Bragg reflection of ocean waves from sandbars

Steve Elgar and B. Raubenheimer
Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA

T. H. C. Herbers
Naval Postgraduate School, Monterey, California, USA

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[1] Resonant Bragg reflection of ocean surface waves by a field of natural shore-parallel sandbars was observed in Cape Cod Bay, MA. Waves transmitted through the bars were reflected strongly from the steep shoreline, and the observed cross-shore variations in the onshore- and offshore-directed energy fluxes are consistent with theory for resonant Bragg reflection, including a 20% decay of the incident wave energy flux that is an order of magnitude greater than expected for wave-orbital velocity induced bottom friction. Bragg reflection was observed for a range of incident wave conditions, including storms when sediment transported toward and away from nodes and antinodes caused by the reflecting waves might result in growth and maintenance of the sandbars. *INDEX TERMS:* 4546 Oceanography: Physical: Nearshore processes; 4560 Oceanography: Physical: Surface waves and tides (1255). **Citation:** Elgar, S., B. Raubenheimer, and T. H. C. Herbers, Bragg reflection of ocean waves from sandbars, *Geophys. Res. Lett.*, *30*(1), 1016, doi:10.1029/2002GL016351, 2003.

1. Introduction

[2] It has been hypothesized that ocean surface waves can be reflected by a field of sandbars, potentially protecting the shoreline from wave attack [*Heathershaw*, 1982, *Davies*, 1982, *Mei*, 1985, *Liu*, 1987, *Bailard et al.*, 1992], and producing partially standing waves that may lead to the growth and maintenance of the sandbars [*Heathershaw*, 1982, *Yu and Mei*, 2000b]. Although the Bragg resonance mechanism for wave reflection has been demonstrated convincingly in the laboratory [*Heathershaw*, 1982, *Davies and Heathershaw*, 1984, *Hara and Mei*, 1987], the corresponding impact of natural sandbars on ocean waves is not known. Multiple shore-parallel sandbars occur along many coasts, including Chesapeake Bay [*Dolan and Dean*, 1985], Lake Michigan [*Evans*, 1940], Georgian Bay [*Boczar-Karakiewicz and Davidson-Arnott*, 1987], the Gulf of Mexico [*Komar*, 1998], and Cape Cod Bay [*Moore et al.*, 2002] (Figure 1). Often, these undulations of the seafloor are nearly uniformly spaced, theoretically causing resonant Bragg reflection of onshore propagating surface waves with wavelengths twice the distance between the bar crests [*Davies*, 1982, *Heathershaw*, 1982, *Mei*, 1985, *Liu*, 1987]. If the beach face is relatively steep, waves transmitted through the bars may undergo partial reflection at the shoreline [*Miche*, 1951, *Elgar et al.*, 1994], followed by re-reflections from the bars, complicating the wave transformation [*Yu and Mei*, 2000a]. Here, we present the first field observations of resonant Bragg reflection of ocean surface waves by shore-parallel sandbars.

2. Field Observations

[3] To test the hypothesis that a field of natural shore-parallel sandbars cause resonant Bragg reflection, colocated bottom-mounted pressure gages and acoustic Doppler current meters were deployed at 5 cross-shore locations in Cape Cod Bay, near Truro, MA, USA (Figure 1). Repeated surveys of the seafloor indicate that the bars did not evolve significantly during the observational period (11 Aug–29 Oct 2001). The 0.5 m high, nearly sinusoidal sandbars extended from approximately 375 to 150 m offshore of the shoreline, and were separated by about 50 m from each other (Figure 1). The seafloor sloped gently (slope $\approx$ 0.0025) across the instrumented transect to a steep beachface (slope $\approx$ 0.10) at the shoreline. The mean water depth along the transect ranged from 1 to 3 m with tidal fluctuations of about ±1 m. Cross- and alongshore tidal flows were less than 0.1 and 0.2 m/s, respectively. Wave heights at the most offshore sensor ranged from 0.02 to 1.5 m. Low energy waves observed when winds were weak consisted primarily of north Atlantic swell (typical frequency $f \approx$ 0.07 Hz) that enters Cape Cod Bay 12 km northwest of the instrumented transect and refracts toward the experiment site, and lower-frequency ($f \approx$ 0.01 Hz) gravity waves known as infragravity waves (Figure 2a). In these benign conditions the dominant source of infragravity waves is believed to be radiation of surf beat from energetic coastlines around the Atlantic ocean basin [*Webb et al.*, 1991, *Herbers et al.*, 1995]. Afternoon sea breezes (wind speeds up to 8 m/s) that blew from the southwest across the 80 km wide Cape Cod Bay generated larger, high frequency ($\approx$0.25 Hz) wind waves (Figure 2a), accompanied by locally forced infragravity waves [*Longuet-Higgins and Stewart*, 1962, *Herbers et al.*, 1994]. Winds from storms usually were from the north or north-east, and did not generate large waves because the fetch was small. Except during a few of the largest wave events, visual observations and estimates of the net cross-shore energy flux from the sensor array indicate there was little wave breaking along the transect, and the surfzone (if any) was confined to a few m wide strip at the shoreline.

[4] Reflection from the shoreline was strong at low frequencies, and decreased with increasing wave energy (Figure 2b), consistent with previous observations [*Elgar et al.*, 1994] and theory [*Miche*, 1951]. Fluctuations in spectral levels at low frequencies (eg, $0.01 \le f \le 0.05$ Hz) observed for small waves (solid curve Figure 2a) are the result of partial standing waves forming cross-shore nodes and antinodes. The locations of nodes and antinodes depend on the water

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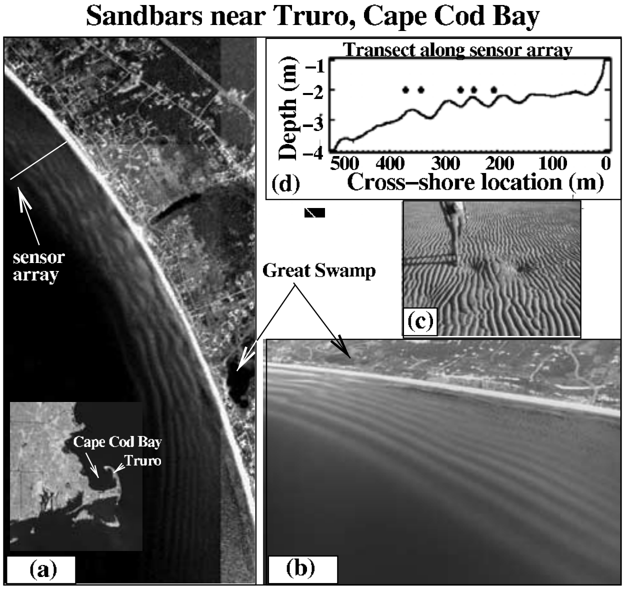

Figure 1. [(a) and (b)] Aerial photographs of the shoreline of Cape Cod Bay near Truro, MA. The field site is located about 12 km southeast of the northern tip of Cape Cod [inset in (a)]. (c) Photograph of wave-orbital ripples on the crest of a sandbar at spring low tide. Scale is given by the person standing near the center. (d) Cross-shore depth profile [along the white line in (a)] showing seafloor elevation relative to mean sea level (curve) and the locations of 5 colocated pressure gage - current meter pairs (symbols). The Great Swamp (or Great Hollow) appears in both aerial photographs, which have horizontal coverage of approximately (a) 3.8 × 8 and (b) 1.8 × 2.6 km. Aerial photographs courtesy of L. Moore (a) and D. Aubrey (b).

depth, and thus the standing wave patterns are blurred in spectra averaged over all tidal stages (eg, Figure 2).

3. Results

[5] Resonant Bragg reflection requires waves with wavelength equal to twice the distance between the crests of the sandbars. If waves propagate at an angle to the sandbars, the resonant wavelength is such that the wavenumber component perpendicular to the bars satisfies the Bragg condition [*Mei*, 1985, *Dalrymple and Kirby*, 1986, *Kirby*, 1993]. For the approximately 50 m bar spacing observed here, resonant waves have frequencies between 0.032 and 0.056 Hz depending on water depth. To exclude wave fields dominated by shore-trapped edge waves [*Ursell*, 1952], analysis is restricted to waves with narrow (less than 20°) directional spread about a mean direction that is within 30° of normal incidence to the sandbar crests.

[6] During low energy conditions, waves at the Bragg resonance frequency reflected from both the bars and the shoreline, as shown in Figure 3a for a 1-hr record with offshore significant wave height H_s = 0.1 m. In this case, the reflection coefficient R^2, defined as the ratio of offshore- to onshore-directed energy flux, was 0.4 onshore of the bars. Similar shoreline reflection was observed in the same water depth about 600 m north in an area with no sandbars. As incident waves partially reflected from the sandbars, the onshore-directed energy flux decreased by about 30% from the seaward to the shoreward side of the bars. This energy loss approximately is balanced by the observed seaward increase of the offshore-directed energy flux (Figure 3a). The corresponding reflection coefficient increases to a maximum value of R^2 = 0.7 at the farthest offshore sensor, suggesting nearly equal contributions from bar and shoreline reflections.

[7] It has been hypothesized [*Yu and Mei*, 2000a] that the cross-shore structure of the wave field depends on the strength of the shoreline reflection R_L^2 and on the phase Θ of the reflected wave relative to the sinusoidally undulating seafloor (ie, on the distance between the shore and the bar field expressed as a phase difference). For shallow water waves at the resonant Bragg frequency propagating over a uniform field of sandbars on a horizontal seafloor, the theoretical onshore- F^+ and offshore- F^- directed energy fluxes, normalized by the onshore flux at the most offshore sensor are given by [*Yu and Mei*, 2000a]

$$F^+ = \frac{1 - R_L \sin(\Theta)\sinh(2X) + \left(1 + R_L^2\right)\sinh^2(X)}{\Delta} \quad (1)$$

$$F^- = \frac{R_L^2 - R_L \sin(\Theta)\sinh(2X) + \left(1 + R_L^2\right)\sinh^2(X)}{\Delta} \quad (2)$$

where $\Delta = 1 - R_L \sin(\Theta)\sinh(2\alpha) + (1 + R_L^2)\sinh^2(\alpha)$. The spatial variable $X = \alpha(1 - x/L)$, where x is distance

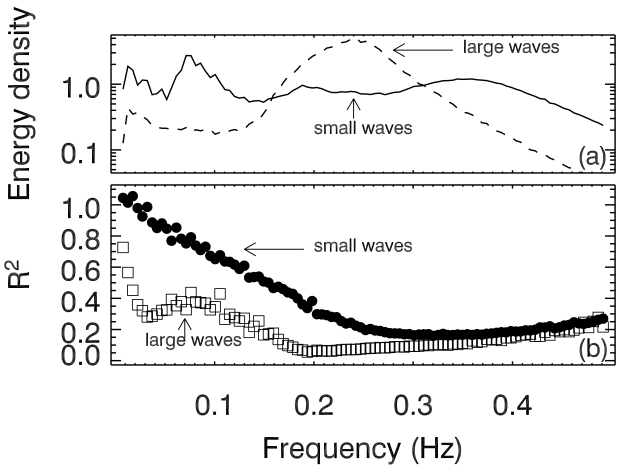

Figure 2. Average (a) energy density and (b) reflection coefficient R^2 versus frequency for small (significant wave height $H_s \leq 0.10$ m, solid curve and filled circles) and large ($H_s \geq 0.25$ m, dashed curve and open squares) waves. Significant wave height is defined as 4 times the standard deviation of sea surface elevation fluctuations in the frequency band from 0.001 to 0.40 Hz. Spectral levels for each 1-hr long data set were normalized by the total variance before averaging. There are 1054 and 96 1-hr records averaged for small and large wave cases, respectively. The observations were made at the most onshore sensor (Figure 1d). Reflection coefficients estimated from observations made in the same water depth in an area with no sandbars 600 m to the north are similar to those shown here, implying the values in panel b are owing to shoreline reflection.

(increasing toward shore) along the bar field of length L, and $\alpha \approx \frac{\pi}{4} m \frac{b}{h}$, with b the amplitude of each of the m sandbars in water depth h. Here, the seafloor (Figure 3d) was approximated as having zero slope, the water depth was estimated as the average depth across the barred region, and the bar amplitude was assumed to be 0.25 m. The reflection coefficient $R^2 = F^-/F^+$ reduces to the theoretical value [*Heathershaw*, 1982, *Davies*, 1982] of bar reflection without a shoreline in the limit of $R_L^2 = 0$. Predictions of the evolution of F^+ and F^- across the bars using the most onshore sensor to specify the shoreline reflection ($R_L^2 = 0.4$ for this case study) are consistent with the observations (Figure 3a). Even though waves with frequencies relatively far from the resonant Bragg frequency reflect from the shoreline (eg, Figure 2), the cross-shore evolution of energy fluxes for waves with wavelengths much shorter or longer than twice the bar crest separation do not have trends similar to those in Figure 3, suggesting the results are not an artifact of the processing or reflection estimation algorithms.

[8] The decay of nonbreaking waves across the bars is stronger at low tide than at high tide (Figure 3b). The average decrease of F^+ for 19 low-tide data records is 21%, compared with a 14% decay for 18 high-tide data records, consistent with theory, which predicts reflection by the bars is strongest at low tide when b/h is maximum. The large difference in the relative amount of offshore-directed energy flux F^- for different tide ranges is owing to stronger shoreline reflection at high tide ($R_L^2 \approx 0.7$) than at low tide ($R_L^2 \approx 0.5$).

[9] In shallow water the theoretical [*Heathershaw*, 1982, *Davies*, 1982] decay in onshore-directed energy flux over small-amplitude bars, neglecting shoreline reflection, is proportional to $(b/h)^2$. Here, a weaker, roughly linear dependence on b/h (for fixed b) is observed that is consistent with theory [*Yu and Mei*, 2000a] that incorporates shoreline reflection (Figure 4).

[10] A reduction in energy flux also may result from bottom friction induced by wave-orbital velocities, especially if sand ripples are present or being formed [*Grant and Madsen*, 1986]. Wave-orbital ripples were observed on the sandbars by SCUBA divers and when the bar crests were exposed during spring low tide (Figure 1c), but crude estimates of energy loss based on a commonly used quadratic friction law [*Grant and Madsen*, 1986] with established friction factors [*Tolman*, 1994, *Ardhuin et al.*, 2001] are 1–2 orders of magnitude smaller than the observed decrease in F^+. Furthermore, the observed seaward increase in F^- cannot be explained by this mechanism. The average decrease in net energy flux ($F^+ - F^-$) across the sandbars is about 7%, a portion of which may be the result of bottom friction associated with wave-orbital ripples.

[11] When offshore wave heights were small (less than 0.25 m, Figures 3a and 3b) there was no wave breaking across the sandbars, and the near-bottom velocities associated with the Bragg reflecting waves likely were too small to cause significant sediment transport. However, sediment mobilized during storms may be transported by near-bottom velocities toward and away from convergences and divergences associated with nodes and antinodes caused by the reflecting Bragg waves, possibly resulting in growth of the sandbars [*Heathershaw*, 1982, *Yu and Mei*, 2000b]. Detection of Bragg resonance in high energy wave conditions is complex because both reflection and wave-breaking contribute to the decay of the incident wave energy flux F^+ across the bars. During a storm with 0.8 m high waves

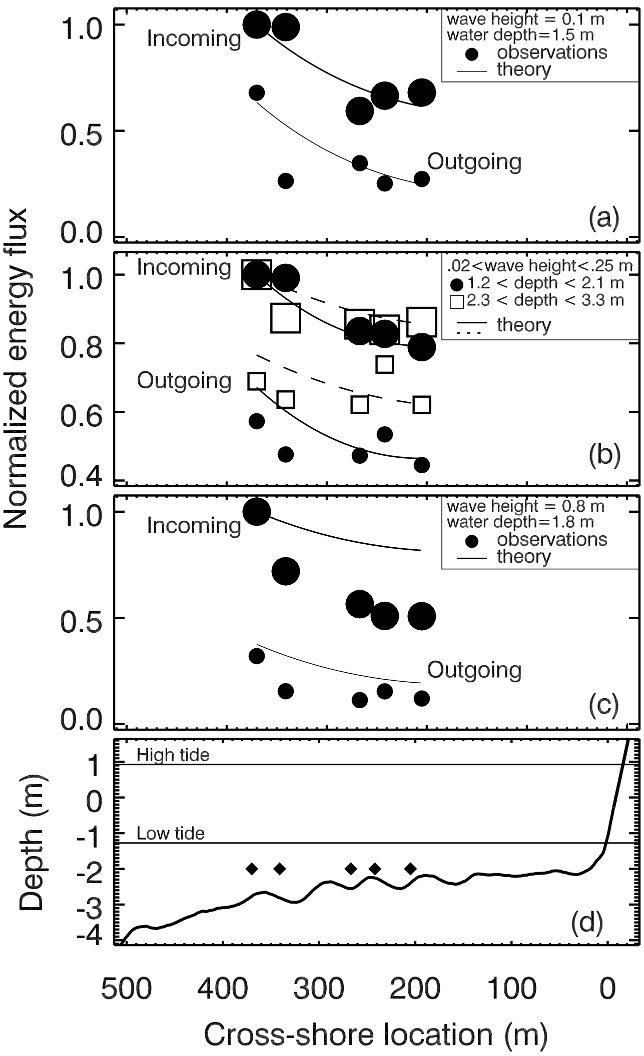

Figure 3. (opposite) Observed (symbols) and predicted (equations 1 and 2) (curves) onshore- and offshore-directed energy flux (normalized by the onshore-directed flux observed at the offshore sensor) versus cross-shore location. (a) A 1-hr period for which the offshore significant wave height H_s was 0.1 m and the average water depth h over the instrument array was 1.5 m. At the resonant Bragg frequency of $f = 0.037$ Hz (wavelength $\approx$ 100 m) a narrow beam of waves arrived at a small incidence angle $\bar{\theta} = 7°$ with directional spread $\sigma_\theta < 10°$. (b) Averages over 19 1-hr runs at low tide ($1.2 \leq h \leq 2.1$ m, filled circles and solid curves), and over 18 1-hr runs at high tide ($2.3 < h \leq 3.3$ m, open squares and dashed curves). Wave heights were $0.02 < H_s < 0.25$ m, and the Bragg frequency ranged from 0.032 to 0.057 Hz. Only data satisfying the directional criteria ($\bar{\theta} < 30°$, and $\sigma_\theta < 20°$) when all 5 sensors were operational are shown. (c) A 1-hr period with larger wave height (H_s = 0.8 m), but similar water depth and directional characteristics (h = 1.8 m, f = 0.042 Hz, $\bar{\theta} < 10°$, and $\sigma_\theta < 20°$). (d) Elevation of the seafloor relative to mean sea level (curve) and sensor locations (symbols) versus cross-shore location.

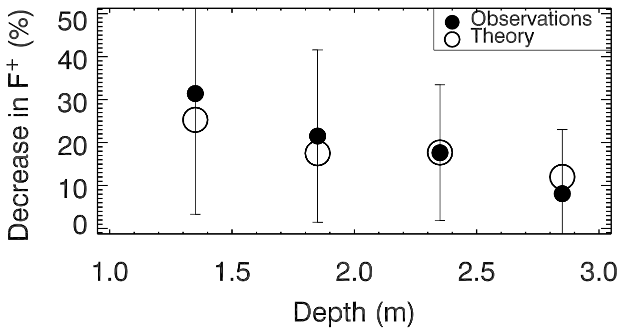

Figure 4. Observed (filled symbols) and predicted (equations 1 and 2) (open symbols) percentage decrease in onshore-directed energy flux across the sandbars versus water depth. The observations and theoretical predictions (one for each of the 37 observed records) were binned into 0.5 m wide depth bins and averaged. The vertical bars are ±1 standard deviation of the observations.

offshore, there was a 50% decrease in F^+, compared with a 20% decrease predicted by nondissipative Bragg reflection theory [*Yu and Mei*, 2000a] (Figure 3c), suggesting that the remaining 30% energy loss is the result of wave breaking. A sharp drop in F^+ between the two most offshore sensors, both for waves at the resonant Bragg frequency (Figure 3c) and for the overall wave field (not shown), indicates that strong dissipation occurred on the outermost bar, followed by unbroken wave propagation to a second surfzone near the shoreline where 80% of the remaining energy flux was lost ($R_L^2 = 0.2$), roughly consistent with established parameterizations of wave breaking in the surfzone [*Thornton and Guza*, 1983, *Whitford*, 1988]. Both reflection and dissipation result in decreased F^+ toward the shoreline, but the observed seaward increase of F^- can be explained only by reflection. The theoretical variation of F^- agrees with the observations (Figure 3c), and suggests the seaward increase in F^- approximately balances the energy loss resulting from reflection from the sandbars. Although the analysis is qualitative and neglects possibly important contributions of nonlinear energy transfers across the spectrum, the results suggest that resonant Bragg reflection occurs in the presence of wave breaking when sediment is more likely to be mobilized and transported.

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S. Elgar, Woods Hole Oceanographic Institution, MS11, Woods Hole, MA 02543, USA. (elgar@whoi.edu)

B. Raubenheimer, Woods Hole Oceanographic Institution, MS12, Woods Hole, MA 02543, USA. (britt@whoi.edu)

T. H. C. Herbers, Naval Postgraduate School, Monterey, CA 93943, USA. (thherber@nps.navy.mil)