag over rough topography is included as a dissipative mechanism in a model for the barotropic tides. Model results suggest that the inclusion of this dissipation mechanism improves hydrodynamical models of the ocean tide. It also substantially increases the amount of modeled tidal dissipation in the deep ocean, bringing dissipation levels there into agreement with recent estimates from TOPEX/Poseidon altimetry data.

Motivation

In this paper we examine two issues. The first is whether including a parameterization for internal wave energy-flux in a model for the barotropic tides improves the simulation of the tides. The second issue is whether the modeled dissipation of the barotropic tides in the deep ocean is realistic when observational estimates are compared to simulations that either include or omit a parameterization of the internal wave energy-flux.

In the first half of the 20$^{\text{th}}$ century, the problem of tidal dissipation was regarded as a solved problem, and it was assumed that there was negligible tidal dissipation in the deep ocean, as *Lamb* [1945] states:

> The tidal currents in mid-ocean are in fact so slight that their frictional effects are unimportant, even from an astronomical standpoint. In shallow water and in narrow seas and estuaries, on the other hand, they become enormously exaggerated as a result of the inertia of the water and the configuration of the ocean bed and the coasts. It appears now to be established [*Taylor*, 1919; *Jeffreys*, 1920] that the total dissipation of energy in such regions, at the expense ultimately of the earth's rotation, is comparable with that which is inferred on astronomical evidence.

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Recent observations, however, have undermined the faith in this view. In particular, results from two independent lines of research have suggested that the dissipation of tidal energy may be significant in the mid-ocean. This new evidence indicates that the conversion of barotropic tidal energy into internal waves (*i.e.* internal tides) by flow over rough topography may indeed be an important energy sink of barotropic tidal energy.

The first line of evidence comes from observations of mixing in the abyssal Brazil Basin. An analysis of tracer dispersion and turbulent microstructure levels indicated that mixing rates were enhanced over the rough topography near the Mid-Atlantic Ridge [*Ledwell et al.*, 2000]. In particular, microstructure derived mixing rates were found to modulate with the spring-neap cycle of the tides. These enhanced mixing rates are thought to be supported by breaking internal waves generated as the barotropic tide flows over rough bathymetry. A second line of evidence comes from tidal energetics estimates made using satellite altimeter data. Using an inverse model to calculate energetics from TOPEX/Poseidon data, *Egbert and Ray* [2000] found that about 1 Terawatt (TW), or between 25–30%, of the total tidal dissipation takes place in the deep ocean, generally near areas of rough topography.

Background

The kinematics of frictional boundary layers are commonly used to model the energy transfer that occurs when flow exerts stress on the seafloor. The work done by the bottom stress is assumed to balance the energy dissipation occurring in the boundary layer, and the expression for the work done by bottom drag is proportional to cubed velocity:

$$W = \rho_0 c_d |\mathbf{u}| \mathbf{u}^2, \tag{1}$$

where $c_d \sim 0.0025$ is the coefficient of drag. In the deep ocean, taking $u = 2$ cm s^{-1}, the work done by tides in the frictional boundary layer is O(0.02 mW m^{-2}), too small to be an appreciable dissipator of tidal energy.

A dissipation mechanism for the barotropic tides via internal wave generation at the continental shelf was proposed by *Baines* [1982]. His estimate that only about 1% of the barotropic tidal energy was converted at continental shelves discouraged further investigation along these lines [*Munk*, 1997]. However, tidal flow is generally directed parallel to the shelfbreak, rather than across it, reducing the generation of internal waves. In contrast, the flow is less constrained by mid-ocean ridges, allowing those areas to be more efficient generators of internal waves [*Munk and Wunsch*, 1998].

The dynamics related to the production of internal waves when steady flow encounters bottom bathymetry are well known, and internal waves produced by steady flow over topography are termed "lee waves". The specific problem of tidal flow over bathymetry has been considered in several studies [*Bell*, 1975a, b; *Baines*, 1982; *Sjöberg and Stigebrandt*, 1992]. The theory proposed by *Bell* [1975a] applies to "subcritical" topography, where the topographic slope is less steep than the ray trajectory of the radiated internal tide. The other models address internal tides generated by steeper "supercritical" topography, with the case of depth discontinuities considered by *Sjöberg and Stigebrandt* [1992].

For the work presented here, a simple relation describing the energy flux lost by the barotropic tide to internal waves is sought. All models referred to above predict an energy flux proportional to squared tidal velocity, though formulations for subcritical and supercritical topography differ drastically. The simplest scaling derived from subcritical theory is:

$$E_f \sim \frac{1}{2}\rho_0 \kappa h^2 N \mathbf{u}^2, \qquad (2)$$

for the energy flux per unit area, where N is the buoyancy frequency and (κ, h) are the wavenumber and amplitude that characterize the bathymetry. We emphasize that (2) is a scale relation, and not a precise specification of internal tide energy-flux. One of the vexing problems of including a parameterization for internal waves in a barotropic tide model is that while internal waves are studied in the frequency domain, numerical modeling is generally performed in the time domain. This makes a parameterization involving multiple frequency constituents difficult to implement. We have neglected a frequency dependence factor of $\omega^{-1}(\omega^2 - f^2)^{1/2}$ in (2), which becomes significant at latitudes where the tidal frequency ω is close to the Coriolis frequency f. While propagating internal tides occur equatorward of the turning latitude where $\omega = f$, bottom-trapped internal tides occur at latitudes where $\omega < f$. Thus, semidiurnal internal tides are trapped poleward of 74.5°, and diurnal internal tides are trapped poleward of 30°. The factor of 1/2 in (2) is retained for aesthetic reasons, allowing us to draw a connection to the "mountain drag" parameterization used in the meteorological literature. It was found by *Palmer et al.* [1986] that atmospheric general circulation models showed an improved circulation pattern when the energy flux of waves produced by winds blowing over orography was accounted for using a relation identical to (2).

In the work presented here, the term "dissipation" refers to the loss of barotropic-tidal energy to bottom friction, as well as to the loss of energy to internal waves. In regions of rough topography, taking horizontal scales of O(10 km) and vertical scales of O(100–300 m) as typical of the roughness, then an energy flux of O(1–10 mW m^{-2}) is estimated to be lost by the barotropic tide to internal waves. Ultimately, internal-wave energy must truly dissipate as turbulence. Wave radiation, however, may carry internal-tide energy to sites far from generation regions, as observed for the internal tide generated at Hawaii [*Ray and Mitchum*, 1996]. This is particularly true of the waves with low vertical modes that carry most of the energy. The extent to which internal tides produce turbulence as they propagate away from their generation sites is not clear, and is the subject of ongoing work.

Implementation

The numerical model equations for the barotropic, shallow-water flow are modified to include the stress associated with wave energy flux in a manner similar to *Palmer et al.* [1986]. The momentum equations are:

$$\frac{\partial U}{\partial t} - fV = -gH\frac{\partial}{\partial x}(\eta - \overline{\eta} + \zeta_l) - c_d|\mathbf{u}|u - \frac{1}{2}\kappa h^2 N u \qquad (3)$$

$$\frac{\partial V}{\partial t} + fU = -gH\frac{\partial}{\partial y}(\eta - \overline{\eta} + \zeta_l) - c_d|\mathbf{u}|v - \frac{1}{2}\kappa h^2 N v, \qquad (4)$$

where $\mathbf{u} = (u, v)$ is the horizontal velocity vector, and $(U, V) = H(u, v)$ are the corresponding transport velocities. The ocean depth is given by H, f is the Coriolis parameter, g is gravity, η is the free-surface elevation, $\overline{\eta}$ is the equilibrium tidal forcing, and ζ_l is the self-attraction and solid-earth load tide [*Farrell*, 1972; *Hendershott*, 1972]. The barotropic model is a simple second-order, finite-difference model of the linear shallow water equations, time-stepped forward using the trapezoidal-leapfrog method [*Killworth et al.*, 1991]. It was configured on a 1/2° Mercator grid from 72°S to 72°N and forced with the 8 largest tidal constituents. The drag on the barotropic flow is in two separate dissipation terms, given by dividing relations (1) and (2) by velocity. The drag of bottom friction is represented by $c_d|\mathbf{u}|\mathbf{u}$. The second term, $(1/2)\kappa h^2 N\mathbf{u}$, is the parameterization used to represent the drag from internal-wave generation. This is not a proper expression of internal-tide stress. Instead, $(1/2)\kappa h^2 N\mathbf{u}$ is better thought of as "Rayleigh" drag, which crudely represents the drag of internal tides over subcritical topography. In regions of supercritical topography, this parameterization still provides drag, though its relation to internal-tide drag is questionable.

The parameterization is only used in water depths greater than 100 m, and the required values of N, κ, and h^2, are set as follows: The buoyancy frequency, N, as a function of longitude, latitude and bottom depth is computed from the *Levitus et al.* [1994] database. The bottom roughness, h^2, is computed from the *Smith and Sandwell* [1997] 1/30° ocean topography database. Over each grid cell, a polynomial sloping surface is fit to the bottom topography (given by $H = a + bx + cy + dxy$), and the residual heights are used to compute h^2, the mean-square bottom roughness averaged over the grid cell. In the gravity wave stress-parameterization used in atmospheric models, κ, the wavenumber of topographic roughness, is generally set to be spatially constant. In this study, we treat κ as a spatially-constant free parameter, whose value is adjusted to minimize the difference between the modeled tides and the observed tides. We find $\kappa \simeq (2\pi/(10 \text{ km}))$ is optimal. In principle, however, κ could be calculated directly from the topography, and be different for the u and v directions, allowing for the anisotropy of bottom topography associated with ridges and faults.

Results

To assess the impact of including the parameterization of internal wave energy-flux on a tidal simulation, the modeled tides were compared to the observed tide [UT-CSR, version 4 by *Eanes and Bettadpur*, 1995]. The comparison is the areally weighted root-mean-square (RMS) difference computed over the deep ocean (defined as water depths greater than 1000 m). It is readily observable in Table 1, that the

Table 1. RMS difference from observed tide [cm]

Tide	Bottom drag only	Bottom drag plus internal-wave drag
M_2	12.0	6.7
S_2	5.9	3.6
N_2	2.1	1.2
K_2	1.4	1.1
K_1	6.3	5.5
O_1	6.1	2.9
P_1	1.7	1.4
Q_1	1.9	1.2

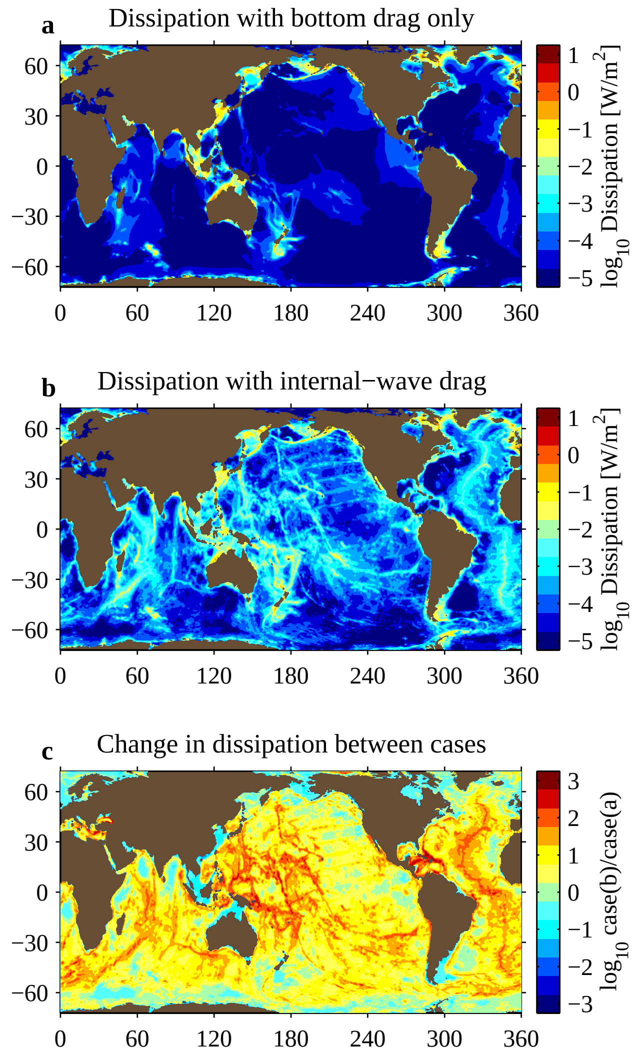

Figure 1. Time-average dissipation in tidal model simulations (a) with bottom drag only, (b) with bottom drag plus internal-wave drag, and (c) the difference between the two cases.

inclusion of the parameterization increases the fidelity of the barotropic tide model. For the 8 constituents modeled, the total RMS is reduced from 16.4 cm to 10.1 cm.

The modeled tidal dissipation was calculated as well. In both model runs the dissipation was about 3.5 TW, in agreement with the measured global tidal dissipation of 3.5 ± 0.1 TW (Table 2). In the simulation without the parameterization of internal waves, less than 1% of the dissipation took place in the deep ocean (deeper than 1000 m), as expected from scaling arguments [*Munk*, 1997]. When the parameterization was included, however, the modeled deep dissipation increased to about 1 TW (30% of the total), of which 0.72 TW is from the M_2 tide, consistent with the estimates made by *Egbert and Ray* [2000]. The model's amount of deep dissipation is very sensitive to our choice of κ, changing nearly linearly with it when κ is $O(2\pi/(10\text{ km}))$. That the modeled deep dissipation is in close agreement with estimates from observations, however, suggests the parameterization is a plausible representation of internal tide energy-flux. Figure 1 shows the spatial distribution of the dissipation. The increased dissipation in the deep ocean appears concentrated along the mid-ocean ridges where the topography in the ocean is the roughest, qualitatively consistent with the estimates by *Egbert and Ray* [2000].

In this paper we have addressed only a very limited aspect of a parameterization for internal waves in a barotropic tide model. A parameterization better suited for supercritical topography is especially needed. However, our estimates indicate that including a parameterization of the internal-wave drag in tide model improves the simulation of the tide. At the same time, the parameterization changes the distribution of dissipation so that about 1 TW of energy is lost from the tides in the deep ocean. We believe that further work may lead to improved general circulation models, where deep ocean mixing-rates should be specified according to predictions of tidal-energy input. Including a parameterization of wave energy-flux in eddy-resolving models may help explain observations of decreased eddy-energy over rough topography in the deep ocean [*Gille et al.*, 2000]. Finally, improved parameterizations of internal tides would aid in the general problem of modeling barotropic ocean variability.

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Table 2. Energy dissipation [Terawatts]

	Bottom drag only	Bottom drag plus internal-wave drag
Total dissipation	3.49	3.54
Deep dissipation	0.02	1.07
Deep / Total	0.7%	30.2%

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