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A transient electric dipole-dipole method for mapping the conductivity of the sea floor

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ABSTRACT

The electrical conductivity of the sea floor is usually much less than that of sea water, and not all electrical measurements made on the sea floor are particularly sensitive to the electrical conductivity value. The analytic impulse and step-on transient responses of two conductive, adjoining half-spaces (with a large conductivity contrast) to an in-line electric dipole-dipole electromagnetic system located on the interface are derived. The shape of the transient at relatively early time is seen to be independent of the conductivity of the more conductive half-space and is indicative of the conductivity of the less conductive half-space. Based on this observation, a simple, practical system can be designed to measure sea floor conductivity.

INTRODUCTION

The use of electrical methods to map the regional geology of the sea floor is still in its infancy, even though targets which could be located by their enhanced electrical conductivity relative to that of their environment have been identified. For example, recent improvements in deep-ocean, near-bottom surveying techniques have led to the chance discovery of a number of polymetallic sulfide mineral deposits in the vicinity of hydrothermal vents on mid-ocean ridges (CYAMEX, 1979; Hekinian et al., 1983; Rona, 1983). The thicknesses, and hence the volumes, of the individual deposits are unknown, although one estimate of size has ranged up to 10^8 m^3 which would be worth in excess of a billion dollars. Recovery of the deposits may never become economic, but a detailed study of their physical properties might increase our understanding of the origin of certain deposits found on land.

Conceptually, one of the simplest mapping tools is the electric dipole-dipole method. Dipole-dipole arrays are used on land both at the static frequency limit and at frequencies or delay times where electromagnetic (EM) induction becomes important. Francis (1984) demonstrated the feasibility of mapping sea floor sulfide deposits using a towed resistivity array. The method works well for sulfide and similar targets which outcrop and have an electrical conductivity of the order of or greater than that of sea water. However, typical sea floor has a conductivity of σ_1 much less than the conductivity σ_0 of sea water, and accurate measurement of its conductivity at the static frequency limit by the method is inhibited. The reason is simple: the electric potential for this case is determined principally by the value of σ_0. A fractional change f in σ_1 causes a change in potential only of the order of $f\sigma_1/\sigma_0$. Under ideal circumstances, changes in σ_1 might be obtained from point to point by making sufficiently accurate voltage measurements, but the topography of the sea floor can be very rugged, particularly in the vicinity of mid-ocean ridges, and topographic noise would probably mask many real conductivity anomalies.

Our solution is to measure the transient dipole-dipole response and determine the sea floor conductivity directly from the shape of the transient. While topographic effects alter the amplitude of that part of the transient due to induction in the sea floor, its shape should be influenced to a lesser extent, because transient shape is a function of the electrical conductivity of a volume of material. In addition, precise location of transmitting and receiving dipoles is less critical for a transient measurement than for a frequency-domain measurement, an obvious advantage when working on the sea floor.

Extending the results of Weir (1980), we derive analytical expressions for the transient dipole-dipole response of a conductive half-space representing the sea over a relatively resistive half-space representing the sea floor. We demonstrate that there are two distinct parts to the transient, a part due to the resistive zone which is seen at relatively early time and which characterizes the conductivity of the zone, and a part at late time associated with a disturbance diffusing through the sea water. Our theory illustrates the fundamental physics of sea floor electrical mapping, and provides functions to check software written to solve more complicated problems.

THEORY

The sketch in Figure 1 is of a coplanar horizontal electric dipole transmitter TX and an electric dipole receiver RX lo-

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cated in an upper half-space of conductivity σ_0 at vertical distances z_t and z_r, repsectively, from the horizontal boundary with a lower half-space of conductivity σ_1. The horizontal separation of the transmitter and the receiver is ρ. Chave and Cox (1982) calculated the electric field at the receiver for a source having a time dependence exp $(i\omega t)$ subject to the approximation that the magnetic effects of displacement currents could be neglected. Assuming zero initial conditions, the Laplace transform $E(s)$ of the electric field may be derived from their work by replacing the product $i\omega$ by the complex Laplace variable s. It is

$$E(s) = \frac{j(s)}{2\pi}\left[F_{TM}(s) + F_{PM}(s)\right], \tag{1}$$

where $j(s)$ is the Laplace transform of the source current dipole moment and the functions F are Hankel transforms defined as

$$F_{TM}(s) = \frac{1}{2\sigma_0}\int_0^\infty \{R_L^{TM} \exp\left[-\beta_0(z_r + z_t)\right] - \exp\left[-\beta_0|z_r - z_t|\right]\}\beta_0 kJ_1'(k\rho)\, dk, \tag{2}$$

where the prime indicates differentiation with respect to the argument k_f, and

$$F_{PM}(s) = -\frac{\gamma_0^2}{2\sigma_0\rho}\int_0^\infty \{R_L^{PM} \exp\left[-\beta_0(z_r + z_t)\right] + \exp\left[-\beta_0|z_r - z_t|\right]\}\left[1/\beta_0\right]J_1(k\rho)\, dk. \tag{3}$$

The functions in expressions (2) and (3) are the independent toroidal *TM* and poloidal *PM* parts of the solution, characterized by the absence of the vertical components of the magnetic and electric field, respectively (Chave, 1984). The corresponding modal reflection coefficients for our double half-space model are

$$R_L^{TM} = (\beta_0\sigma_1 - \beta_1\sigma_0)/(\beta_0\sigma_1 + \beta_1\sigma_0), \tag{4}$$

and

$$R_L^{PM} = (\beta_0 - \beta_1)/(\beta_0 + \beta_1), \tag{5}$$

where in the ith medium $\beta_i^2 = k^2 + \gamma_i^2$ and $\gamma_i^2 = \mu_0\sigma_i s$.

If both transmitter and receiver lie on the interface between the two half-spaces ($z_t = z_r = 0$), then expressions (2) and (3) take on simpler forms which, by symmetry, are invariant under an exchange of the half-space indices. The expressions become

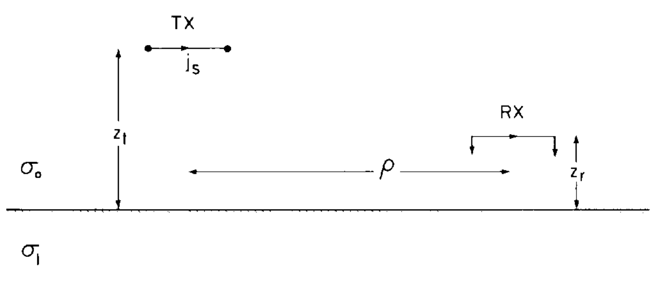

FIG. 1. The sea floor transient electric dipole-dipole method: Definition of symbols.

$$F_{TM}(s) = -\int_0^\infty \frac{\beta_0\beta_1}{\beta_0\sigma_1 + \beta_1\sigma_0} kJ_1'(k\rho)\, dk, \tag{6}$$

and

$$F_{PM}(s) = -\frac{1}{\rho(\sigma_0 - \sigma_1)}\int_0^\infty (\beta_0 - \beta_1)J_1(k\rho)\, dk. \tag{7}$$

If the conductivity σ_0 of the upper half-space is significantly greater than the conductivity σ_1 of the lower half-space, such that terms of order σ_1/σ_0 and higher may be neglected in comparison with terms of order unity, then the expressions further simplify to

$$F_{TM}(s) = -\frac{1}{\sigma_0}\int_0^\infty \left[\beta_0 - \frac{\gamma_1^2}{\beta_1}\right] kJ_1'(k\rho)\, dk, \tag{8}$$

and

$$F_{PM}(s) = -\frac{1}{\sigma_0\rho}\int_0^\infty \left[\beta_0 - \beta_1\right]J_1(k\rho)\, dk. \tag{9}$$

The Hankel transforms may now be evaluated using standard integrals, in particular, Sommerfeld's integral which is

$$\int_0^\infty [k/\beta_i] \exp(-\beta_i z)J_0(k\rho)\, dk = \exp(-\gamma_i R)/R, \tag{10}$$

where $R^2 = \rho^2 + z^2$, and the relatives of this integral listed in Appendix B of Chave and Cox (1982), all considered for the special case $z = 0$. The final expression for the electric field becomes

$$E(s) = \frac{j(s)}{2\pi\sigma_0\rho^3}\left[(\sqrt{\tau_0 s} + 1)\exp(-\sqrt{\tau_0 s}) + (\tau_1 s + \sqrt{\tau_1 s} + 1)\exp(-\sqrt{\tau_1 s})\right], \tag{11}$$

where the electromagnetic diffusion time constant for the ith medium is

$$\tau_i = \rho^2\mu_0\sigma_i. \tag{12}$$

The time constant increases with the square of the transmitter-receiver separation and with the given conductivity, as expected for a purely diffusive process.

Expression (11) may be inverted to the time domain, if the Laplace transform $j(s)$ of the source current dipole moment is specified. A current I switched on at time $t = 0$ for a time Δt in a transmitter of length $\Delta\ell$ has the source transform

$$j^I(s) = I\Delta\ell\Delta t, \tag{13}$$

and the corresponding impulse response may be obtained from standard tables of inverse Laplace transforms as

$$E^I(t) = \frac{I\Delta\ell\Delta t}{4\sigma_0\rho^3 t}\left[\frac{1}{2}\sqrt{\left(\frac{\tau_0}{\pi t}\right)^3}\exp(-\tau_0/4t) + \sqrt{\left(\frac{\tau_1}{\pi t}\right)^3}\left(\frac{\tau_1}{4t} - 1\right)\exp(-\tau_1/4t)\right]. \tag{14}$$

A current I switched on at time $t = 0$ and held constant has the transform

$$j^S(s) = I\Delta\ell/s, \tag{15}$$

and the corresponding step response is

$$E^S(t) = \frac{I\Delta\ell}{2\pi\sigma_0\rho^3}\left[\sqrt{\frac{\tau_0}{\pi t}}\exp(-\tau_0/4t) + \text{erfc}(\sqrt{\tau_0/4t})\right.$$

$$+ \sqrt{\frac{\tau_1}{\pi t}} \left(\frac{\tau_1}{2t} + 1 \right) \exp(-\tau_1/4t) + \operatorname{erfc}(\sqrt{\tau_1/4t}) \Bigg]. \quad (16)$$

The function erfc is the complementary error function.

It is convenient to convert the step and impulse responses to dimensionless forms. Dimensionless time x is defined as the ratio of the true time t to the lower half-space time constant τ_1. The parameter α is the conductivity ratio σ_0/σ_1. A unit of electric field is the late-time step response $E^S(\infty)$ or, equivalently, the electric field of a static dipole given by

$$E^S(\infty) = I\Delta\ell/\pi\sigma_0\rho^3. \quad (17)$$

The dimensionless step and impulse responses are

$$\frac{E^I(x)}{E^S(\infty)} = \frac{\Delta x}{4\sqrt{\pi}x} \left[\frac{1}{2} \sqrt{\left(\frac{\alpha}{x}\right)^3} \exp(-\alpha/4x) + \sqrt{\left(\frac{1}{x}\right)^3} \left(\frac{1}{4x} - 1 \right) \exp(-1/4x) \right] \quad (18)$$

and

$$\frac{E^S(x)}{E^S(\infty)} = \frac{1}{2} \left[\sqrt{\frac{\alpha}{\pi x}} \exp(-\alpha/4x) + \operatorname{erfc}(\sqrt{\alpha/4x}) + \sqrt{\frac{1}{\pi x}} \left(\frac{1}{2x} + 1 \right) \exp(-1/4x) + \operatorname{erfc}(\sqrt{1/4x}) \right] \quad (19)$$

respectively, where Δx is the dimensionless time interval $\Delta t/\tau_1$.

DISCUSSION

Graphs of the dimensionless impulse response, expression (18) with Δx set to unity, and the dimensionless step response, expression (19), are displayed in Figures 2 and 3, respectively, for values of the conductivity ratio σ_0/σ_1 of 10, 30, 300, and 1 000. The large initial peak in the impulse response is caused by an EM disturbance diffusing through the lower half-space and it always arrives at the same dimensionless time for all conductivity contrasts. In terms of true time t, this signal occurs later and becomes broader or more dispersed as the half-space conductivity or the transmitter-receiver separation increases. The second peak associated with the upper half-space always occurs at a later time and is small compared to the initial peak.

The step responses displayed in Figure 3 show an initial increase to about three-quarters of the static dipole value caused by propagation through the lower half-space, which always occurs at the same dimensionless time. The signal traveling in the upper half-space arrives later. The late time limit is the static dipole field.

Clearly, the positions in true time of the initial peak in the impulse response and the initial rise in the step response are direct measures of the lower half-space conductivity provided the transmitter-receiver separation is known.

For many practical values of the contrast in conductivity between the sea water and the sea floor, the separation in time between the two parts of the transient response, and hence the resolution of sea floor conductivity, is substantial. The electrical conductivity of sea water is about 3.2 S/m below the warm, saline surface layer that lies above the main thermocline. Typical conductivities of water-saturated sediments are about ten times smaller, while basement rocks have even lower values.

The size of any practical sea floor transient EM system must be dictated by practicality and the amplitude and wavelength of typical sea floor topography, with smaller separa-

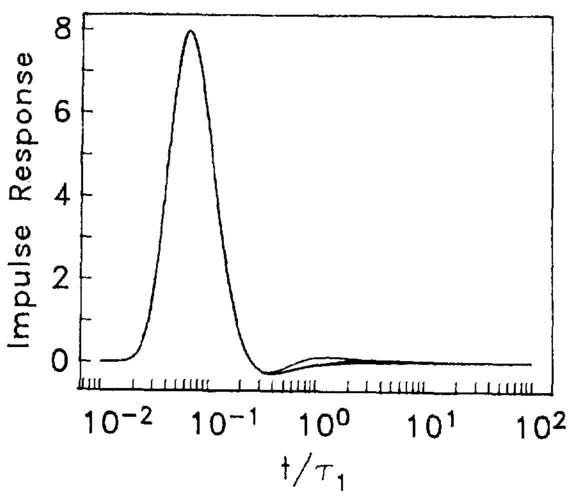

FIG. 2. The normalized impulse response computed from expression (18), with Δx set to unity, for a range of values of the conductivity contrast. Due to the dimensionless representation, the impulse propagating in the lower medium always arrives at the same dimensionless time and has the same width and amplitude for all conductivity contrasts. The much smaller amplitude impulse traveling in the upper medium arrives at later dimensionless times as the conductivity contrast increases.

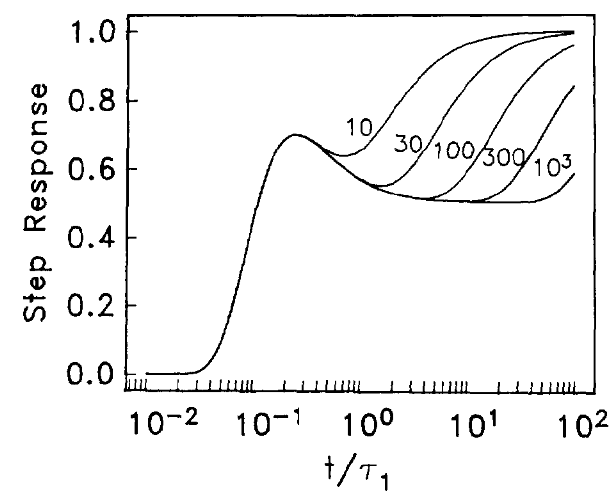

FIG. 3. The normalized step-on response computed from expression (19) for a range of values of the conductivity contrast. The initial step to about three-quarters of the static limit occurs at the same dimensionless time and is due to propagation in the lower medium. The secondary step at progressively later time for increasing conductivity contrast is due to propagation in the upper medium.

tions being required over particularly rugged terrain. A submersible based system will be limited to scales of about 10 m for safety reasons, but it should be possible to tow a longer (> 100 m) array from a surface ship. The time constants, defined in expression (12), associated with the 10 to 100 m scale are from 40 μS to 4 mS for the sea floor and from 400 μS to 40 mS for sea water. A practical system can be assembled easily either by modifying a conventional land technique that measures the turn-off step response, or by utilizing the impulse response. The latter can be accomplished through the pseudo-random binary sequence methodology pioneered by Duncan et al. (1980).

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