

## Modeling the alongshore current on barred beaches

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**Abstract.** Mean alongshore currents observed on two barred beaches are compared with predictions based on the one-dimensional, time- and depth-averaged alongshore momentum balance between forcing (by breaking waves, wind, and 10–100 km scale alongshore surface slopes), bottom stress, and lateral mixing. The observations span 500 hours at Egmond, Netherlands, and 1000 hours at Duck, North Carolina, and include a wide range of conditions with maximum mean currents of 1.4 m/s. Including rollers in the wave forcing results in improved predictions of the observed alongshore-current structure by shifting the predicted velocity maxima shoreward and increasing the velocity in the bar trough compared with model predictions without rollers. For these data, wave forcing balances the bottom stress within the surfzone, with the other terms of secondary importance. The good agreement between observations and predictions implies that the one-dimensional assumption holds for the range of conditions examined, despite the presence of small alongshore bathymetric nonuniformities. With stronger bathymetric variations the model skill deteriorates, particularly in the bar trough, consistent with earlier modeling and laboratory studies.

### 1. Introduction

Models of the alongshore current in the nearshore often are based on the one-dimensional (1-D), depth-integrated, and time-averaged alongshore momentum balance between wind, wave, and tidal forcing, bottom stress, and lateral mixing [Longuet-Higgins, 1970; Thornton and Guza, 1986, among many others]. Breaking waves are the most important forcing mechanism in the surfzone, although wind [Whitford and Thornton, 1993; Feddersen *et al.*, 1998] and tidal [Houwman and Hoekstra, 1998] forcing can contribute significantly. Predictions of 1-D models, in which alongshore variations in waves and bathymetry are neglected, compare favorably with observations on (near) planar beaches [e.g., Thornton and Guza, 1986]. However, on barred beaches, predictions [e.g., Church and Thornton, 1993] of a strong, narrow current jet on the seaward side of the bar crest and near-zero flow in the bar trough differ from the observed broadly distributed (in the cross-shore) current, with max-

imum near or shoreward of the bar crest [Feddersen *et al.*, 1998], or even in the deepest part of the trough [Church and Thornton, 1993].

Model failure may result from alongshore variations in bathymetry and the neglect of rollers in the wave forcing [e.g., Lippmann *et al.*, 1995; Reniers and Battjes, 1997]. Alongshore bathymetric variations with length scale of  $O(100\text{ m})$  often are observed in the nearshore [e.g., Lippmann and Holman, 1990] and may cause alongshore variations in wave height, wave direction, and mean water level (set-up). Model simulations [Sancho *et al.*, 1995; Slinn *et al.*, 2000] and laboratory observations [Haller *et al.*, 1997] show that these alongshore variations may alter significantly the nearshore current field, especially in the bar trough. Similarly, when the bathymetry had strong alongshore nonuniformities, the 1-D alongshore momentum balance assumption did not hold on the barred beach near Duck, North Carolina [Feddersen *et al.*, 1998]. Including rollers in the wave forcing causes a spatial lag between the dissipation of wave energy and the transfer of momentum to steady currents, thus shifting the alongshore current maximum shoreward (although not necessarily into the deepest part of the trough) and increasing the velocity in the trough [e.g., Lippmann *et al.*, 1995; Reniers and Battjes, 1997; Kuriyama and Nakatsukasa, 2000]. Comparisons with laboratory observations show that 1-D models with rollers are more accurate than models without rollers [Reniers and Battjes, 1997]. However, roller-based alongshore current models have not been tested over a wide range of field conditions.

Here, predictions of a 1-D alongshore current model are compared with field data from two barred beaches, Egmond aan Zee (Netherlands) and Duck, North Carolina (USA). The data span 500 (Egmond) and 1000 (Duck) hour time periods when alongshore variations in morphology were small.

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Model alongshore currents (section 2) are controlled by three free parameters (apparent bed roughness, wave-front slope, and eddy viscosity). Parameter values are obtained with model-data fitting at Egmond (section 3) and Duck (section 4). With a roller-based parameterization of the wave forcing, the model accurately describes the cross-shore structure of the mean alongshore current, supporting the validity of the 1-D assumption. The relative importance of the different forcing terms, bottom stress, and lateral mixing to the alongshore current and the effect of strong alongshore nonuniformities in the bathymetry are discussed in section 5. Conclusions are summarized in section 6.

## 2. Model Formulation

### 2.1. Wave Model

The wave model consists of two coupled differential equations describing the time-averaged (over many wave periods) wave and roller energy balances. With the assumption that the wave field is narrow-banded in frequency and direction, the wave energy balance is

$$\frac{d}{dx} \left( \frac{1}{8} \rho g H_{\text{rms}}^2 c_g \cos \bar{\theta} \right) = -D_{br} - D_{bf}, \quad (1)$$

where  $x$  is the cross-shore coordinate,  $H_{\text{rms}}$  is the root-mean-square wave height,  $\rho$  is water density,  $g$  is gravitational acceleration,  $c_g$  is the group velocity,  $\bar{\theta}$  is the mean wave angle, and  $D_{br}$  and  $D_{bf}$  are breaking-wave dissipation and bottom friction that are modeled according to *Battjes and Janssen* [1978] and *Nielsen* [1983], respectively. Standard parameter values are used throughout ( $\alpha = 1$ ,  $\gamma = 0.5 + 0.4 \tanh(33s_0)$  [Battjes and Stive, 1985], where  $\alpha$  is a dissipation parameter,  $\gamma$  is related to the maximum wave height, and  $s_0$  is the deep water wave steepness). Bottom friction is not important in the surfzone where wave breaking dominates the dissipation. Linear wave theory is used to calculate  $c_g$  and Snell's law is used to determine  $\bar{\theta}(x)$  from offshore measurements.

The energy balance for rollers is [Stive and De Vriend, 1994]

$$\frac{d}{dx} (2E_r c \cos \bar{\theta}) = -D_r + D_{br}, \quad (2)$$

where  $E_r$  is the roller energy density (set to zero at the offshore boundary),  $c$  is the phase speed, and  $D_r$  is the roller dissipation, given by [Duncan, 1981; Deigaard, 1993]

$$D_r = \frac{2gE_r \sin \beta}{c}, \quad (3)$$

where the wave-front slope  $\beta$  usually is assumed to be 0.1 or less [e.g., Walstra et al., 1996]. The wave model is solved with a standard forward stepping scheme using the observed bathymetry, and offshore values of  $H_{\text{rms}}$ , wave period  $T$ ,  $\bar{\theta}$ , and water level  $\zeta$ .

### 2.2. Current Model

The depth- and time-averaged alongshore current velocity  $\bar{v}$  is obtained from the 1-D depth-integrated and time-

averaged alongshore momentum balance between wave, wind, and tidal forcing, and bottom stress and lateral mixing

$$-\frac{1}{\rho} \frac{dS_{yx}}{dx} + \frac{\tau_y^w}{\rho} - gh \frac{d\zeta}{dy} = c_f \langle |\bar{u}|v \rangle - \frac{d}{dx} \left( \nu h \frac{d\bar{v}}{dx} \right), \quad (4)$$

where  $S_{yx}$  is the off-diagonal component of the radiation stress tensor [Longuet-Higgins and Stewart, 1964],  $\tau_y^w$  is the alongshore wind stress,  $h$  is water depth,  $d\zeta/dy$  is the tidally induced 10–100 km scale alongshore slope of the mean sea surface,  $c_f$  is a drag coefficient,  $\bar{u}$  and  $v$  are the total instantaneous horizontal velocity vector and the instantaneous alongshore velocity, respectively,  $\langle \rangle$  represents a time average over many wave periods, and  $\nu$  is the depth-averaged eddy viscosity. Alongshore variations in morphology, waves, and currents are neglected, as are Earth rotation, variation of water density, and fluid acceleration (i.e.,  $d\bar{v}/dt$ ).

The wave forcing is the cross-shore gradient of  $S_{yx}$ . Using linear theory and assuming waves to be narrow-banded in frequency and direction,  $S_{yx}$  is

$$S_{yx} = \frac{1}{8} \rho g H_{\text{rms}}^2 \frac{c_g}{c} \cos \bar{\theta} \sin \bar{\theta} + 2E_r \cos \bar{\theta} \sin \bar{\theta}, \quad (5)$$

where the terms on the right-hand side are the wave and roller contribution, respectively. Using (1) with  $D_{bf} \ll D_{br}$ , (2) and (5) yield

$$\frac{dS_{yx}}{dx} = -\frac{\sin \bar{\theta}}{c} D_r. \quad (6)$$

The drag coefficient  $c_f$  is parameterized with the Manning-Strickler equation [e.g., Sleath, 1984]

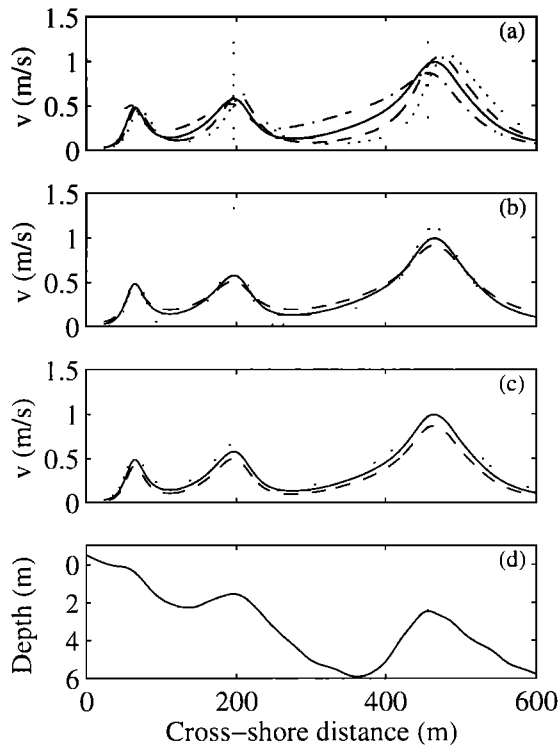
$$c_f = 0.015 \left( \frac{k_a}{h} \right)^{1/3}, \quad (7)$$

where  $k_a$ , the apparent bed roughness, is assumed to be cross-shore constant and time-independent, based on *Houwman and Van Rijn's* [1999] coupled bedform-fluid modeling. The value of  $k_a$  is chosen to fit  $\bar{v}$  observations, and the sensitivity of the results to the form of  $c_f$  is discussed in section 5.2. The velocity moment in the bottom stress formulation is parameterized as [Feddersen et al., 2000]

$$\langle |\bar{u}|v \rangle = \sigma_T \bar{v} [1.16^2 + (\bar{v}/\sigma_T)^2]^{1/2}, \quad (8)$$

where  $\sigma_T^2$  is the wave-orbital velocity variance, calculated from  $H_{\text{rms}}$ ,  $h$ , and  $T$  using linear wave theory. This empirical parameterization adequately represents  $\langle |\bar{u}|v \rangle$  for the directionally spread random wave field at both Duck and Egmond (skill  $r^2 = 0.99$  and a best fit slope of 0.97), confirming that (8) estimates measured moments more accurately than estimates based on either the weak-current ( $\langle |\bar{u}|v \rangle \sim \sigma_T \bar{v}$ ) or strong-current ( $\langle |\bar{u}|v \rangle \sim |\bar{v}| \bar{v}$ ) assumptions [Feddersen et al., 2000].

Lateral mixing is included in (4) as a diffusion term [Longuet-Higgins, 1970]. Important sources of lateral mixing in the surfzone are breaking-induced turbulence [Battjes,



**Figure 1.** Alongshore current  $\bar{v}$  versus cross-shore distance showing the effect of changing the wave-front slope, eddy viscosity, and apparent bed roughness. In all panels, the offshore conditions are  $H_{rms} = 1.5$  m,  $T = 8$  s, and  $\theta = 30^\circ$ , with no wind or tidal forcing. The solid curve in Figures 1a–1c is the standard run ( $\beta = 0.1$ ,  $\nu = 0.5$  m<sup>2</sup>/s,  $k_a = 0.03$  m). Other curves correspond to changes in one parameter with the others held constant: (a) no roller (dotted line),  $\beta = 0.2$  (dashed line), and  $\beta = 0.05$  (dash-dotted line), (b)  $\nu = 0$  m<sup>2</sup>/s (dotted line) and  $\nu = 1$  m<sup>2</sup>/s (dashed line), and (c)  $k_a = 0.015$  m (dotted line) and  $k_a = 0.06$  m (dashed line). (d) The depth profile was measured at Egmond. The dotted vertical lines in each plot indicate the bar crest positions.

1975], depth variation in the cross-shore and alongshore velocities [Svendsen and Putrevu, 1994], and shear waves [Özkan-Haller and Kirby, 1999]. However, the cross-shore distribution of  $\nu$  is not understood well, and for simplicity, a cross-shore constant and time-independent  $\nu$  is assumed.

The modeled cross-shore distribution of  $D_r$ ,  $c$ ,  $\bar{\theta}$ , and  $\sigma_T$ , and the observed bathymetry, alongshore wind stress, and large-scale sea-surface slope are input into the current model, which is solved iteratively. The offshore and shoreline boundary condition for (4) are  $d\bar{v}/dx = 0$ , implying no diffusion of momentum through the boundaries.

### 2.3. Free parameters

The influence of wave-front slope  $\beta$ , eddy viscosity  $\nu$ , and the apparent bed roughness  $k_a$  on the model  $\bar{v}$  is examined using typical Egmond waves and bathymetry (Figure 1). With  $k_a = 0.03$  m, (7) results in  $c_f \approx 3.5 - 4.0 \times 10^{-3}$  across the inner bar and trough. With no roller, narrow current jets are located on the seaward side of each bar and near the shoreline, with near-zero currents in the troughs, qualita-

tively similar to the model predictions of Church and Thornton [1993] (Figure 1a). The roller shifts the maximum mean alongshore current  $\bar{v}_{max}$  onshore and broadens the current jets by increasing  $\bar{v}$  in the troughs (compare dotted with solid curves in Figure 1a). The magnitude of the onshore shift in  $\bar{v}_{max}$  and increase in trough  $\bar{v}$  is related to the advection length of the roller, which increases with decreasing  $\beta$  (Figure 1a) and increasing  $c$  (e.g., with increasing wave period and water depth over the bar crest). Lateral mixing smooths the cross-shore distribution of  $\bar{v}$  without shifting the location of  $\bar{v}_{max}$  (Figure 1b). An increase in  $k_a$  decreases the magnitude of  $\bar{v}$ , without significantly altering the cross-shore shape of  $\bar{v}$  (Figure 1c). A doubling of  $k_a$  to 0.06 m results in only a 15–20% decrease in  $\bar{v}$ , consistent with (7).

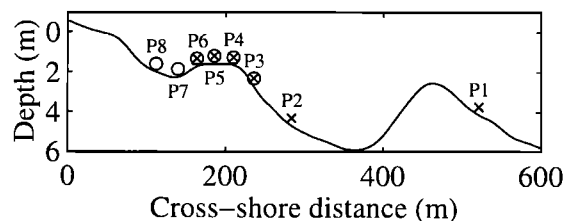
## 3. Egmond

### 3.1. Observations

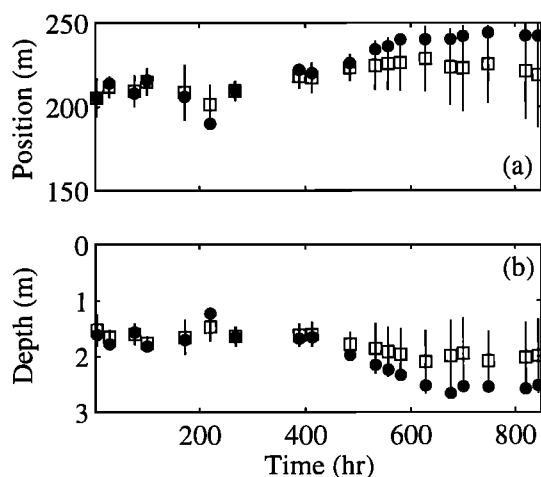
Data were collected during October and November 1998 near Egmond aan Zee, Netherlands. The site is characterized by two well-developed bars (Figure 2). Offshore wave conditions ( $H_{rms}$ , significant zero-downcrossing period, and  $\bar{\theta}$ , energy-weighted mean direction [Kuik et al., 1988]) were measured by a directional wave buoy, located in 16-m depth, 5 km offshore. Winds measured 10 m above mean sea level at position P1 (Figure 2) are used to estimate wind stress. Estimates of alongshore water level gradients are obtained from observations at two tidal stations separated in the alongshore by 30 km and centered around Egmond.

Bidirectional current meters and pressure sensors were deployed on a cross-shore transect across the inner bar with an additional pressure sensor (P1) offshore of the outer bar (Figure 2). Data from P1–P6 were acquired for  $\sim 34$  min per hour (starting at each whole clock hour) at a sampling rate of 2 or 4 Hz. At P7 and P8, 10-min average velocities were stored. For comparison with  $\bar{v}$  at P3–P6, the first three 10-min values of each hour are averaged. Current meter elevations above the bed were less than 1 m and varied as the morphology changed. Data are discarded when the current meters were within 0.2 m of the bed. The median grain size at the inner bar was  $\sim 225$   $\mu$ m.

Spatially extensive nearshore bathymetric surveys were obtained every few days with an amphibious vehicle. The inner-bar crest, originally located 200 m from the shoreline, migrated 40 m farther offshore (Figure 3a). Bathymetric alongshore nonuniformities were always detectable (Fig-



**Figure 2.** Depth relative to mean sea level versus cross-shore distance on October 16, 1998, at Egmond and locations of current meters (circles) and pressure sensors (crosses).



**Figure 3.** (a) Location of the inner bar crest location on the main measurement transect (solid circles) and location of the bar crest averaged over a 350-m wide alongshore region centered on the main transect (squares) and (b) depth above the inner bar crest at the main transect (solid circles) and averaged over a 350-m wide alongshore region (squares) versus time at Egmond. The vertical lines are  $\pm$  one standard deviation. Time = 0 corresponds to October 15, 1998, 09:00 MET.

ure 3) but were most pronounced after  $t \approx 500$  hours when a broad cross-shore channel developed close to the measurement transect.

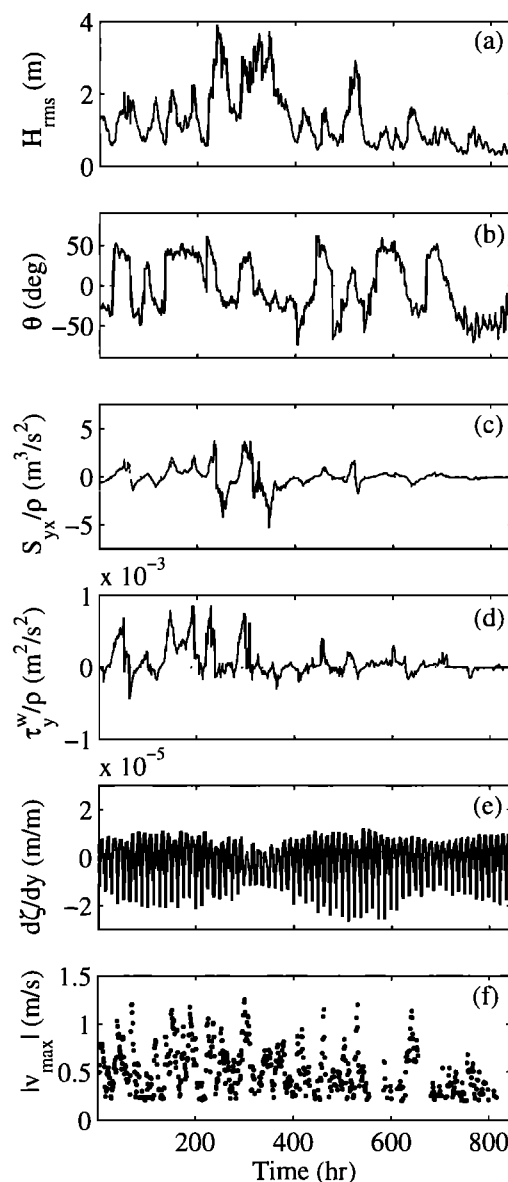
A wide range of conditions were encountered. In 16-m depth,  $H_{\text{rms}}$  ranged between 0.2 and 3.9 m (Figure 4a), significant periods between 3.9 and 10.8 s, and  $\bar{\theta}$  between  $\pm 45^\circ$  (Figure 4b), where positive  $\bar{\theta}$  indicates waves incident from the southwest.  $S_{yx}/\rho$ , computed with (5) and  $E_r = 0$ , ranged from  $-5.3$  to  $3.7 \text{ m}^3/\text{s}^2$  (Figure 4c), and wind stress estimates  $\tau_y^w/\rho$ , computed from the observed wind speed and direction with a standard formulation and a drag coefficient of 0.002, varied between  $-4.4$  and  $8.6 \times 10^{-4} \text{ m}^2/\text{s}^2$  (Figure 4d). The neap (spring) tidal range was  $\sim 1.4$  (2.1) m. The alongshore surface slope varied semidiurnally owing to tides and reached maximum values of  $\pm 2 \times 10^{-5} \text{ m/m}$  (Figure 4e). The observed  $|\bar{v}_{\text{max}}|$  ranged from 0.1 to 1.3 m/s (Figure 4f). During the first 500 hours,  $\bar{v}_{\text{max}}$  was located near or shoreward of the bar crest (P4-P6), and afterwards  $\bar{v}_{\text{max}}$  was located mainly in the deepest part of the trough (P7). A tidal variation in the  $\bar{v}_{\text{max}}$  location [Thornton and Kim, 1993] was not observed.

### 3.2. Model-Data Comparison

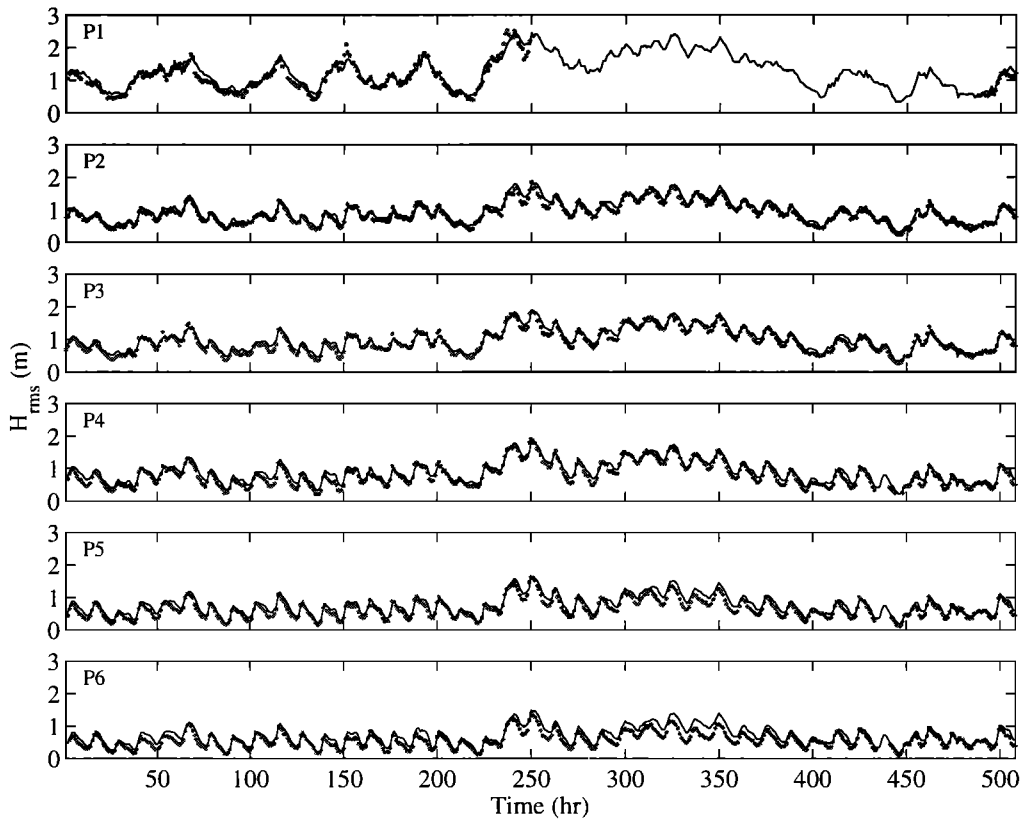
The 1-D wave and current models assume alongshore uniformity in morphology, waves, and currents, so the model-data comparison is restricted to the first 500 hours of observations. Modeled and observed  $H_{\text{rms}}$  agree (Figure 5) with skill  $r^2 > 0.89$  at all sensors and show the transition from  $H_{\text{rms}}$  that are closely related to offshore  $H_{\text{rms}}$  (compare P1 in Figure 5 with Figure 4a) to depth-limited and tidally modulated  $H_{\text{rms}}$  over the inner bar (e.g., P5 and P6). Root-mean-square errors  $\epsilon_{\text{rms}}$  for individual sensors vary between 0.10 and 0.16 m, with an average of 0.13 m for all

sensors. Wave heights at the inner bar crest (P5 and P6) are overestimated by 0.10–0.15 m, with larger errors at low tide than at high tide, and maximum differences of  $\sim 0.4$  m near  $t \approx 340$  hours (Figure 5). Examples of the observed and predicted cross-shore distribution of  $H_{\text{rms}}$  at high, mid, and low tide are given in Figures 6a–6c, respectively.

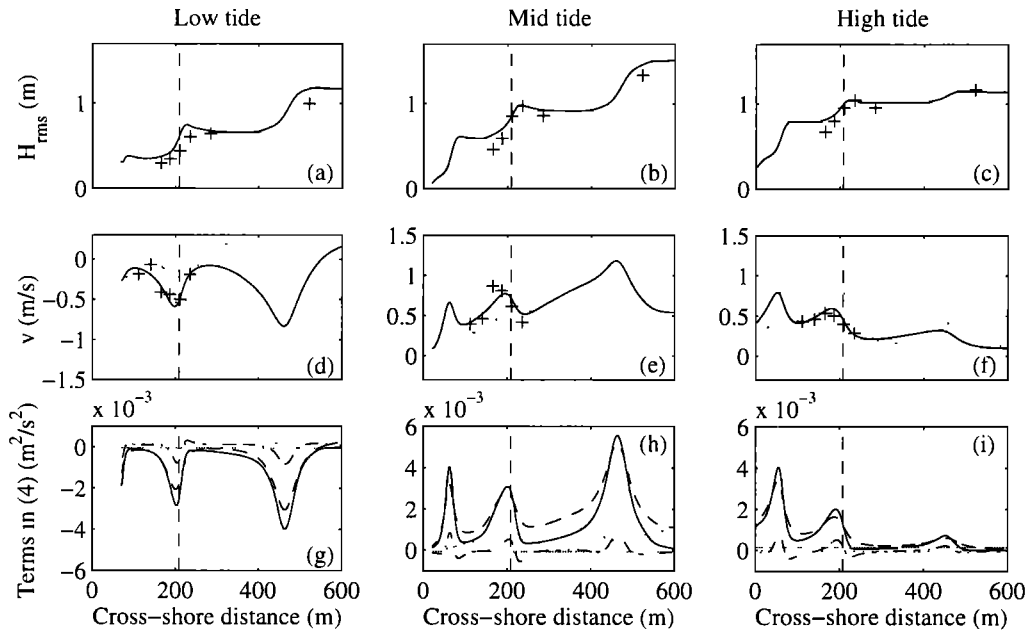
Good agreement between measured and modeled  $\bar{v}$  is obtained with  $\beta = 0.05$ ,  $\nu = 0.5 \text{ m}^2/\text{s}$ , and  $k_a = 0.022$  m. The  $\beta = 0.05$  is about the midrange of values cited by Walstra *et al.* [1996], and  $\nu = 0.5 \text{ m}^2/\text{s}$  is roughly consistent with the  $\nu$  parameterization used by Özkan-Haller and Kirby [1999] with  $M = 0.5$ , which for the present conditions yielded



**Figure 4.** (a) Offshore root-mean-square wave height  $H_{\text{rms}}$ , (b) offshore wave angle  $\bar{\theta}$ , (c) offshore incident wave radiation stress  $S_{yx}/\rho$ , (d) alongshore wind stress  $\tau_y^w/\rho$ , (e) large-scale alongshore surface slope  $d\zeta/dy$ , and (f) absolute maximum alongshore velocity  $|\bar{v}_{\text{max}}|$  versus time at Egmond. The 599 shown values in Figure 4f correspond to situations with at least four active current meters and  $|\bar{v}_{\text{max}}| > 0.2 \text{ m/s}$ .



**Figure 5.** Measured (symbols) and modeled (lines)  $H_{rms}$  from offshore (P1) to onshore (P6) versus time at Egmond.



**Figure 6.** Measured (symbols) and modeled (lines) (a)-(c)  $H_{rms}$  and (d)-(f)  $\bar{v}$  (solid line, roller; dotted line, no roller) at Egmond, and (g)-(i) terms of the alongshore momentum balance (solid line, wave forcing (roller model); horizontal gray line, wind forcing; dotted line, tidal forcing; dashed line, bottom stress; and dash-dotted line, lateral mixing) versus cross-shore distance. Columns from left to right: low tide ( $t = 76$  hours), mid tide ( $t = 52$  hours), and high tide ( $t = 56$  hours). The location of the inner-bar crest,  $x = 208$  m, is shown with a vertical dashed line.

**Table 1.** Alongshore Current Model Error Statistics at Egmond ( $k_a = 0.022$  m,  $\nu = 0.5$  m<sup>2</sup>/s)<sup>a</sup>

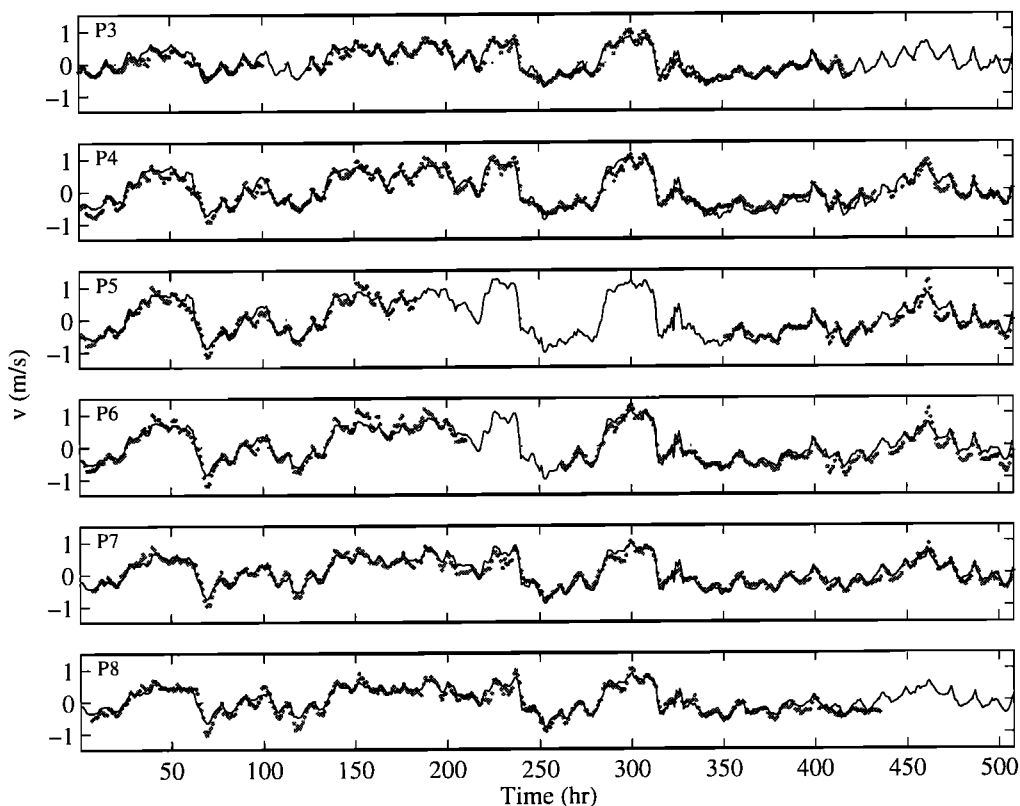
	Roller ( $\beta = 0.05$ )			No Roller		
	$\epsilon_{\text{rms}}$ , m/s	$m$	$r^2$	$\epsilon_{\text{rms}}$ , m/s	$m$	$r^2$
P3	0.14	0.93	0.88	0.25	1.32	0.85
P4	0.17	1.03	0.90	0.29	1.39	0.90
P5	0.15	0.95	0.91	0.19	0.75	0.89
P6	0.17	0.86	0.92	0.29	0.53	0.90
P7	0.13	1.00	0.91	0.19	0.58	0.88
P8	0.14	0.86	0.90	0.19	0.63	0.88

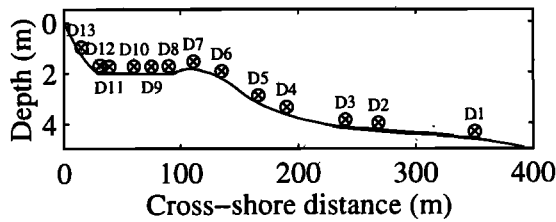
<sup>a</sup>Here,  $\epsilon_{\text{rms}}$  is the root mean square error between modeled and observed  $\bar{v}$ ,  $m$  and  $r^2$  are the slope and correlation coefficient of least squares fits between modeled and observed  $\bar{v}$ . A value of  $m > 1$  corresponds to model overprediction of the observed  $\bar{v}$ .

a range of cross-shore averaged  $\nu$  between 0.1–0.9 m<sup>2</sup>/s. With  $k_a = 0.022$  m, bar-crest  $c_f$  ranges depending on the tide from  $2.6 \times 10^{-3}$  to  $5.2 \times 10^{-3}$ , within the range of previously determined surfzone  $c_f$  [Feddersen et al., 1998; Garcez-Faria et al., 1998]. Average  $c_f$  are 5–15% lower in the trough than on the bar crest. With these free model parameter values, the cross-shore distribution of  $\bar{v}$  is repro-

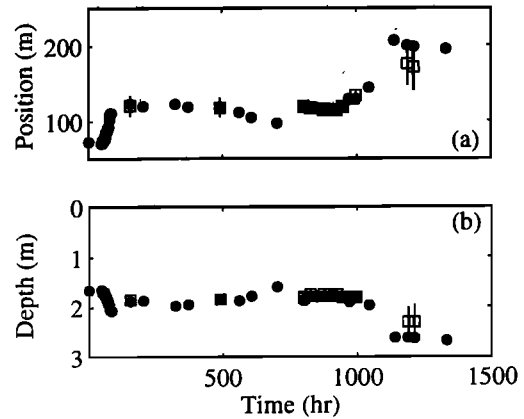
duced accurately (Figure 7) with skill  $r^2 \geq 0.88$ , best-fit slopes  $m$  between 0.86 and 1.03, and  $\epsilon_{\text{rms}} < 0.2$  m/s (Table 1, roller run). The predicted  $\bar{v}_{\text{max}}$  location is between the bar crest at P4 and the shoreward side of the bar at P6 (e.g., Figures 6d–6f), with a tidally induced cross-shore variation of 10–30 m, similar to the distance between sensors, possibly explaining why a tidal variation in  $\bar{v}_{\text{max}}$  location was not observed. The relatively poor  $H_{\text{rms}}$  prediction at P5 and P6 at  $t \approx 340$  hours (Figure 5) does not result in poor  $\bar{v}$  prediction (Figure 7). The good agreement between modeled and measured  $\bar{v}$  indicates that the use of cross-shore constant and time-independent values of  $\beta$ ,  $\nu$ , and  $k_a$  at Egmond is reasonable.

The effect of the roller is shown by running the model with the same  $\nu = 0.5$  m<sup>2</sup>/s and  $k_a = 0.022$  m but without the roller (Table 1, no roller run). Neglecting the roller causes an immediate transfer of momentum from organized wave motion to  $\bar{v}$ , resulting in overprediction of  $\bar{v}$  seaward of the bar crest (P3–P4; best-fit slope  $m \approx 1.3$ –1.4) and underprediction of  $\bar{v}$  on the shoreward side of the bar and in the trough (P6–P8;  $m \approx 0.5$ –0.6). In comparison with the roller run,  $\epsilon_{\text{rms}}$  are increased 25–80%. The predicted  $\bar{v}_{\text{max}}$  location is  $\sim 10$  m seaward of the bar crest, with about a 10 m tidal variation, less than predicted with rollers. The degraded model performance without rollers is most marked at mid and high tide (Figures 6d–6f), when the roller-induced onshore shift in  $\bar{v}_{\text{max}}$  location and the increase in trough  $\bar{v}$  are largest.

**Figure 7.** Measured (symbols) and modeled (lines)  $\bar{v}$  from offshore (P3) to onshore (P8) versus time at Egmond. Error statistics are given in Table 1, roller run.



**Figure 8.** Depth relative to mean sea level versus cross-shore distance on September 14, 1994, at Duck and locations (symbols) of colocated current meters and pressure sensors.



**Figure 9.** (a) Location of the bar crest location on the main measurement transect (solid circles) and location of the bar crest averaged over a 350-m wide alongshore region centered on the main transect (squares) and (b) mean depth above the bar crest at the main transect (solid circles) and averaged a 350-m wide alongshore region (squares) versus time at Duck. Time = 0 corresponds to September 1, 1994, 01:00 EST. The vertical lines are  $\pm$  one standard deviation.

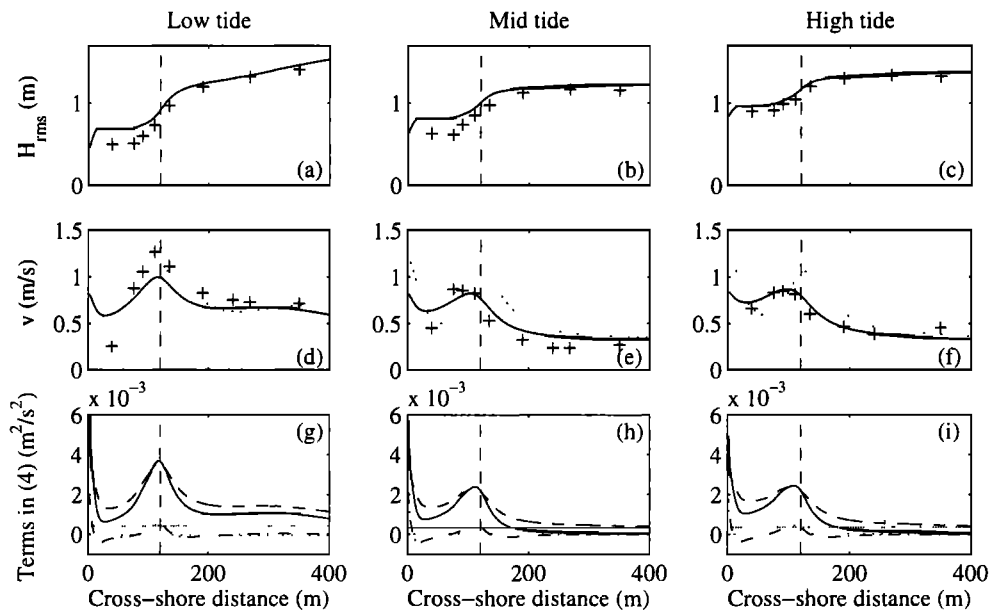
## 4. Duck

### 4.1. Observations

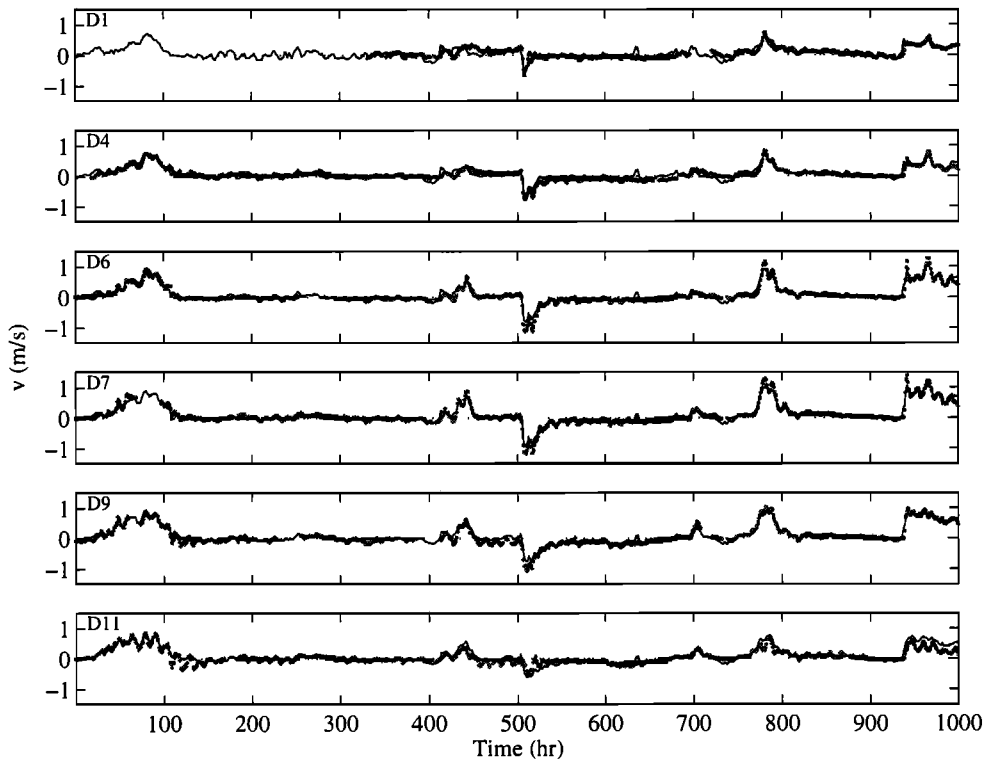
The Duck data were collected from September 1 until October 31, 1994, during the Duck94 experiment at the U.S. Army Corps of Engineers Field Research Facility (FRF) near Duck, North Carolina, on a barrier island in the Atlantic Ocean. Wave statistics ( $H_{rms}$ , peak period, and energy-weighted  $\bar{\theta}$  [Kuik *et al.*, 1988]) in 8-m water depth were estimated from a two-dimensional (2-D) array of 15 bottom-mounted pressure sensors [Long, 1996]. Wind stress estimates are obtained from winds measured 19.5 m above mean sea level at the end of the nearby FRF pier. Alongshore surface slopes are computed from two pressure sensors separated by 30 km (centered around Duck) along the 5-m isobath [Lentz *et al.*, 1999]. Pressure and velocity observations were obtained at 13 cross-shore positions (Figure 8) extending from near the shoreline to 4.5-m depth. Details on data acquisition and processing are given by Elgar *et al.*

[1997], Gallagher *et al.* [1998], and Feddersen *et al.* [1998]. The mean grain size along the transect varied between 180–250  $\mu\text{m}$  [Gallagher *et al.*, 1998].

Spatially extensive bathymetric surveys obtained occasionally with an amphibious vehicle and cross-shore depth profiles obtained continuously with altimeters [Gallagher *et al.*, 1998] show that the bar migrated offshore 120 m during the experiment but only 50 m during the first 1000 hours



**Figure 10.** Measured (symbols) and modeled (lines) (a)-(c)  $H_{rms}$  and (d)-(f)  $\bar{v}$  (solid line, roller; dotted line, no roller) at Duck, and (g)-(i) terms of the alongshore momentum balance (solid line, wave forcing (roller model); horizontal gray line, wind forcing; dotted line, tidal forcing; dashed line, bottom stress; and dash-dotted line, lateral mixing) versus cross-shore distance. Columns from left to right: low tide ( $t = 780$  hours), mid tide ( $t = 776$  hours), and high tide ( $t = 785$  hours). The location of the inner-bar crest,  $x = 120$  m, is shown with a vertical dashed line.



**Figure 11.** Measured (symbols) and modeled (lines) alongshore mean current  $\bar{v}$  versus time at Duck from offshore (D1) to onshore (D11). Error statistics are given in Table 2, roller run.

(Figure 9a). Alongshore nonuniformities in the bar were small for  $t < 1000$  hours (Figure 9), when a broad cross-shore channel developed near the measurement transect.

In 8-m depth, 750 m from shore,  $H_{\text{rms}}$  ranged between 0.1 and 2.9 m, peak periods between 3 and 16 s, and  $\bar{\theta}$  between  $\pm 50^\circ$ . Estimates of  $S_{yx}/\rho$ , computed with (5) and  $E_r = 0$ , ranged between  $-1.0$  and  $0.9 \text{ m}^3/\text{s}^2$ , less than 25% of the maximum values at Egmond. Wind stress estimates,  $\tau_y^w/\rho$ , varied between  $-2.5$  and  $7.1 \times 10^{-4} \text{ m}^2/\text{s}^2$ . The tide was semidiurnal with a spring range of 1 m. Compared with Egmond, the alongshore surface slopes were small, generally less than  $3 \times 10^{-6} \text{ m/m}$ . Alongshore current maxima reached absolute values up to 1.4 m/s, and the largest  $|\bar{v}_{\text{max}}|$  (greater than  $\sim 0.5 \text{ m/s}$ ) were located near the bar crest [Feddersen *et al.*, 1998]. The  $|\bar{v}_{\text{max}}|$  location was tidally modulated, shifting from near the bar crest at low tide to 30 m farther shoreward at high tide.

#### 4.2. Model-Data Comparison

Model-data comparison is restricted to the first 1000 hours, when alongshore bathymetric nonuniformities were small (Figure 9). Excluding calm conditions (offshore  $H_{\text{rms}} \leq 0.5 \text{ m}$ ), the wave height prediction error  $\epsilon_{\text{rms}}$  varies between 0.04 and 0.11 m, with an average of 0.08 m for all sensors, slightly less than at Egmond. Skill  $r^2$  exceeds 0.8 from D1 to D10, demonstrating that the observed tidally modulated wave field at the bar crest and in the trough (not shown, but similar to Egmond, Figure 5) is reproduced accurately. Similar to Egmond, the differences in the trough are

smallest at high tide (0–0.1 m) and increase to 0.1–0.2 m at low tide (Figures 10a–10c).

With the Egmond  $\bar{v}$  model parameters ( $\beta = 0.05$ ,  $\nu = 0.5 \text{ m}^2/\text{s}$ , and  $k_a = 0.022 \text{ m}$ ), modeled and measured  $\bar{v}$  (not shown) are correlated ( $r^2 \geq 0.75$ ), but the predictions are smaller than the observations (best-fit  $m < 1$ ). Reducing  $k_a$  to 0.0125 m (a 20%  $c_f$  reduction compared with Egmond) results in improved model predictions (Figure 11 and Table 2, roller run). The reasons for the  $k_a$  differences are unknown and the parameterization and variation of  $c_f$  are not understood (see section 5.2). With  $k_a = 0.0125 \text{ m}$ ,  $c_f$  on the bar crest ranges depending on the tide from  $2.4 \times 10^{-3}$  to  $3.3 \times 10^{-3}$ , with an average of  $2.7 \times 10^{-3}$ . Trough  $c_f$  are 3–10% lower. These  $c_f$  are similar to previous surfzone-averaged estimates of  $c_f = 3.3 \times 10^{-3}$  [Feddersen *et al.*, 1998]. The best-fit slopes  $m$  range from 0.86 to 1.07 (D5–D12), and  $\epsilon_{\text{rms}}$  is 0.1 m/s, increasing to 0.2 m/s close to the shore (Table 2). The approximately constant best-fit slopes (with values near one) across the bar into the trough indicate that the cross-shore structure of the alongshore mean current is reproduced reasonably well, except near the shoreline (D13) where the observed  $\bar{v}$  is overpredicted by  $\sim 25\%$ . Consistent with observations, the predicted  $\bar{v}_{\text{max}}$  location is near the bar crest at low tide, shifting 15–25 m shoreward at high tide.

The  $\bar{v}$  model performance is tidally modulated (Figures 10d–10f). At high and mid tide, the predicted cross-shore structure of  $\bar{v}$  agrees well with the observations (Figures 10e–10f). At low tide (Figure 10d) the  $\bar{v}_{\text{max}}$  location is repro-



**Table 2.** Alongshore Current Model Error Statistics at Duck ( $k_a = 0.0125$  m,  $\nu = 0.5$  m<sup>2</sup>/s)<sup>a</sup>

	Roller ( $\beta = 0.05$ )			No Roller		
	$\epsilon_{rms}$ , m/s	$m$	$r^2$	$\epsilon_{rms}$ , m/s	$m$	$r^2$
D1	0.11	0.82	0.76	0.11	0.94	0.77
D2	0.10	0.85	0.83	0.11	0.91	0.84
D3	0.09	0.87	0.88	0.09	0.92	0.88
D4	0.10	0.86	0.88	0.12	1.02	0.88
D5	0.12	0.96	0.87	0.11	1.01	0.85
D6	0.10	0.86	0.96	0.18	1.14	0.92
D7	0.10	0.88	0.96	0.14	0.87	0.92
D8	0.09	0.90	0.97	0.19	0.70	0.89
D9	0.10	0.88	0.96	0.20	0.61	0.89
D10	0.11	1.01	0.91	0.12	0.75	0.92
D11	0.17	1.03	0.80	0.15	0.91	0.80
D12	0.21	1.07	0.77	0.19	1.08	0.79
D13	0.23	1.27	0.75	0.33	1.74	0.81

<sup>a</sup> To exclude the frequent periods of near zero flow (e.g., Figure 11), the statistics were based on observations only with  $|\bar{v}_{max}| > 0.2$  m/s and at least five active current meters between (and including) D5 to D12. The number of these observations ranged from 133 at D5 to 352 at D8.

duced accurately, but the overall modeled  $\bar{v}$  distribution is broader than observed. Similar discrepancies occur during other tidal cycles with strong currents. The different breaker types (e.g., plunging or spilling) that may occur at different tide levels are not accounted for and may contribute to model errors. With  $k_a = 0.0125$  m and  $\nu = 0.5$  m<sup>2</sup>/s, but without the roller, the model (Table 2, no roller run) overpredicts  $\bar{v}$  on the seaward side of the bar (D6,  $m \approx 1.1$ ) and strongly underpredicts  $\bar{v}$  in the trough (D8-D11,  $m \approx 0.6-0.9$ ). Similar to Egmond, the model improvement from including the roller into the wave-forcing parameterization is greatest at mid and high tide, when roller advection distances are largest (Figures 10e-10f).

## 5. Discussion

### 5.1. Importance of Dynamical Terms

Although Egmond and Duck differ in incident wave conditions and morphology (i.e., double and single bar systems), the alongshore momentum balances are similar. For example, in the three cases at different tidal stages, the wave forcing largely balances the surfzone bottom stress, and wind forcing, tidal forcing, and lateral mixing are secondary terms but cannot in general be neglected (Figures 6g-6i and 10g-10i). Lateral mixing is most important near the current jets, where it reduces  $|\bar{v}_{max}|$  and broadens  $\bar{v}(x)$ . Seaward of the surfzone, the bottom stress balances the wind and tidal forcing at Egmond and roughly balances wind forcing at Duck (Figures 6g-6i and 10g-10i). The low-tide wave forcing typ-

ically is stronger, narrower, and farther offshore than the high-tide wave forcing (e.g., Figures 10g-10i), resulting in different alongshore current distributions (Figure 10d-10f).

Estimates of the importance of the model terms (4) are obtained by spatially averaging the absolute value of each term across the bar-trough region (Egmond:  $110 \leq x \leq 235$  m, P3-P8; Duck:  $30 \leq x \leq 165$  m, D5-D12) for each run, and calculating the rms value for the entire experiment (Table 3). The rms wave and wind forcing at Duck are each half those at Egmond, reflecting the less energetic conditions at Duck. At both sites the waves are the dominant forcing mechanism and largely balance the bottom stress (Table 3, roller run). Wind forcing is 20–25% of the wave forcing over the bar-trough region at both sites. Tidal forcing is, on average, as significant as wind forcing at Egmond but is negligible at Duck [Feddersen et al., 1998; Lentz et al., 1999]. Tidal forcing at Egmond results in semidiurnal currents with amplitude 0.3–0.5 m/s when wind forcing is weak. At both sites, rollers alter the spatial structure of the wave forcing but not the rms wave forcing integrated over the bar-trough region (Table 3, compare roller with no roller runs). However, without rollers, the rms lateral mixing approximately doubles owing to the narrower spatial distribution of wave forcing and  $\bar{v}$ .

The wave forcing with and without rollers also was averaged separately over the bar crest (Egmond:  $165 \leq x \leq 235$  m, P3-P6; Duck:  $90 \leq x \leq 165$  m, D5-D8) and trough (Egmond:  $110 \leq x < 165$  m, P3-P6; Duck:  $30 \leq x < 90$  m, D8-D12) regions (not shown). At both sites, the rms wave forcing without rollers is largely concentrated at the bar crest, and the trough rms wind and wave forcing are approximately equal. The roller changes the wave-forcing distribution, causing the bar and trough rms wave forcing to be approximately equal, resulting in improved model-data  $\bar{v}$  agreement.

### 5.2. The $c_f$ Parameterizations

The ratio of roughness to depth  $k_a/h$  is believed to influence  $c_f$  and is utilized in the Manning-Strickler  $c_f$  parameterization (7) implemented here. The cross-shore and temporally constant  $k_a$  is suggested by coupled bedform-fluid modeling [Houwman and Van Rijn, 1999] but is inconsistent with  $k_a$  estimates based on vertical profile measurements of

**Table 3.** Root-Mean-Square Values ( $\times 10^{-3}$  m<sup>2</sup>/s<sup>2</sup>) of Terms in the Alongshore Momentum Balance Integrated Over the Bar-Trough Region

	Egmond		Duck	
	Roller	No Roller	Roller	No Roller
Wave forcing	1.28	1.27	0.61	0.58
Wind forcing	0.25	0.25	0.14	0.14
Tidal forcing	0.20	0.20	0.02	0.02
Bottom stress	1.41	1.34	0.71	0.71
Lateral mixing	0.19	0.44	0.08	0.18

**Table 4.** Selected Alongshore Current Model Error Statistics With Different  $c_f$  Parameterizations ( $\beta = 0.05$ ,  $\nu = 0.5 \text{ m}^2/\text{s}$ )

	$c_f \sim k_a/h$ (7)			$c_f = \text{Constant}$		
	$\epsilon_{\text{rms}}$ , m/s	$m$	$r^2$	$\epsilon_{\text{rms}}$ , m/s	$m$	$r^2$
Egmond <sup>a</sup>						
P3	0.14	0.93	0.88	0.18	0.63	0.84
P5	0.15	0.95	0.91	0.19	1.07	0.89
P8	0.14	0.86	0.90	0.18	0.72	0.88
Duck <sup>b</sup>						
D1	0.11	0.82	0.76	0.12	0.65	0.75
D7	0.10	0.88	0.96	0.10	0.86	0.97
D11	0.17	1.03	0.80	0.18	1.02	0.79

<sup>a</sup> $k_a = 0.022 \text{ m}$  and  $c_f = 3.2 \times 10^{-3}$ .

<sup>b</sup> $k_a = 0.0125 \text{ m}$  and  $c_f = 2.7 \times 10^{-3}$ .

the alongshore current at Duck, where  $k_a$  was found to vary from 0.01–2.1 m ( $k_a = 30z_a$ , Garcez-Faria et al. [1998, Table 1]). Therefore, although (7) with cross-shore constant and time-independent  $k_a$  yields accurate  $\bar{v}$  predictions,  $k_a$  may not be modeled accurately.

For simplicity, the drag coefficient  $c_f$  commonly is assumed to be cross-shore constant [e.g., Thornton and Guza, 1986; Reniers and Battjes, 1997] and not to depend on  $k_a/h$  as used here (7). At Egmond, cross-shore constant  $c_f = 3.2 \times 10^{-3}$  (the bar-trough average from (7)) leads to a moderate increase in  $\epsilon_{\text{rms}}$  of several cm/s (Table 4). The bar crest (P5)  $\bar{v}$  is overpredicted slightly, and the offshore (P3) and trough (P8)  $\bar{v}$  are underpredicted significantly (Table 4). With the bar-trough average  $c_f = 2.7 \times 10^{-3}$ , the offshore (D1)  $\bar{v}$  at Duck also is underpredicted, but the bar (D7) and trough (D11)  $\bar{v}$  model-data error and regression slope  $m$  are similar to those with (7) (Table 4). The constant  $c_f$  Duck  $\bar{v}$  predictions in the trough may not be as degraded as at Egmond because the bar-trough depth difference at Duck is smaller than at Egmond (compare Figure 2 with Figure 8), which according to (7) leads to a smaller difference between bar and trough values of  $c_f$ . The offshore  $\bar{v}$  underprediction with constant  $c_f$  at both Egmond and Duck indicates that the offshore  $c_f$  should be reduced relative to the bar-trough average, consistent with integrated momentum balances [Feddersen et al., 1998]. This suggests that, in general, a cross-shore varying  $c_f$ , reduced in the trough and offshore and increased at the bar crest, such as parameterized by (7), yields improved  $\bar{v}$  predictions.

Although (7) yields accurate  $\bar{v}$  predictions when  $k_a$  is chosen to match observations, the appropriate a priori value of  $k_a$  is not known. The average offshore  $S_{yx}$  is overestimated owing to the assumption of a narrow-band in  $f - \theta$  wave field (appendix A). This causes a positive bias in the model wave forcing and thus an overestimation of  $k_a$  and hence  $c_f$ . At Duck,  $S_{yx}$  is overestimated on average by  $\sim 60\%$  (ap-

pendix A) and thus  $c_f$  using a more accurate  $S_{yx}$  would be roughly 60% of the values used here. The  $S_{yx}$  overestimation will differ from site to site and may contribute to the difference in  $k_a$  at Egmond and Duck.

The  $k_a$  used here with (7) should not be interpreted as an apparent roughness height. The Duck  $k_a$  is slightly less than the rms physical roughness, but the expected amplification from wave-current interaction is  $O(10 - 100)$  [Houwman and Van Rijn, 1999]. Wave breaking may also increase  $c_f$  [e.g., Church and Thornton, 1993] and thus contribute to the increased bar-trough  $c_f$  relative to offshore, where wave breaking was infrequent. The form of (7),  $c_f \sim h^{-1/3}$ , indirectly may incorporate breaking-wave effects. Given the bias in the wave forcing, the problematic assumptions of (7), and the possible inconsistency between modeled  $k_a$  and the physical roughness and expected amplification factor, the  $k_a$  in (7) is best interpreted as a free model parameter that when chosen appropriately yields accurate  $\bar{v}$  predictions and  $c_f$  values within the range of previously determined  $c_f$ .

### 5.3. Two-Dimensional Conditions

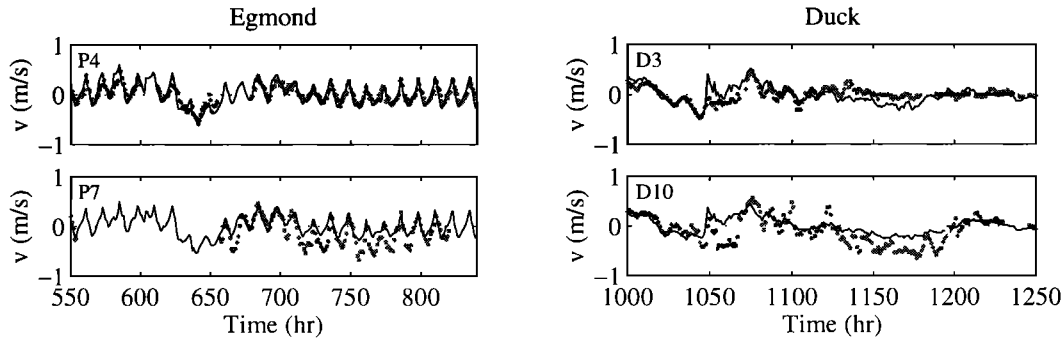
The high skill and best-fit slopes near one between measured and modeled currents implies that the assumption of 1-D morphology, waves, and currents is valid for the range of conditions examined ( $t < 500$  hours at Egmond,  $t < 1000$  hours at Duck). Apparently, 2-D terms (such as local alongshore pressure gradients) are small relative to the 1-D terms. Larger morphological nonuniformities that developed later did not affect  $H_{\text{rms}}$  predictions ( $\epsilon_{\text{rms}} = 0.08$  and 0.09 m for all sensors at Egmond and Duck, respectively). In contrast,  $\bar{v}$  model skill is lower at Egmond after a rip channel developed near the sensors. Model-data agreement seaward of the rip channel (P4) is good (Figure 12, Table 5), indicating that the 2-D terms are negligible in this region. The predictive skill of the model is lower closer to shore, where  $\epsilon_{\text{rms}}$  almost doubles and the best-fit  $m$  reduces to 0.5 (Figure 12, Table 5). Furthermore, the model performance is strongly tidally modulated. At high tide, modeled and measured  $\bar{v}$  agree well, but at low tide the model un-

**Table 5.** Selected Alongshore Current Model Error Statistics for 1-D and 2-D Periods ( $\beta = 0.05$ ,  $\nu = 0.5 \text{ m}^2/\text{s}$ )

	1-D <sup>a</sup>			2-D <sup>b</sup>		
	$\epsilon_{\text{rms}}$ , m/s	$m$	$r^2$	$\epsilon_{\text{rms}}$ , m/s	$m$	$r^2$
Egmond ( $k_a = 0.022 \text{ m}$ )						
P4	0.17	1.03	0.90	0.10	1.10	0.79
P7	0.13	1.00	0.91	0.21	0.48	0.65
Duck ( $k_a = 0.0125 \text{ m}$ )						
D3	0.09	0.87	0.88	0.14	0.89	0.51
D10	0.11	1.01	0.80	0.26	0.49	0.43

<sup>a</sup>Egmond:  $t < 500$  hrs; Duck:  $t < 1000$  hrs.

<sup>b</sup>Egmond:  $550 < t < 850$  hrs; Duck:  $1000 < t < 1250$  hrs.



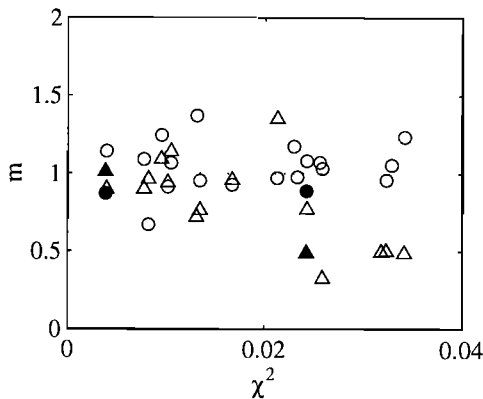
**Figure 12.** Measured (symbols) and modeled (lines)  $\bar{v}$  with 2-D conditions versus time at (left) Egmond and (right) Duck.

derpredicts  $\bar{v}$  by a factor of 3. Video time exposure images and visual observations revealed no wave breaking at high tide (i.e., the flows were tidal) and some breaking at low tide but not in the rip channel. Also, the mean cross-shore currents at P4-P7 are tidally modulated with near-zero flow at high tide and offshore directed flows of 0.6 m/s at low tide, suggesting a rip current active on a semidiurnal scale. Similar to Egmond, model skill at Duck also degrades under 2-D conditions ( $1000 < t < 1250$  hours), particularly in the bar trough (D10, Figure 12, Table 5).

To examine further the effect of 2-D bathymetry on  $\bar{v}$  model performance at Egmond, error statistics were computed for each period between two profile measurements and compared with a nondimensional metric of bathymetric nonuniformity, quantified for each survey as the spatially averaged and normalized squared difference between the depth  $d(x, y)$  and the alongshore ( $y$ ) averaged cross-shore depth profile  $\bar{d}(x)$

$$\chi^2 = \frac{1}{L_x L_y} \int_0^{L_x} \int_0^{L_y} \left( \frac{d(x, y) - \bar{d}(x)}{\bar{d}(x)} \right)^2 dy dx. \quad (9)$$

where  $L_x$  and  $L_y$  are the cross-shore and alongshore length of the survey region, respectively. Averaged over  $110 \leq x \leq 235$  m and  $-175 \leq y \leq 175$  m ( $y = 0$  is the



**Figure 13.** Best-fit slope  $m$  seaward of bar crest (circles) and in the bar trough (triangles) versus  $\chi^2$ . Open symbols, Egmond P4 and P7; filled symbols, Duck D3 and D10. The two  $\chi^2$  values at Duck are averaged over the 1-D and 2-D period.

main transect),  $\chi^2$  varies between 0.004–0.021 during the 1-D period ( $t < 500$  hours) and increases to 0.033 afterward. Seaward of the bar (P4), the best-fit  $m$  is independent of  $\chi^2$  and scatters around 1 (open circles in Figure 13). Comparable model performance is found in the trough for  $\chi^2 < \approx 0.02$ , but  $m$  dropped to  $\approx 0.5$  for larger  $\chi^2$  (open triangles in Figure 13). During the Duck experiment,  $\chi^2$  ( $30 \leq x \leq 190$  m,  $-175 \leq y \leq 175$  m) was far less variable with  $0.0038 \pm 0.0008$  (mean  $\pm$  standard deviation) during the 1-D period ( $t < 1000$  hours) and  $\chi = 0.024 \pm 0.001$  afterward. The bar and trough best-fit  $m$  for the 1-D and 2-D period (i.e., Table 5) are comparable to those at Egmond for the same  $\chi^2$  (Figure 13). On the basis of the  $\chi^2 = 0.02$  criterion, the 1-D assumption was valid for  $\sim 60$  and 70% of time during the Egmond and Duck experiments, respectively. Note that  $\chi^2 = 0.02$  corresponds to an alongshore depth variability of 14% of the mean depth. Further work is needed to determine if  $\chi^2$  is a robust indicator of 2-D effects.

## 6. Conclusions

Using standard literature values [Battjes and Stive, 1985], the cross-shore wave height distribution is reproduced accurately, with an average root mean square error of 0.13 and 0.08 m at Egmond and Duck, respectively. The error in wave height is tidally modulated with increased error at low tide. For a wide range of conditions at both locations, the cross-shore structure of the mean alongshore current is predicted well with a 1-D alongshore current model (i.e., 2-D effects are ignored). Essential to the accurate prediction is the inclusion of rollers in the wave forcing, confirming earlier laboratory results. Rollers shift the velocity maxima shoreward and increase the alongshore flow in the trough, resulting in predictions of the mean alongshore current consistent with field observations. Within the surfzone, the model primarily balances wave forcing with bottom stress. Wind forcing, lateral mixing, and at Egmond tidal forcing, are secondary, but not negligible.

The agreement between measured and modeled mean alongshore current implies that the assumption of a time and cross-shore independent apparent bed roughness (chosen to fit the observations), wave front slope, and eddy viscosity is reasonable, and that weak nonuniformities in morphology are unimportant to the alongshore current, consistent with

previous integrated 1-D alongshore momentum balances at Duck. The Egmond roller slope and eddy viscosity work well at Duck, but for unknown reasons, bed roughness at Duck decreases almost 50% (a 20% reduction in drag coefficient). The appropriate roughness without fitting to observations is unclear. With appropriate roughness a cross-shore varying drag coefficient yields improved predictions compared with a constant drag coefficient. Additional data-model comparisons suggest that the 1-D balance does not hold, especially in the bar trough, when the bathymetry is strongly alongshore inhomogeneous, consistent with earlier model and laboratory studies.

## Appendix A: Radiation Stress Estimates

The radiation stress component  $S_{yx}$  in a directionally spread random wave field seaward of the surfzone is, according to linear theory [Battjes, 1972],

$$S_{yx} = \int_0^\infty \int_{-\pi}^\pi E(f, \theta) \frac{c_g(f)}{c(f)} \sin \theta \cos \theta d\theta df, \quad (\text{A1})$$

where  $E(f, \theta)$  is the frequency-directional ( $f$ - $\theta$ ) energy spectrum, and  $c_g(f)$  and  $c(f)$  are the frequency dependent group and phase velocities, respectively. For a monochromatic (single frequency) and unidirectional (single wave-angle) plane wave of height  $H$ , (A1) reduces to

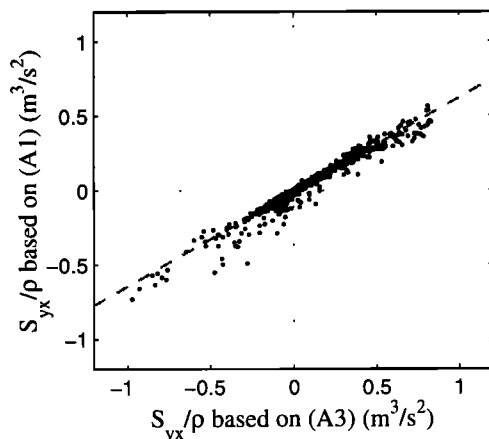
$$S_{yx} = \frac{1}{8} \rho g H^2 \frac{c_g}{c} \sin \theta \cos \theta, \quad (\text{A2})$$

where  $c_g$  and  $c$  are evaluated at the monochromatic wave frequency.

For simplicity, alongshore current models that incorporate random waves often approximate  $S_{yx}$  similar to (A2),

$$S_{yx} = \frac{1}{8} \rho g H_{\text{rms}}^2 \frac{c_g(\bar{f})}{c(\bar{f})} \sin \bar{\theta} \cos \bar{\theta}, \quad (\text{A3})$$

where  $c_g$  and  $c$  are evaluated at the peak or mean (energy-



**Figure A1.** Offshore  $S_{yx}/\rho$  based on equation (A1) versus  $S_{yx}/\rho$  based on equation (A3) at Duck. The dashed line is the best-fit line with slope 0.63. The correlation  $r = 0.98$  (1332 data points).

weighted) wave frequency  $\bar{f}$  and  $\bar{\theta}$  is a peak or mean (energy-weighted) wave angle. This  $S_{yx}$  approximation (A3) is accurate only for a wave field that is narrow banded in frequency and direction.

Accurate  $S_{yx}$  estimates based on (A1) were obtained by applying a moment-estimation method [Elgar *et al.*, 1994] to data from the FRF 8-m depth pressure sensor array [Long, 1996]. Corresponding  $S_{yx}$  estimates based on (A3) were made using array estimates of  $H_{\text{rms}}$ , mean (energy-weighted)  $\bar{f}$ , and mean (energy-weighted)  $\bar{\theta}$  [Kuik *et al.*, 1988]. These two estimates are correlated (Figure A1), but estimates based on (A3) overpredict  $S_{yx}$  based on (A1) by  $\sim 60\%$ . Egmond  $S_{yx}$  estimates based on (A1) were not available, so for consistency, (A3) with  $\bar{\theta}$  was used for both Egmond and Duck.

The tendency for (A3) to overpredict  $S_{yx}$  can be understood by considering a single-frequency but directionally spread wave field with a top-hat (with width  $\Delta$ )  $\theta$  distribution,

$$S(\theta) = \begin{cases} \frac{1}{2\Delta}, & \bar{\theta} - \Delta \leq \theta \leq \bar{\theta} + \Delta \\ 0, & \text{otherwise} \end{cases}. \quad (\text{A4})$$

The ratio of (A1) to (A3) is

$$\frac{\frac{1}{2\Delta} \int_{\bar{\theta}-\Delta}^{\bar{\theta}+\Delta} \sin \theta \cos \theta d\theta}{\sin \bar{\theta} \cos \bar{\theta}} = \frac{\sin \Delta \cos \Delta}{\Delta}, \quad (\text{A5})$$

which is  $\leq 1$  for all  $\Delta$ . For narrow directional spreads ( $\Delta \rightarrow 0$ ), the ratio (A5) approaches one. The directional width  $\Delta$  can be related to the directional spread  $\sigma_\theta$  [Herbers *et al.*, 1999], and for the Duck 8-m depth  $\sigma_\theta = 30^\circ \pm 5^\circ$ ,  $\Delta \approx 55^\circ$ , and the ratio (A5)  $\approx 0.5$ , roughly consistent with the best-fit slope of 0.63. For any  $S(\theta)$  symmetric about  $\bar{\theta}$ , the ratio of (A1) to (A3) is  $\leq 1$ . The overprediction of  $S_{yx}$  by (A3) biases the optimal alongshore current model parameters (i.e., increases  $k_a$ , and hence  $c_f$ ).

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