

CONCENTRATION AND DEPTH FIELD DETERMINED BY THE LIGHT TRANSMITTED THROUGH A DYED SOLUTION

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Abstract

In this paper we analyse a method to determine the dye concentration and the depth of a dyed solution by measuring the reduction in the intensity of light transmitted through the dyed solution, as compared to that transmitted through a clear solution. In other contexts the basis of this technique is often referred to as "optical thickness" and gives a measure of the average concentration of the dye over the whole fluid thickness h . However, for layered fluid flows, we may instead measure the depth of a layer by marking one layer with a uniform concentration of dye so that the attenuation due to the dye is confined to that layer and thus measurements of the optical thickness give the depth of the dyed layer. In this paper we present a detailed description of procedures required to derive the dye concentration (averaged along light rays) from an intensity field, and, ultimately, the thickness of the dyed layer. A number of new techniques to improve the accuracy and reliability of these measurements are also introduced. Finally the technique is illustrated with measurements of baroclinic vortices over a slope.

1. Introduction

Measurements of optical thickness have been used for a number of years to measure layer thickness (Linden & Simpson [13]; Holford [8]; Holford & Dalziel [9]) and cross-tank mean concentration

(Barnett [1], [2]; Linden & Redondo [11]; Redondo & Linden [14]; Linden *et al.* [12], Redondo & Yague [15]; Gilmour [6]; Gilmour & Woods [7], Hacker *et al.* [10], Verlaan [16]) based on image processing routines developed by Dalziel in 1989 as part of the image processing system *DigImage* (Dalziel [5]).

The discussion in this paper centres around the use of standard video equipment in conjunction with digital image processing techniques to obtain the required optical thickness measurements. Some of the recently developed methodologies may also be used to improve purely visual, qualitative representations of the dye concentration field and also to improve the appearance of other visualisations such as shadowgraph.

2. Light attenuation theory

2.1. Simple attenuation

The classical absorption theory of Lambert-Beer gives

$$\frac{d\hat{I}}{dz} = -\eta\hat{I}, \quad (1)$$

with the rate of absorption η being some function of the concentration, $f(c)$, say. If we suppose that the concentration of the dye is constant along the light ray (*i.e.* $c(z) = c$) integration of (1) along the ray gives the attenuation ratio

$$\frac{\hat{I}(h,c)}{\hat{I}(0,c)} = e^{-f(c)h}, \quad (2)$$

where h is the distance travelled by the light ray in the dyed water and $\hat{I}(0,c)$ is the illuminating light intensity at the start of the ray. The function $f(c)$ is called absorbency of the dye and here it is assumed to be only a function of the dye concentration c . For low dye concentration $f(c)$ may be approximated as linear in c ,

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$$f(c) = Ac + b, \quad (3)$$

where $A > 0$ determines how increasing the concentration affects the attenuation and $b > 0$ represents the attenuation due to the undyed water. The linear nature of (1) allows us to separate these two components and write

$$\frac{\hat{I}(h,c)}{\hat{I}(h,0)} = \frac{\hat{I}(h,c)}{\hat{I}(0,c)} \bigg/ \frac{\hat{I}(h,0)}{\hat{I}(0,0)} = e^{-Ach}, \quad (4)$$

thus allowing us to use the light transmitted through an undyed solution instead of the light entering the dyed solution in order to determine c or h .

2.2. Wavelength dependence

The absorbency of the dye $f(c)$ introduced in the previous section is not only a function of the dye concentration but is also a function of the wavelength of the light because the dyed solution absorbs different wavelengths of light in different ways. We may examine the impact of this effect by assuming that the intensity of the light source has some fixed spectrum $S(\mathbf{I})$ such that the intensity it produces at any given wavelength may be expressed as $I_0(\mathbf{I}) = I_0 S(\mathbf{I})$. Similarly we shall take the response of the camera as $G_0 G(\mathbf{I})$, where G_0 is the video camera gain (assumed constant for the present discussion). Then the output from the video camera will be

$$V = G_0 I_0 \int_0^\infty S(\lambda) G(\lambda) P(c, \lambda) d\lambda, \quad (5)$$

where $P(c, \mathbf{I}) = e^{-f(c, \mathbf{I})h}$ is the absorption function of a dye which obeys (1) and we have assumed the response of the video camera to be linear in the intensity. Using equation (5) and a very simple model for $S(\mathbf{I})$, $G(\mathbf{I})$ and the absorption function of the dye $P(c, \mathbf{I}) = e^{-f(c, \mathbf{I})h}$ (for details see Cenedese & Dalziel, [3]) the ratio calculated in equation (2) becomes

$$\frac{V}{V_0} = \frac{1 + e^{-Ach}}{2}. \quad (6)$$

Thus the relationship between the attenuation V/V_0 and the depth is no longer an exponential in contrast to that suggested by (2). While the above example is contrived, we note that the attenuation of many dyes may be modelled approximately as a

constant plus an exponential term for weak solutions.

In the above example we could recover an exponential relationship by replacing V and V_0 with $V' = V^{-1/4} G_0 I_0$ and $V'_0 = V_0^{-1/4} G_0 I_0$. However, such transformations are not, in general, possible and would require a precise knowledge of the absorption spectrum $f(c, \mathbf{I})$ and video camera gain G_0 . Instead we may use filters to select some range of wavelengths where $P(c, \mathbf{I})$ is (approximately) independent of \mathbf{I} . One way of achieving this is to place a suitably coloured filter, with spectral response $F(\mathbf{I})$, in front of the light source or camera such that the camera output becomes

$$V = G_0 I_0 \int_0^\infty S(\lambda) F(\lambda) G(\lambda) P(c, \lambda) d\lambda. \quad (7)$$

A suitably chosen filter will then recover an exponential relationship for V/V_0 .

3. Coloured light calibration

As discussed in §2.2, in order to have an exponential relationship between intensity and depth, we must choose a region where the dye absorption of intensity is independent of the wavelength. The absorption spectrum for different dyes suggests which kind of filter is better for each dye. For red organic food dye the absorption spectrum suggests a filter that selects a wavelength of approximately 5100 Angstroms (e.g. a green or blue filter). The absorption of the light in the range of wavelengths chosen by the filter is almost constant as required in §2.2. The calibration of the depth with the presence of a filter between the flow and the video camera produces a relationship between the I/I_0 and the depth h that is almost exponential with an asymptote at $y \gg 0$. This result confirms the theoretical prediction of the wavelength dependence of the light absorption through a dyed solution and the effect of the wavelength selection by mean of coloured light. The exponential relationship between I/I_0 and the depth h will be extensively used in the experiments described in §4 in order to determine the depth of the interface perturbation.

4. Application

The method described in this paper has been applied to determine the depth field of vortices produced by a source or a sink of buoyant fluid in a two layer rotating environment over a sloping bottom (Cenedese & Linden, [4]). The presence of a sloping bottom simulated the β -effect for the ambient fluid so that the shallowest part of the tank corresponds to the “northern” shore of the northern hemisphere topographic β -plane. East is to the right looking onshore, west is to the left and south is the deepest end. The digitised images $I(x,y)$ of the experiments were recorded directly and for each image the ratio I/I_1 was calculated, where $I_1(x,y)$ is the intensity of the background flow just before the start of the experiment. For the determination of the interface perturbation depth generated by the vortices developing in the top layer it was not necessary to know the digitised intensities I_0 of the background illumination without any dye in the upper layer. The total depth of the top layer H_{tot} is given by the initial top layer depth H_1 plus the interface perturbation depth H' . Thus, if we have a dye which behaves exponentially, equation (2) shows

$$H_{tot} = -\frac{1}{f(c)} \ln \frac{I}{I_0}, \quad (8)$$

$$H_1 = -\frac{1}{f(c)} \ln \frac{I_1}{I_0}, \quad (9)$$

and the interface perturbation depth is given by

$$\begin{aligned} H' &= H_{tot} - H_1 = \\ &= -\frac{1}{f(c)} \left(\ln \left(\frac{I}{I_0} \right) - \ln \left(\frac{I_1}{I_0} \right) \right) =, \quad (10) \\ &= -\frac{1}{f(c)} \ln \frac{I}{I_1} \end{aligned}$$

which is independent of the background images without dye $I_0(x,y)$. The application of equation (2) is strictly valid only when the relationship between the depth and the intensity ratio I/I_0 is exponential. Therefore the procedure described in §3 was necessary in order to apply the equation (10) and avoid the need to record the $I_0(x,y)$ image. Measurement of I_0 would have been impractical for these experiments because the two

layer stratification is obtained with a technique that takes few hours to be completed and the aperture required for the camera to record an image of I_0 would render the images I and I_1 much less intense and more prone to contamination by noise.

Once the digitised video sequence had been transformed into absolute intensities, the ratio I/I_1 was utilised to find the depth field of the interface perturbation $H'(x,y)$. This depth perturbation due to the injection (anticyclones) or withdrawal (cyclones) of upper layer fluid is shown in figures 1 (a), (b).

5. Conclusions

A description of the procedures involved to calculate the depth or the concentration field from the reduction in intensity of light transmitted through a dyed solution were presented. A technique to obtain an exponential relationship between intensity and depth was presented. The description of this technique involves the study of the absorption spectrum of the different types of dye used in order to choose the appropriate filter to put between the video camera and the flow. The precise conduction of these procedures brings an accurate determination of the concentration and depth fields. The method described in this paper is more sophisticated than the one used in Holford & Dalziel (1996) and gives substantially improved accuracy in the depth measurements.

This method was applied to determine the depth field of vortices produced by a source or a sink and allowed to have an accurate knowledge of the shape and size of these structures. Following the depth contours was possible to measure the westward velocity of the vortices and therefore compare it with the speed U_s of the linear topographic Rossby wave. A comparison between cyclonic and anticyclonic vortices revealed asymmetry in the quantitative properties of the vortices and also in the qualitative transition from the single to the multiple regime. The shape of the interface perturbation generated by the two different type of vortices appears to be substantially different as shown in figures 1 and also the shape and the orientation of the vortex depends on the sign of the vorticity field. Quasi-geostrophic motion must, by definition, be exactly the same for cyclones and anticyclones of the same

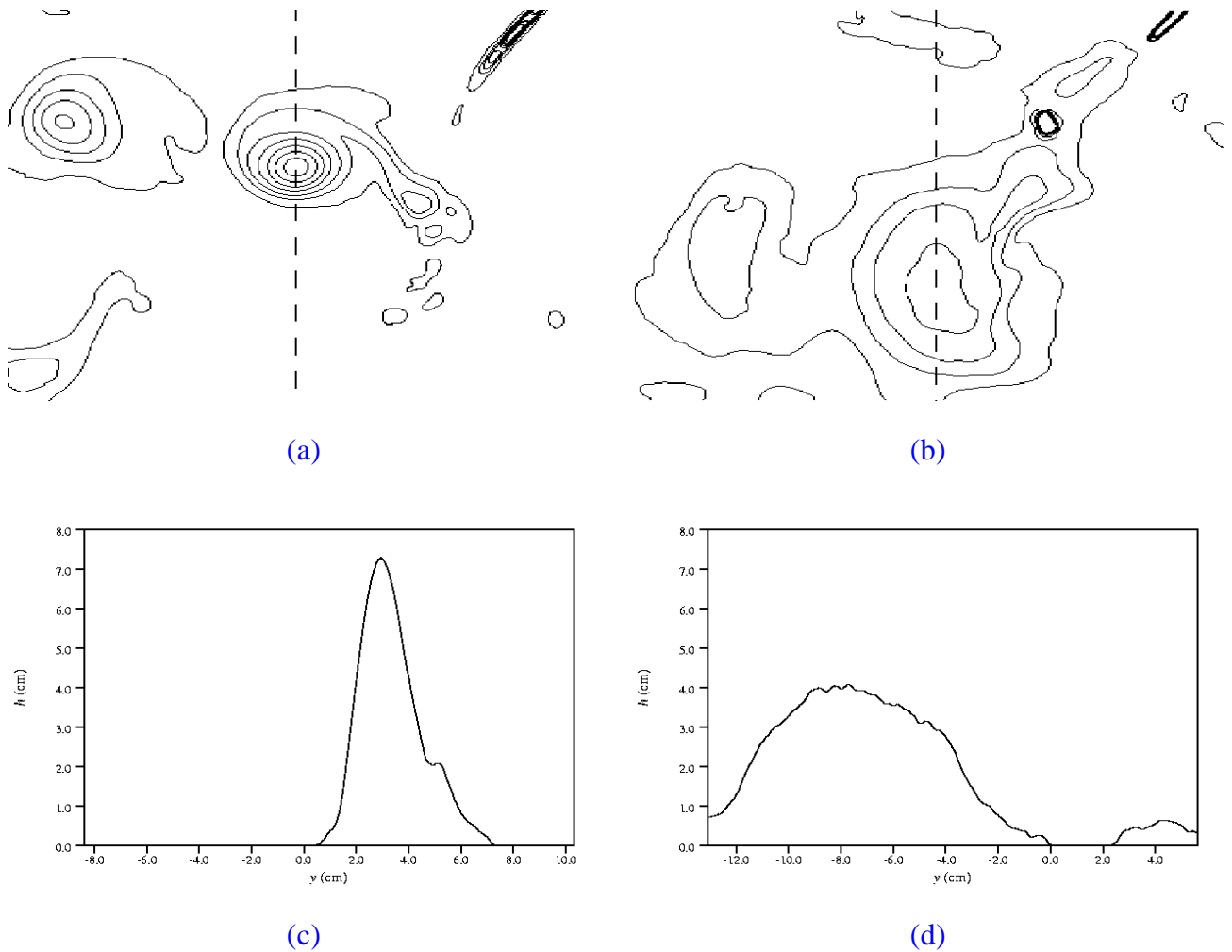


Figure 1. Interface perturbation depth fields after 94 sec of the multiple (a) cyclones ($f=3.3 \text{ s}^{-1}$, $Q=3.00 \text{ cm s}^{-1}$) and (b) anticyclones ($f=2.0 \text{ s}^{-1}$, $Q=4.50 \text{ cm s}^{-1}$). The sink/source is positioned on the right side of the picture and the vortices are moving westward. The isobaths are represented by continuous lines and are drawn every cm. The figure (c) and (d) show the depth sections along the dashed lines in figure (a) and (b), respectively.

vorticity structure, therefore the observed asymmetry between cyclones and anticyclones implies that ageostrophic effects were present in the flow.

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AVI Files: There are two. The first, `Cyclones`.AVI corresponds to figure 1a but with the lower layer depth shown as false colour and the upper layer depth as contours. The second, `AntiCycl`.AVI corresponds to figure 1b with the lower layer depth as false colour and upper layer depth as contours. Both sequences are displayed at 1 frame per second, starting with a title frame for three seconds.