A Note on Interior Pathways in the Meridional Overturning Circulation

by

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Abstract

A simple oceanic model is presented for source-sink flow on the β-plane to discuss the pathways from source to sink when transport boundary layers have large enough Reynolds numbers to be inertial in their dynamics. A representation of the flow as a Fofonoff gyre, suggested by prior work on inertial boundary layers and eddy driven circulations in two-dimensional turbulent flows, indicates that even when the source and sink are aligned along the same western boundary the flow must intrude deep into the interior before exiting at the sink. The existence of interior pathways for the flow is thus an intrinsic property of an inertial circulation and is not dependent on particular geographical basin geometry.

1. Introduction

The climatically important meridional overturning circulation of the world's ocean can be conceptualized in its simplest form as sinking of dense water in the North Atlantic that proceeds to flow in the abyss to fill the deep basins of the rest of the global ocean. In its earliest theoretical representation the pathway of the dense water was portrayed as occurring in a narrow, deep western boundary layer (Stommel and Arons, 1960). Subsequent developments of the dynamics of the Meridional Overturning Circulation (MOC) essentially considered the deep western boundary current as a simple, pipe-like conduit in the global abyssal circulation joining the polar source waters to their eventual more southern and temporary reservoirs on their pathway to eventual return to the polar North Atlantic. It is easy to show that if the western boundary current were viscous and linear, the flow from source to sink would not penetrate the interior. That exercise is left to the reader.

That simple picture has come under increased scrutiny as a result of both observational and theoretical reasons. Bower *et. al.* 2009 found evidence from RAFOS float trajectories emanating from the Labrador Sea that followed pathways that were rarely limited to a deep western boundary current. Only about 8% of the floats followed the simple path southward in a western boundary current.

From a theoretical perspective a western boundary current with a high Reynolds number, i.e., that is essentially inertial rather than viscous, and so preserving potential vorticity, requires inflow from the interior to its east (Greenspan 1962). The necessity of such an interior westward flow has been interpreted (Pedlosky, 1965) as necessary to prevent Rossby Wave energy from radiating into the interior. For the wind driven circulation of the upper ocean that westward flow is produced, at least over a major part of the western boundary layer's path, by interior flow in the subtropical gyres driven by the wind stress.

In the model under discussion the putative boundary layer flow is driven by a source in the northwest corner of the basin and a sink in the southwest corner, crudely modeling the result of polar sinking of Atlantic water in the Arctic and "pulled" southward by upwelling to the surface of deep water in the Southern ocean (See, for example, Marshall and Speer, 2012). For source-sink flow it is not *a priori* clear what the mechanism would be to provide that westward containing flow unless the source-driven flow generates its own interior westward current. The suggestion has also been made that such a circulation may be eddy driven (Lozier *et. al.*)

The calculation presented in this paper utilizes a simple Fofonoff model. It has been shown (Bretherton and Haidvogel, 1976) that the end state of a highly turbulent flow, preserving energy but minimizing enstrophy, would lead naturally to that model and this paper takes up that suggestion and applies it to a Fofonoff model modified by a source and a sink both on the western boundary. It is shown that although the Fofonoff model supports western boundary currents the resulting source-driven circulation naturally generates interior pathways to provide the containment required by Greenspan's theorem. This suggests that the presence of interior pathways in such high Reynolds number circulations is an intrinsic feature of the dynamics and not related to any inability of the flow to follow the boundary because of the curvature of the boundary. Further, when the source strength is strong enough so that the solution is not of boundary layer type interior pathways fill the gyre.

Section 2 presents the model and provides the analytical solution. Section 3 presents and discusses the results as a function of the strength of the forcing source flow.

2. The model and solution

Consider a rectangular model basin of north-south extent *L*, and east-west extent Lx_e. The basin is filled with constant density fluid over a flat bottom. At the northwest corner of the basin a source of fluid enters meridionally with flux *S* through a narrow opening in the northern boundary flush against the western wall of the basin. At the southwestern boundary a similar sink of fluid of the same strength extracts the fluid from the basin. The question of interest is the pathway taken between the source and sink.

As described above, the Fofonoff model for the flow is applied. If ψ is the streamfunction for the flow such that the velocities in the *x* and *y* directions are *u,v* respectively, $u=-\dfrac{\partial\psi}{\partial y}, v=\dfrac{\partial\psi}{\partial x}$ while the streamfunction ψ satisfies

$$\nabla^2\psi+\beta y=a^2\psi\,, \tag{2.1}$$

and where β is the planetary vorticity gradient and a^2 is the Fofonoff parameter that we will relate to the source strength. On the southern and northern boundaries of the basin, i.e. at $y = 0$ and L, ψ is zero. The streamfunction also vanishes on the eastern boundary at $y = Lx_e$. However, on the western boundary at $x = 0$, ψ=*S*. Note that with $a^2>0$ the solution to (2.1) will be stable to finite amplitude perturbations (Arnol'd.1965)

We introduce the following scaling to reduce the problem to non-dimensional form. The stream function is scaled by *S*, *x* and *y* are scaled by *L*. Then the characteristic velocity $U=S/L$ is used to define *a* from the relation $a^2={}^{\beta}\!/_{U}$ so that the dimensionless interior westward flow in the limit of very large β is unity. That leads to the nondimensional form of (2.1)

$$\nabla^2\psi - K^2\psi = -K^2 y \tag{2.2}$$

where all quantities are nondimensional and where

$$K^2 = \beta L^3 / S \tag{2.3}$$

For large values of K^2 the classical Fofonoff gyre would appear in the absence of the source and sink and boundary layer solutions of (2.2) would show a uniform interior flow girdled on western, northern and eastern boundaries by thin layers of thickness K^{-1}. Such solutions are particularly apt when the source strength is very weak and so K^2 is large. To consider more general solutions it is useful to take advantage of the homogeneous boundary conditions on the southern and northern boundaries, i.e. at $y = 0$, 1 respectively and represent the solution as a sine series in y. The solution that also satisfies the condition of zero streamfunction on the eastern boundary, $x = x_e$ while yielding a unit value on the western boundary at $x = 0$ can be easily found as,

$$\psi = \sum_{n=1}^{N} \psi_n(x) \sin n\pi y,$$

$$\psi_n = -\frac{2}{n\pi}(-1)^n (K^2 / K_n{}^2)\left[1 - \frac{\sinh(K_n x)}{\sinh(K_n x_e)} + \frac{\sinh(K_n (x - x_e))}{\sinh(K_n x_e)}\right] - \frac{2}{n\pi}\left\{1 - (-1)^n\right\}\frac{\sinh K_n (x - x_e)}{\sinh(K_n x_e)} \tag{2.4 a,b}$$

where $K_n{}^2 = K^2 + n^2\pi^2$. The final term on the right hand side of (2.4) is only present when the source is considered. Note that (2.4) is valid for both positive and negative values of K^2. For the calculations shown in the next section the sum in (2.4) was terminated after 100 terms in the series.

3. Results and discussion

Figure 1 shows the result of the solution of (2.2) in the absence of a source and sink, i.e., the classic Fofonoff gyre. The value of K^2/π^2 is 400. In this case, for the Fofonoff free mode with no forcing , the value of K is arbitrary and the value chosen for the calculation of Figure1 is chosen for comparison with the same value of K for the forced flow in Figure 2. The solution obtained is, for this large value of K^2 indistinguishable from the asymptotic boundary layer solution of Fofonoff(1954). The interior flow is uniform in y and westward. Boundary layers on western, northern and eastern boundaries complete the recirculation. Streamfunction values are shown to illustrate the direction of flow.

The nature of the circulation is quite different when the solution is driven by a source –sink pair located on the northwest and southwest corners of the basin as shown in Figure 2. The interior flow is very much the same as in Figure 1. However, the flow in the western boundary current is directed southward and is fed only *indirectly* by the source. The flow issuing from the source flows, almost in its entirety, along the northern boundary and joins the interior through an eastern boundary layer. As a consequence the western boundary current starts southward as a rather weak current and builds in transport strength as the current is continuously fed from the interior by the westward flowing current, which by Greenspan's theorem, is required to hold the boundary current at the western wall. If the source were at some other location, e.g. along the northern boundary or the eastern boundary, the resulting flow would be similar to Figure 2 except that the circulation would start at the location of the source but otherwise resemble the flow in Figure 2.

If the source strength is increased as in Figure 3 so that the value of K^2/π^2 is 20, the solution loses its purely boundary layer character. The major part of the flow from the source now sweeps eastward in the interior before turning and reaching the sink in the southwest corner through pathways that are largely in the interior. There is still a β induced asymmetry in the flow. The interior eastward flow occupies a smaller meridional extent compared to the westward flow but the character of the solution is no longer boundary layer-like and Greenspan's theorem is no longer rigorously relevant. It's also clear that the development of interior pathways is not related to any curvature of the western boundary or a separation-induced phenomenon but is rather intrinsic to the source-sink flow on the β plane when the flow is inertial, i.e. when friction is not strong enough to allow a simple frictional western boundary current conduit directly from source to sink. Figure 4 shows the circulation for a somewhat smaller $K^2/\pi^2 = 10$.

If we imagine the circulation in Figures 3 and 4 as rough models of the abyssal flow in the Atlantic west of the Mid Atlantic Ridge and consider a characteristic velocity for the interior flow of the order of a few cm/sec the pattern of Figures 3 and 4 are probably more appropriate than the boundary layer form of Figure 2. In Figure 3, with its value of $K^2/\pi^2 = 20$, a characteristic interior velocity is of the order of $U = \beta L^2 / K^2$ which for L = 1000 km and $\beta = 10^{-13} cm^{-1} \sec^{-1}$ yields a value of U of about 5 cm/sec which seems reasonable for the interior abyssal flow. We note that the tendency towards interior pathways will increase the time to traverse the gyre from source to sink with obvious implications for the overturning circulation of which it is a part. It also emphasizes that the MOC is a three dimensional dynamical structure not limited to a meridional vertical plane.

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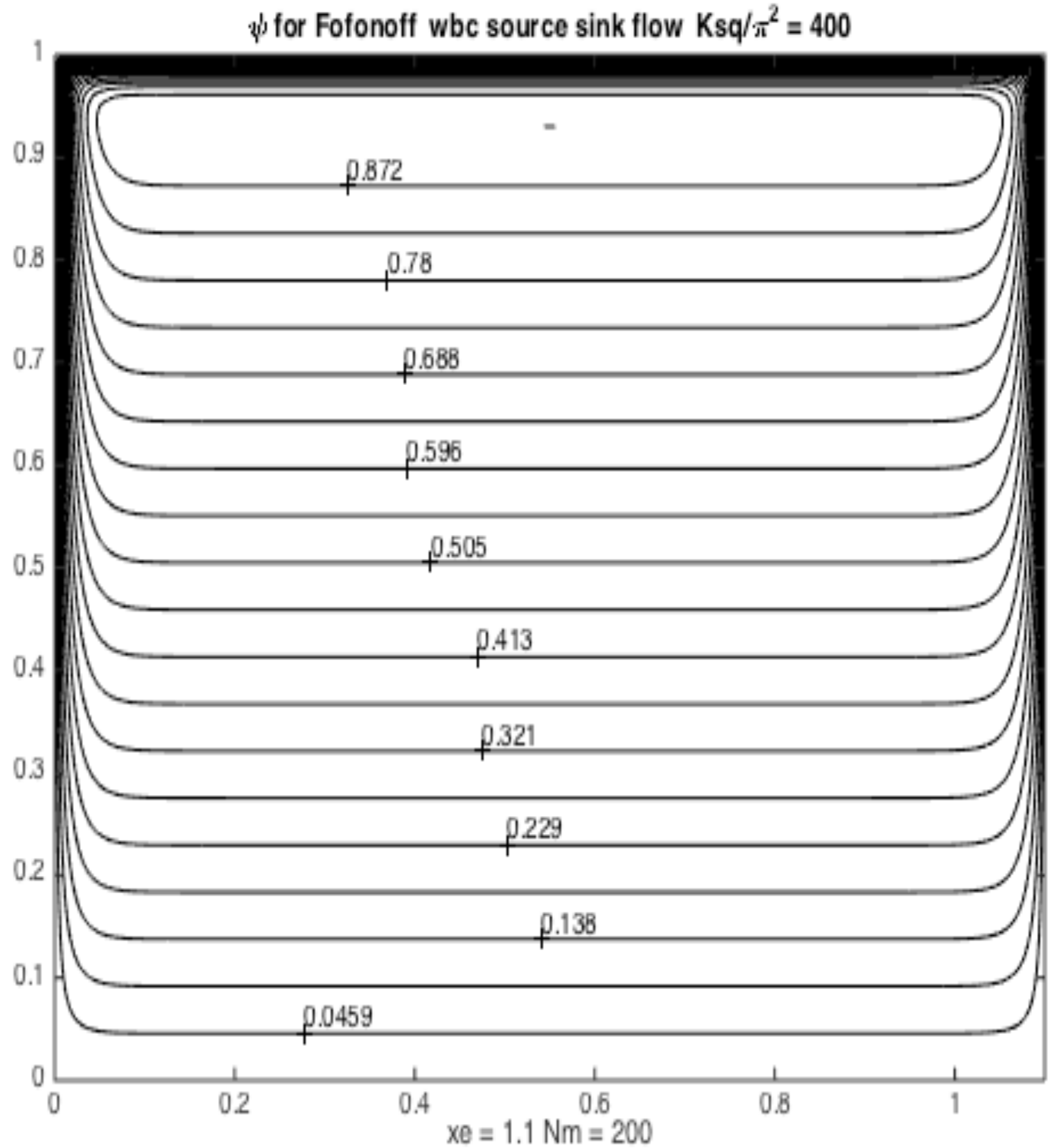
Figures

Figure 1. The classical Fofonoff anticyclonic gyre with no source or sink, i.e. $S = 0$. $K^2 = 400\pi^2$.

Figure 2. The flow as in Figure 1 but now with a source in the northwest corner and a sink in the southwest corner.

Figure 3. The source-sink flow as in Figure 2 but for a smaller value of $K^2 = 4\pi^2$

Figure 4. As in Figure 3 but for $K^2 / \pi^2 = 10$.


ψ for Fofonoff wbc source sink flow Ksq/π² = 400
0.872
0.78
0.688
0.596
0.505
0.413
0.321
0.229
0.138
0.0459
xe = 1.1 Nm = 200

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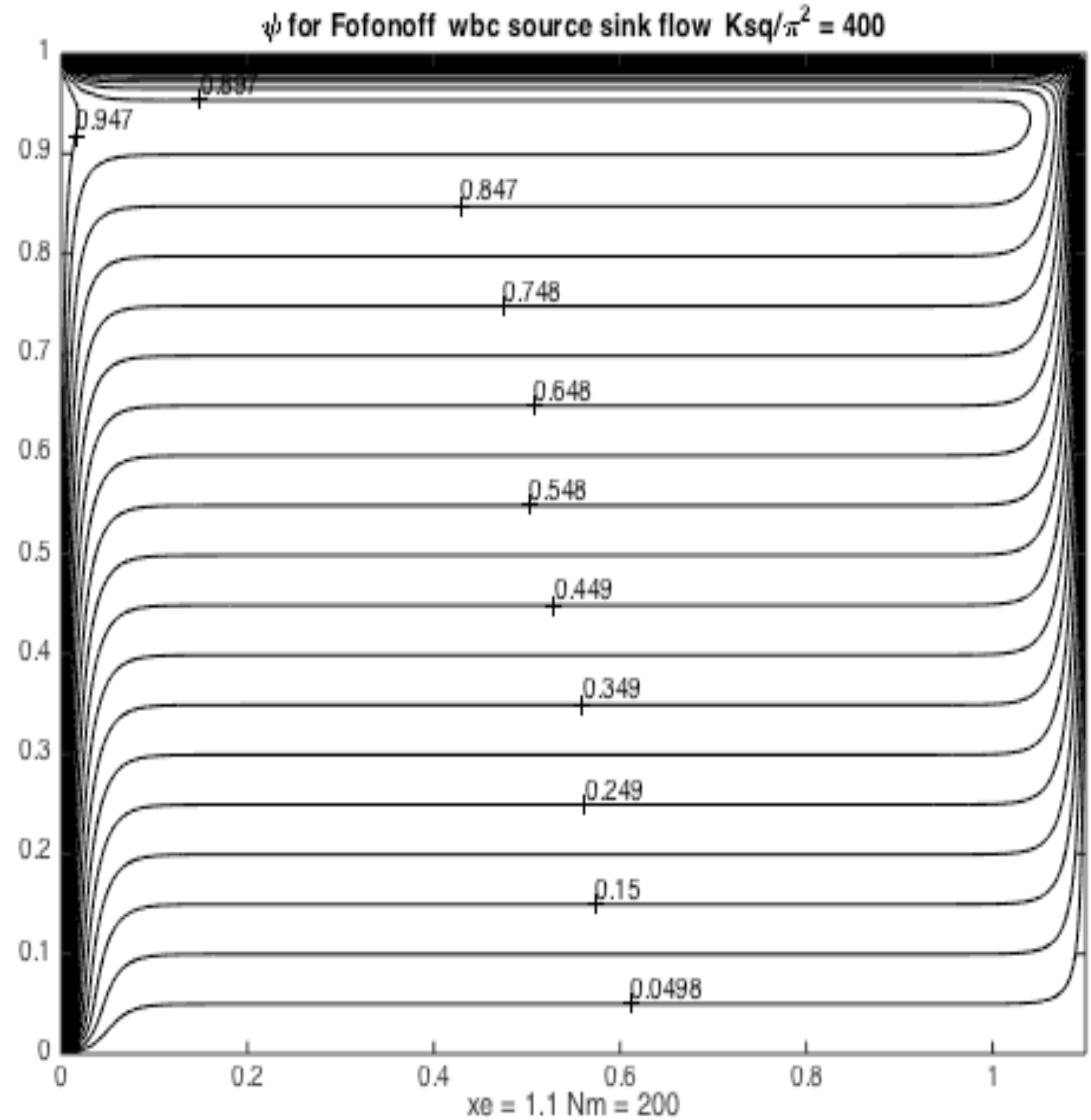

Figure 2. The flow as in Figure 1 but with a source in the northwest corner and a sink in the southwest corner.

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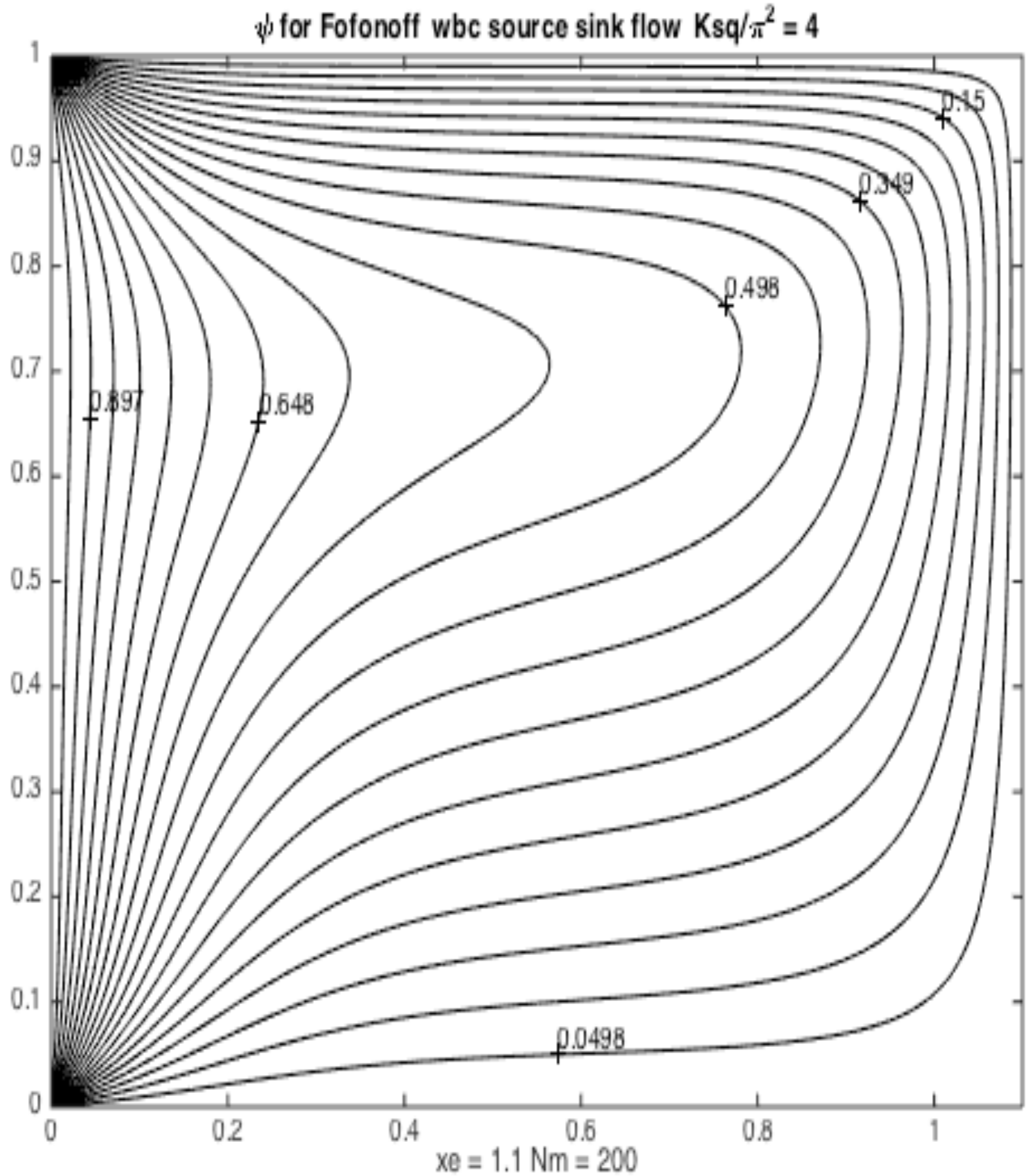

Figure 3. The source-sink flow as in Figure 2 but for a smaller value of $K^2 = 4\ \pi^2$

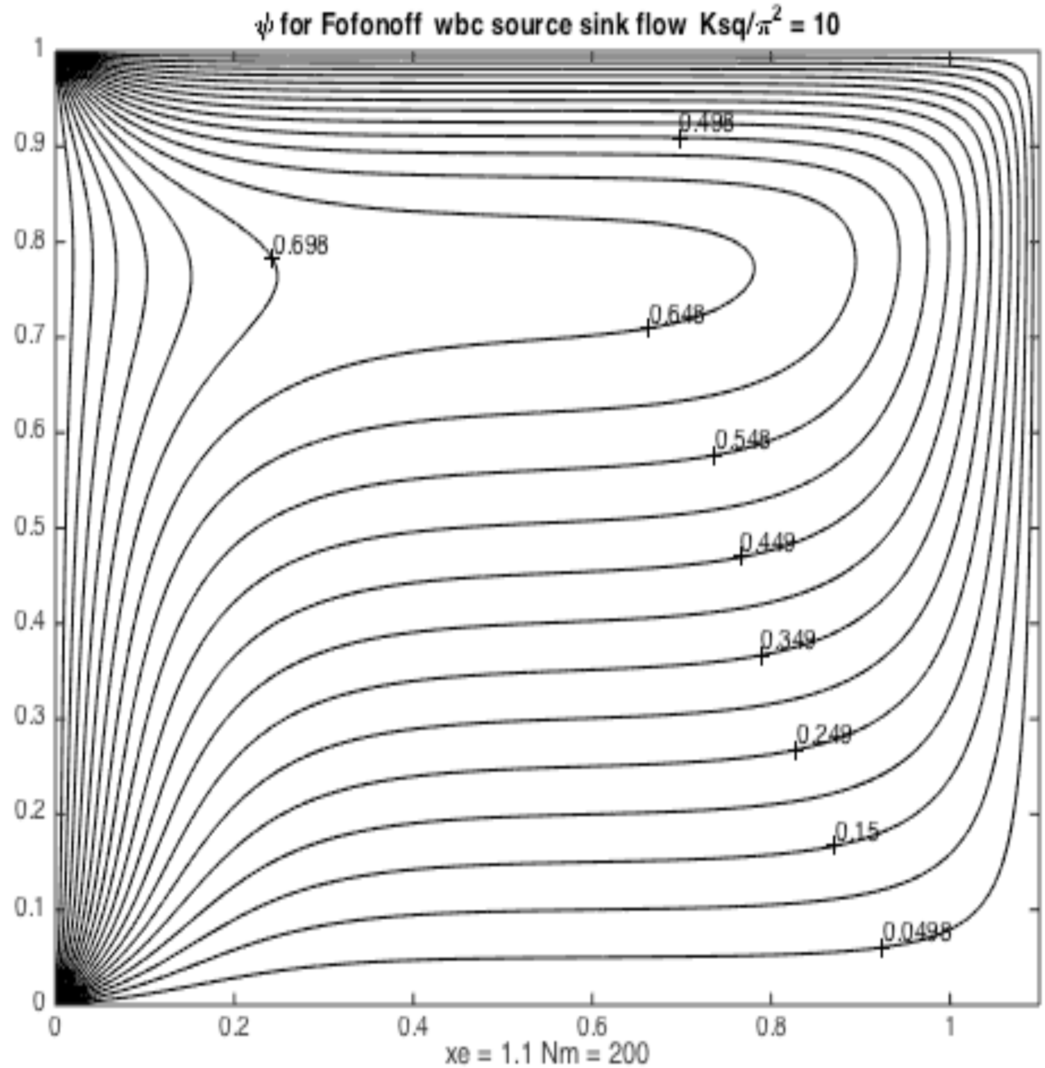

Figure 4. As in Figure 3 but for $K^2 / \pi^2 = 10$.

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