Environmental Hydraulics - Stratified and Rotating flows

Density Current Down a Slope in a Rotating Fluid

C. Cenedese[1], J.A. Whitehead[1] & T. A. Ascarelli[2]

[1]Woods Hole Oceanographic Institution, 360 Woods Hole Rd. Woods Hole, MA 02543, USA.

[2]Universita' di Roma "La Sapienza". Via Eudossiana, 18. 00184 Roma, Italy.

ABSTRACT

The occurrence of density currents as a result of interaction between two or more fluids of different densities is widespread in nature and several examples of density currents are known within the field of hydraulic and environmental engineering. The behavior of a dense fluid released down a slope has been investigated in the laboratory in a rotating tank filled with fresh water. We systematically varied four fundamental parameters: the Coriolis parameter, f, the slope inclination, s, the flow rate of the dense fluid, Q, and the reduced gravity, $g' = g(\rho_2 - \rho_1) / \rho_1$ where ρ_2 is the density of the dense fluid and ρ_1 is the density of the environmental fresh water. Over a wide range of parameter values three flow types are found: laminar flow, waves, and eddies. Regime diagrams illustrate the range of dimensionless numbers that produce these three different flow types, and the parameters indicate some aspects of the dynamics for their formation. Kelvin-Helmholtz instability seems the most likely candidate for wave generation that is observed for values of the Froude number Fr ³ 1. A theoretical solution for the downslope velocity field has been found using a steady state model that takes into account the boundary layers both over the slope and on the interface between the two fluids. Comparison between the theoretical and the experimental results are discussed.

INTRODUCTION

The descent of dense water from the continental shelves into the deep ocean over the continental slope is an important component of the thermohaline circulation. Sources of deep water formation are found mainly in the polar regions, and outflows to the main oceans are found in the Denmark Strait (Dickson and Brown[1]) and in the Weddell Sea (Killworth[2]) in Antarctica (Whitehead[3]), or in proximity of enclosed seas where evaporation contributes to increase the density of the enclosed waters that therefore flow down into the open seas through a strait like the Mediterranean outflow (Price et al.[4]). The dynamic and behavior of such dense currents have been

"Proceedings of the 2001 International Symposium on Environmental Hydraulics

modeled in the past both theoretically and experimentally. In a simplified so-called "stream tube" model (Smith[5], Price et al.[4], Price and Baringer[6], Baringer and Price[7]) the dense plume flows down slope balancing buoyancy, Coriolis and friction forces. Laboratory experiments have also confirmed that cyclonic vortices of overlying fluid can be generated by either a process in which the upper layer columns get stretched (Lane-Serf and Baines[8-9], Etling et al.[10], Whitehead et al.[11]) or by baroclinic instability (Etling at al.[10]). Furthermore, the descent of dense water down a slope is not always associated with eddy formation. Shapiro and Zatsepin[12] observed that the dense bottom layer becomes unstable to growing waves and a stability analysis showed that the stability threshold depends on the Froude number. Recently, numerical models have included the dynamic interaction of the dense flow down a slope with the above less dense layer (Jiang and Garwood[13-14-15], Spall and Price[16], Swaters[17]) and shown that a strong coupling between the two layers can lead to the formation of cyclonic vortices. Similar vortex formation was observed near the Denmark strait (Bruce[18], Krauss [19]).

The purpose of this work is to investigate the behavior of such dense flow down a slope over an extremely wide range of parameters and to quantitatively identify the transition between the three main regimes previously observed: the laminar, the wave and the eddy regime. Although these regimes have been previously observed in different studies, a comprehensive study is still lacking that shows the fundamental parameters for each regime.

THEORETICAL MODEL FOR A LAMINAR FLOW

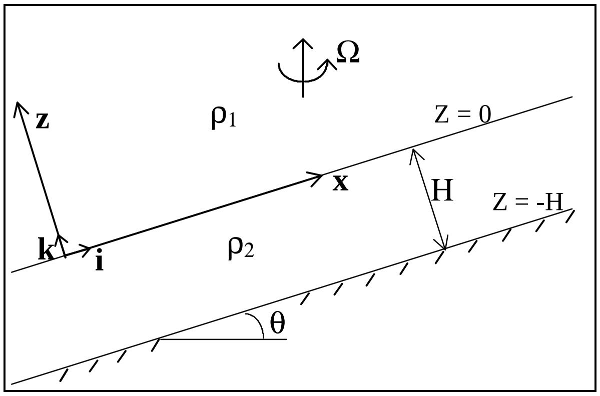


Figure 1. Sketch of the model configuration.

We started by considering a two layers fluid, each of constant uniform density ρ_n, $n = 1,2$, as shown in figure 1 where $\rho_2 > \rho_1$. Those two layers lie over a sloping bottom having a slope $s = tan\theta$, where θ is the angle between the slope and the horizontal. The interface between the two fluids is positioned parallel to the bottom slope and the upper layer is considered to be infinitely deep. In the rotating frame of reference (Ω) we adopt the tilted coordinate ***i, j, k*** shown in figure 1, the rigid sloping bottom is defined by the boundary $z = -H$, where H is the depth of the lower layer in the tilted frame of reference, while the interface is defined by the surface $z = 0$. The velocity has components u_n, v_n, w_n, $n = 1,2$, parallel to the x, y, z– axes, respectively. Considering $u_n(z)$, $v_n(z)$ and $w_n = 0$, the linearised steady Boussinesq momentum equations have the following solutions when considering a non-slip bottom boundary condition, $v_2 = 0$ and $u_2 = 0$ for $z = -H$, and that the velocity and the shear are continuous on the interface, $z = 0$, between the two layers

$$u_1 = v_p\left[-J\cos(az)e^{-az} + K\sin(az)e^{-az} + 0.5\sin(az)e^{-az}\right], \tag{1}$$

$$v_1 = v_p\left[J\, sin(az)e^{-az} + K\, cos(az)e^{-az} + 0.5\, cos(az)e^{-az} \right], \tag{2}$$

$$u_2 = v_p\left[-J\, cos(az)e^{-az} + K\, sin(az)e^{-az} + 0.5\, sin(az)e^{az} \right], \tag{3}$$

$$v_2 = v_p\left[J\, sin(az)e^{-az} + K\, cos(az)e^{-az} - 0.5\, cos(az)e^{az} + 1 \right], \tag{4}$$

where $J = -0.5\, sin(2aH)e^{-2aH} + sin(aH)e^{-aH}$, $K = 0.5\, cos(2aH)e^{-2aH} - cos(aH)e^{-aH}$, $a = (f cos\theta/2\nu)^{0.5}$, $v_p = g' tan\theta/f$ and $f = 2\Omega$ is the Coriolis parameter. Similar solutions were derived earlier by Nagata et al.[20].

EXPERIMENTAL APPARATUS

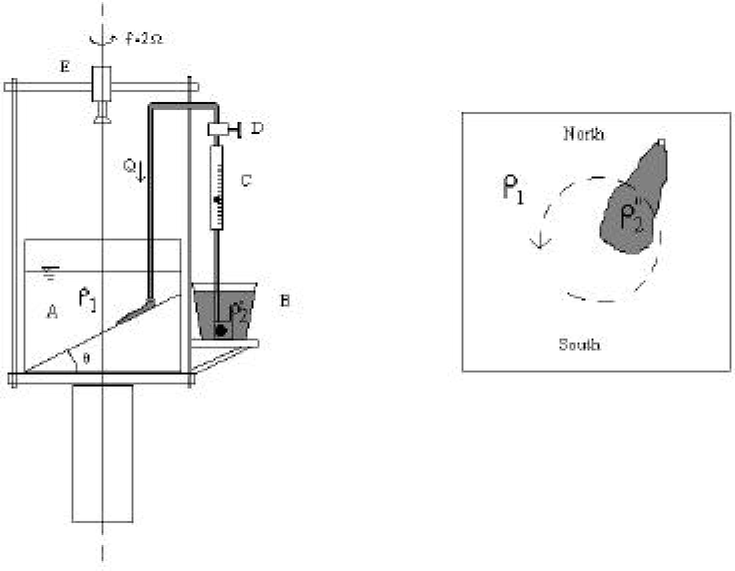
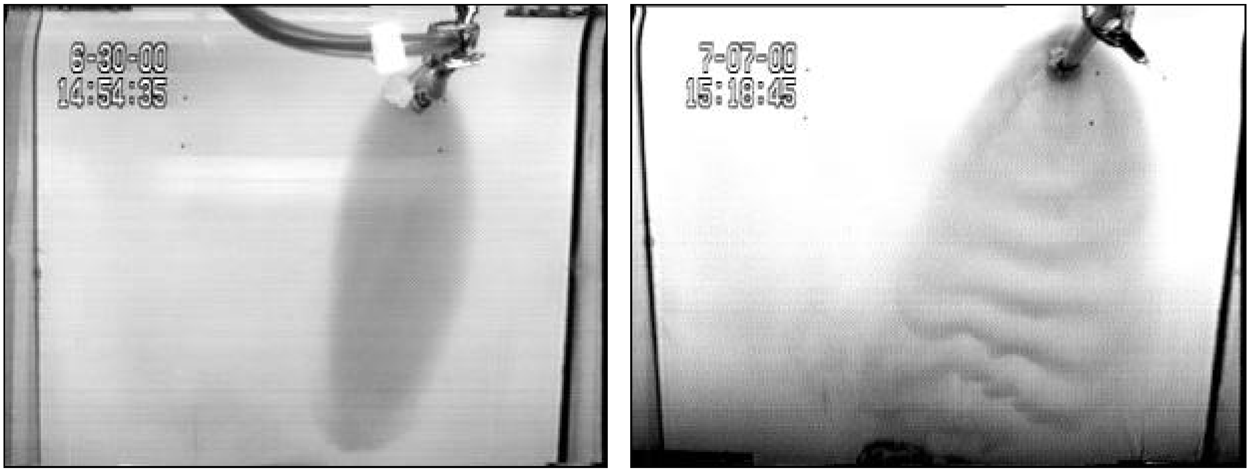


Figure 2. Sketch of the experimental apparatus. Side view (left) and plan view of the tank (right). (A) fresh water tank, (B) dense reservoir, (C) flow-meter, (D) regulation valve for the dense flow, (E) the video-camera, ρ_1 and ρ_2 are the density of the fresh water and dense water, respectively.

The experiments were conducted in a glass tank of depth 60 cm and base 61 cm mounted concentrically on a 1m diameter, rotating turntable with a vertical axis of rotation (figure 2). The tank had a bottom slope, $s = tan\theta$, to simulate the β-effect. The shallowest part of the tank corresponded to the "northern" shore of the northern hemisphere topographic ß-plane. East was to the right looking onshore, west was to the left and south was the deepest end. The tank was filled with fresh water of density ρ_1. A reservoir (B) of salted and dyed water of density $\rho_2 > \rho_1$ was placed on the rotating table and connected to a nozzle via a submergible pump and a plastic tube. The nozzle was covered by a piece of sponge so as to reduce the mixing between the dense fluid and the fresh environment water, and it was positioned on the right side (looking upslope) of the shallowest part of the tank at approximately 1-2 cm from the bottom. A video camera (E) was mounted above the tank and fixed to the turntable so that measurements could be obtained in the rotating frame. The dense current was made visible by dying the fluid with food coloring. The motion of dye was also observed from a side view. The flow at the free surface was observed by adding buoyant paper pellets. The depth of the fresh water on the shallowest side of the tank was kept constant at 10 cm for every slope inclination and the Coriolis parameter f was varied between 0.0 and 2.2 s^{-1}. The slope s of the bottom was set at four different values $s = tan\theta = 0.25, 0.30, 0.40, 0.50$. The dense fluid was forced at a flow rate Q that ranged between 0.45 and 41.60 cm^3s^{-1}. The nozzle inner radius was kept constant at 0.3 cm. The buoyancy forces are described by the reduce gravity $g' = g(\rho_2 - \rho_1) / \rho_1$, where g is the gravitational acceleration, and g' was varied between 2 and 20 cm s^{-2}.

QUALITATIVE RESULTS

After the dense current was released at the nozzle on the right side (looking upslope) of the shallowest part of the tank, it started flowing downslope. The southernmost part of the front presented a characteristic "nose" that was thicker than the fluid behind and had the characteristic described by Simpson[21] with a region of high mixing behind the crest. The width of the current grew rapidly while descending the slope until it reached an approximately constant value some distance downstream. For the experiment with $f = 0$ the dense current descended southward while when $f > 0$ the current underwent a geostrophic adjustment. Over the wide range of parameter values investigated, three flow types were found: laminar flow, waves, and eddies.



(a) (b)

Figure 3. (a) Laminar flow for a very small value of the Coriolis parameter $f = 0.04$ s^{-1}, $Q = 1.22$ cm^3s^{-1}, $s = 0.25$ and $g' = 2$ cm s^{-2}. (b) Waves regime observed for $f = 0.04$ s^{-1}, $Q = 12$ cm^3s^{-1}, $s = 0.5$ and $g' = 2$ cm s^{-2}.

Laminar flow. The dense fluid behind the nose presented constant thickness and a very sharp interface with the fresh water layer, indicating that no mixing was occurring behind the nose (figure 3a).

Waves. The dense fluid flowed downslope and after a "laminar flow" period it started developing a wave-like disturbances on the interface between the dense and fresh fluids (figure 3b). The waves traveled in the same direction of, and faster than, the mean flow. In the plane orthogonal to the bottom slope the dense current was thicker in correspondence of the crest of the waves and thinner in correspondence of the waves' trough. The waves amplitude grew as the waves were traveling downslope and they eventually broke up in most of the experiments, inducing mixing between the dense salty water and the lighter fresh water.

Eddies. In some experiments the downslope movement of the dense fluid caused the periodic formation of cyclonic eddies in the fresh water layer above. Similar behavior has been found and discussed Lane-Serf and Baines[8-9], Etling et al.[10]. The cyclonic eddies in the fresh water column caused the lower dense layer to acquire a dome shape with a height larger than the dense current thickness.

QUANTITATIVE RESULTS

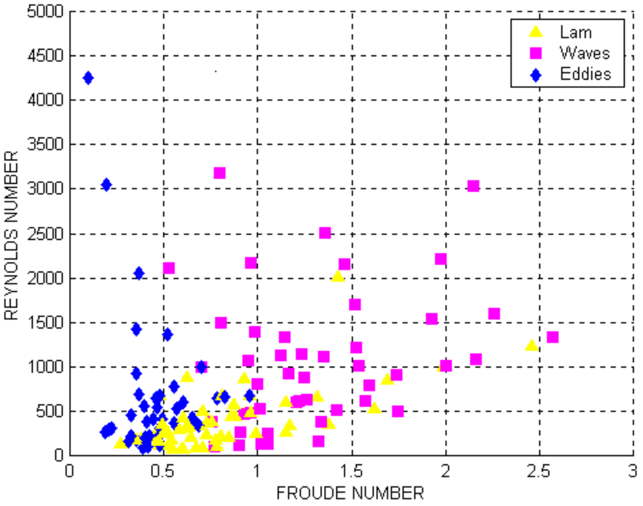


Figure 4. Value of the Froude number $Fr = U/(g'H)^{0.5}$, where $U=(u_2^2 + v_2^2)^{0.5}$, versus values of the Reynolds number $Re = UH/\nu$. The wave regime occurs mainly for values of *Fr* ³ 1 , while the laminar and eddies regime occur for $Fr < 1$.

As expected the eddies were present for the largest values of the Coriolis parameter. The wave regime was observed for larger values of f when increasing the bottom slope s or the reduced gravity g'. From equations (3) and (4) the velocities in the lower layer are directly proportional to $v_p = g's / f$, therefore increasing f will decrease the velocity of the dense current while increasing s and g' will increase its value. Hence we computed the value of the Froude number $Fr = U / (g'H)^{0.5}$, where $U = (u_2^2 + v_2^2)^{0.5}$, to verify if the gravity wave speed $(g'H)^{0.5}$ is the critical velocity the dense current has to reach to generate waves in the bottom dense layer. Unfortunately, the values of the velocities in the dense flow were not measured directly for all the experiments. Therefore, for each experiment we calculated the Froude number utilizing equations (3) and (4). From figure 4 it is clear that the wave regime occurred for values of the *Fr ³ 1*, indicating that indeed the gravity wave speed is the critical velocity the dense current has to reach in order for the waves to form.

CONCLUSIONS

All the experiments performed in this study revealed that increasing the slope inclination and the reduced gravity allowed the formation of waves for increasing values of f. The wave regime occurred for values of the *Fr ³ 1*. Furthermore, increasing the rotation frequency induced the formation of the eddies.

REFERENCES

[1] Dikson, R. and Brown, J. 1994 The production of North Atlantic deep water: sources, rates and pathways. *J. Geophys. Res.*, 99, 12319-12341.

[2] Killworth, P. 1977 Mixing in the Weddell Sea continental slope. *Deep-Sea Res.*, 24, 427-448.

[3] Whitehead, J.A. 1987 Dense water off continents. *Nature*, 327, 656.

[4] Price J. F., Baringer, M.O., Lueck, R.G., Johnson, G.C., Ambar, I., Parrilla, G., Cantos, A., Kennelly, M.A. and Sanford, T.B. 1993 Mediterranean outflow mixing dynamics. *Science*, **259**, 1277-1282.

[5] Smith, P.C. 1975 A streamtube model for bottom boundary currents in the ocean. *Deep-Sea Res.*, 22, 853-873.

[6] Price, J.F. and Baringer, M.O. 1994 Outflows and deep water production by marginal seas. *Prog. Oceanogr.*, 33, 161-200.

[7] Baringer, M.O. and Price, J.F. 1997 Momentum and energy balance of the Mediterranean outflow. *J. Phys. Oceanogr.*, **27**, 1678-1692.

[8] Lane-Serf, G.F. and Baines, P.G. 1998 Eddy formation by dense flows on slopes in a rotating fluid. *J. Fluid Mech.*, 363, 229-252.

[9] Lane-Serf, G.F. and Baines, P.G. 2000 Eddy formation by overflows in stratified water. *J. Phys. Oceanogr.*, 30, 327-337.

[10] Etling, D., Gelhardt, F., Schrader, U., Brennecke, F., Kuhn, G., Chabert d'Hieres, G. and Didelle, H. 2000 Experiments with density currents on a sloping bottom on a rotating fluid. *Dyn. Atmos. Oceans*, 31, 139-164.

[11] Whitehead, J.A., Stern, M.E., Flierl, G.R. and Klinger, B.A. 1990 Experimental Observations of Baroclinic eddies on a sloping bottom. *J. Geophys. Res.*, 95, 9585-9610.

[12] Shapiro, G.I. and Zatsepin, A.G. 1996 Gravity current down a steeply inclined slope in a rotating fluid. *Ann. Geophysicae*, 15, 366-374.

[13] Jiang, L. and Garwood, W.Jr. 1995 A numerical study of three-dimensional bottom plumes and southern ocean continental slope. *J. Geophys. Res.*, 100, 18471-18488.

[14] Jiang, L. and Garwood, W.Jr. 1996 Three-dimensional simulations of overflows on continental slopes. *J. Phys. Oceanogr.*, 26, 1224-1233.

[15] Jiang, L. and Garwood, W.Jr. 1998 Effect of topographic steering and ambient stratification on overflows on continental slope. A model study. *J. Geophys. Res.*, 103, 12375-12396.

[16] Spall, M.A. and Price, J.F. 1998 Mesoscale variability in Denmark Strait: the PV outflow hypothesis. *J. Phys. Oceanogr.*, 28, 1598-1623.

[17] Swaters, G.E. 1998 Numerical simulation of the baroclinic dynamics of density-driven coupled fronts and eddies on a sloping bottom. *J. Geophys. Res.*, 103, 2945-2962.

[18] Bruce, J.G. 1995 Eddies southwest of the Denmark strait. *Deep-Sea Res.*, 42, 13-29.

[19] Krauss, W. 1996 A note on overflow eddies. *Deep-Sea Res.*, 43, 1661-1667.

[20] Nagata, Y., Kimura, R., Honji, H., Yamazaki, Y., Kawaguchi, K. and Hosoyamada, T. 1993 Laboratory experiments of dense water descending a continental slope. *Deep ocean circulation, physical and chemical aspects.* Ed. T. Teremoto, Elsevier, 333-350.

[21] Simpson, J.E. 1997 *Gravity currents in the environment and the laboratory, second edition*, Cambridge University Press, chapt. 11, 140-163.