JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 106, NO. C3, PAGES 4625–4627, MARCH 15, 2001

Coastal profile evolution at Duck, North Carolina: A cautionary note

Steve Elgar

Woods Hole Oceanographic Institution, Woods Hole, Massachusetts

Abstract. The Hurst exponents of 10.8-year-long time series of cumulative bed level observed between the shoreline and $\sim$ 8-m water depth on an ocean beach are shown to be consistent with bed-level time series described by a sinusoid with a 10.8-year period plus white noise. Thus, for these observations, Hurst exponents cannot distinguish self-organized morphological evolution from the hypothesis that nearshore morphology on monthly to decadal timescales is a forced response to small-scale physical processes driven by waves and currents.

1. Introduction

Southgate and Möller [2000] (hereineafter referred to as S&M) calculated the Hurst exponents [*Hurst,* 1956; *Mandelbrot and Wallis,* 1969a] of time series of cumulative bed level derived from 10.8 years of observations of cross-shore depth profiles extending from the shoreline to approximately 8-m water depth near Duck, North Carolina [*Lee and Birkemeier,* 1993; *Larson and Krauss,* 1994]. Similar to previous studies [*Lee et al.,* 1998], S&M subdivide the nearshore zone from the shoreline to 800 m offshore into four regions: dune, inner bar, outer bar, and upper shoreface (S&M, Figure 13). On the basis of the Hurst exponents, S&M conclude that the dune and upper shoreface cumulative bed-level time series on timescales of 1–12 and 1–20 months, respectively, may be the result of nonlinear self-organized behavior, and that the bathymetry in the inner and outer bar regions exhibits self-organized behavior over timescales greater than 12–24 months. Verifying these results is important because they have "major implications for modeling coastal morphodynamics on different timescales" (S&M, p. 11,506), because models based on the small-scale physics of sediment transport forced by waves and currents will not predict the behavior of large-scale self-organized morphology.

Here, a model consisting of a sinusoid plus Gaussian noise is shown to predict accurately the cross-shore structure of the Hurst exponent of the observed cumulative bed-level time series. The model is linear, suggesting that the Hurst exponent cannot be used to distinguish the hypothesis that the observed bed-level fluctuations are caused by nonlinear self-organized behavior from the alternative hypothesis that the bed-level time series are a Gaussian random process with a periodic component. Determination of underlying physical mechanisms from relatively short time series using Hurst exponents [*Mandelbrot and Wallis,* 1969a, 1969b, 1969e], principal oscillation pattern analysis [*Penland,* 1985], and nonlinear forecasting [*Sugihara and May,* 1990] has been shown to be difficult. Therefore the conclusion (S&M) that modeling approaches based on details of small-scale physical processes or on external forcing by waves and currents cannot make accurate predictions of the evolution of nearshore morphology may not be justified.

Paper number 2000JC000493.
0148-0227/01/2000JC000493$09.00

2. Alternative Hypothesis

A white noise process (neighboring values in time are uncorrelated) has a Hurst exponent of 0.0, Brownian motion (each value in a Brownian time series is the sum of all previous values plus Gaussian noise, so neighboring values are correlated) has a Hurst exponent of 0.5, and fractal (e.g., self-organized) systems have a Hurst exponent greater than 0.5 [*Hurst,* 1956; *Mandelbrot and Wallis,* 1969a, 1969b, 1969c, 1969d, 1969e]. Thus, if a time series of bed level is white noise, the cumulative bed level is a Brownian process and has Hurst exponent = 0.5.

Southgate and Möller [2000] obtained Hurst exponents of $\sim$ 0.8 in the dune and upper shoreface regions of the nearshore zone and of $\sim$ 0.5 in the inner and outer bar regions. The Hurst exponents estimated from the observations can be reproduced by a bed-level time series $y(x, t)$, where x is cross-shore location and t is time, consisting of a sinusoidal component with cross-shore varying amplitude $a(x)$ plus white Gaussian noise $N(t)$, given by

$$y(x,t) = a(x)\sin(2\pi ft) + N(t). \tag{1}$$

The frequency $f = 1/130$ corresponds to one full cycle (10.8-year period) for 130 monthly samples, the same as the data analyzed by S&M.

A 130-point long (same as S&M) time series of white noise ($a(x) = 0$) bed level is shown in Figure 1a, and the corresponding cumulative bed-level time series is shown in

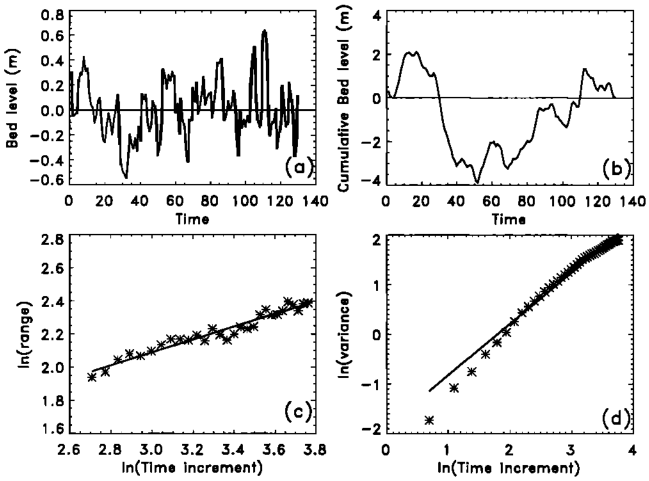

Figure 1. (a) Bed level and (b) cumulative bed level versus time. The bed-level time series is given by equation (1) with $a(x) = 0$. The natural logarithms of the (c) range and (d) variance versus the natural logarithm of the time increment (calculated in the same way as given by S&M). Symbols are the calculated values, the solid lines are least squares fits. The data shown are 1 example selected arbitrarily from 1000 realizations.

Figure 1b. These bed-level and cumulative bed-level time series are similar to those observed in the inner and outer bar regions of the nearshore zone (compare Figures 1a and 1b with S&M's Figure 4, upper (profile 188 at 200 m) and second-from-bottom (profile 62 at 200 m) panels). The corresponding natural logarithms of the range and of the variance versus the natural logarithm of the time increment are shown in Figures 1c and 1d here and in S&M's Figures 8 and 9 (second-from-the-top panels). The range is the difference between the maximum and minimum value of $y(x, t)$ over a specified time interval, called the time increment.

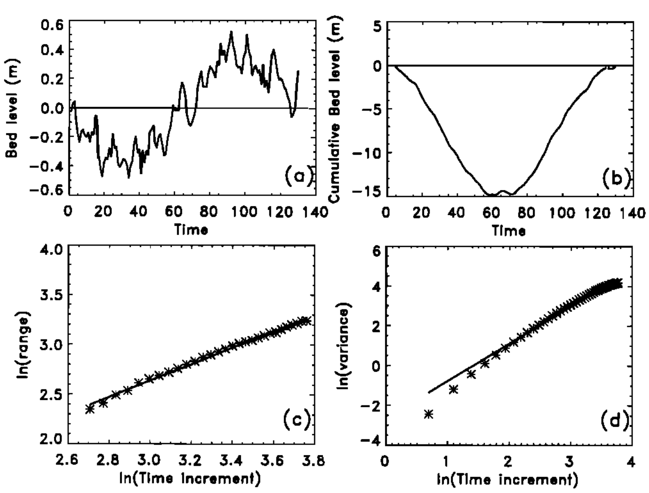

Figure 2. (a) Bed level and (b) cumulative bed level versus time. The bed-level time series is given by equation (1) with the variance of the noise equal to 1/5 the variance of the sinusoid. The natural logarithms of the (c) range and (d) variance versus the natural logarithm of the time increment (calculated in the same as given by S&M). Symbols are the calculated values, the solid lines are least squares fits. The data shown are 1 example selected arbitrarily from 1000 realizations.

The Hurst exponent (calculated from a fit to the points in Figures 1c and 1d in the same manner as given by S&M, also described by *Hastings and Sugihara* [1993]) for the cumulative bed-level time series for this case of white noise bed levels is 0.4 using the range and 0.5 using the variance, consistent with Brownian cumulative bed-level fluctuations. There are large statistical fluctuations in Hurst exponents estimated from the range, with 95% of values from one thousand 130-point time series falling between about -0.3 and 0.6 (mean value $\approx$ 0.5), consistent with the results of *North and Halliwell* [1994]. The 95% limits are much larger than the fluctuations in Hurst exponents estimated from 80 inner bar region and 80 outer bar region time series by S&M (their Table 1), possibly because the S&M time series are not statistically independent.

A 130-point time series (i.e., one cycle of the 10.8-year period) where the variance of the noise is 1/5 the variance of the sinusoid is shown in Figure 2a, and the corresponding cumulative bed-level time series is shown in Figure 2b. These bed-level and cumulative bed-level time series are similar to those observed in the upper shoreface region of the nearshore zone (compare Figures 2a and 2b with S&M's Figure 4, second-from-top (profile 188 at 600 m) and bottom (profile 62 at 600 m) panels). The corresponding natural logarithms of the range and of the variance versus the natural logarithm of the time increment are shown in Figures 2c and 2d here and in S&M's Figures 8 and 9 (lower panels). The Hurst exponent for the cumulative bed-level time series for this case of a sinusoid plus noise is 0.8 using the range and 0.9 using the variance. There are relatively small statistical fluctuations in Hurst exponents estimated from the range, with 95% of values from one thousand 130-point time series falling between 0.7 and 0.9, consistent with the results of *North and Halliwell* [1994] and S&M (their Table 1).

The observed cross-shore structure of the Hurst exponent (S&M's Figures 6 and 7) can be reproduced by the sinusoid plus noise model if $a(x)$ has the correct cross-shore depen-

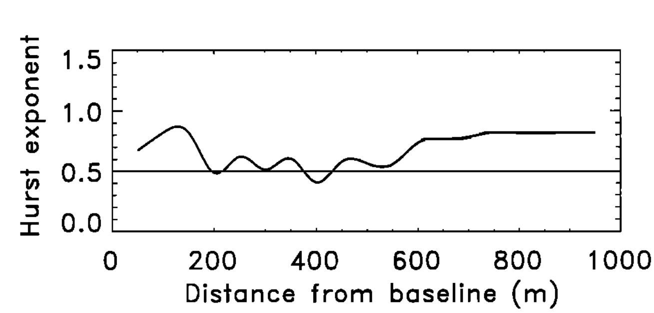

Figure 3. Hurst exponent versus distance from the baseline. At each cross-shore location a bed-level time series consisting of a sinusoid with 10.8-year period plus white Gaussian noise was generated. The noise variance ranged from approximately 20% to 900% of the variance of the sinusoid, with larger noise producing smaller Hurst exponents.

dence (Figure 3). Time series (130 points, one cycle) with noise levels with variance from 1/5 to 9 times the sinusoidal component produce Hurst exponents from $\sim$0.9 to 0.5.

3. Conclusions

Previous investigations [*Southgate and Möller,* 2000] suggest that Hurst exponents estimated from bed-level time series obtained from 10.8 years of observations of cross-shore depth profiles extending from the shoreline to $\sim$ 8-m water depth near Duck, North Carolina, provide circumstantial evidence that nearshore bathymetric evolution is a self-organized process. The results presented here demonstrate that the Hurst exponents can be reproduced by a model consisting of a sinusoid with 10.8-year period plus white Gaussian noise, suggesting that for the time series of bed level observed at Duck, North Carolina, the Hurst exponent is not capable of distinguishing a nonlinear self-organized system from a linear Gaussian random process with a periodic component.

Acknowledgments. This research was supported by the Office of Naval Research, the Army Research Office, and the National Ocean Partnership Program. B. Raubenheimer made valuable comments. This is WHOI contribution 10208.

References

Hastings, H., and G. Sugihara, *Fractals: A User's Guide for the Natural Sciences,* Oxford Univ. Press, New York, 1993.

Hurst, H., Methods of using long-term storage in reservoirs, *Proc. Inst. Civ. Eng., 5,* 519–577, 1956.

Larson, M., and N. Kraus, Temporal and spatial scales of beach profile change, Duck, North Carolina, *Mar. Geol., 117,* 75–94, 1994.

Lee. G., and W. Birkemeier, *Beach and Nearshore Survey Data: 1985–1991 CERC Field Research Facility,* U.S. Army Corps of Eng., Washington, D.C., 1993.

Lee, G., R. Nicholls, and W. Birkemeier, Storm-driven variability of the beach-nearshore profile at Duck, North Carolina, USA, 1981-1991, *Mar. Geol., 148,* 163–177, 1998.

Mandelbrot, B., and J. Wallis, Computer experiments with fractional Gaussian noises, 1, Averages and variances, *Water Resour. Res., 5,* 228–241, 1969a.

Mandelbrot, B., and J. Wallis, Computer experiments with fractional Gaussian noises, 2, Rescaled ranges and spectra, *Water Resour. Res., 5,* 242–259, 1969b.

Mandelbrot, B., and J. Wallis, Computer experiments with fractional Gaussian noises, 3, Mathematical appendix, *Water Resour. Res., 5,* 260–267, 1969c.

Mandelbrot, B., and J. Wallis, Some long-run properties of geophysical records, *Water Resour. Res., 5,* 321–420, 1969d.

Mandelbrot, B., and J. Wallis, Robustness of the rescaled range R/S in the measurement of noncyclic long run statistical dependence, *Water Resour. Res., 5,* 967–988, 1969e.

North, C., and D. Halliwell, Bias in estimating fractal dimension with the rescaled-range (R/S) technique, *Math. Geol. 26,* 531–555, 1994.

Penland, C., Random forcing and forecasting using Principal Oscillation Pattern analysis, *Monthly Weather Rev., 117,* 2165-2185, 1985.

Southgate, H., and I. Möller, Fractal properties of coastal profile evolution at Duck, North Carolina, *J. Geophys. Res., 105,* 11,489–11,507, 2000.

Sugihara, G., and R. May, Nonlinear forecasting as a way of distinguishing chaos from measurement error in time series, *Nature, 344,* 734-741, 1990.

S. Elgar, Applied Ocean Physics and Engineering, Woods Hole Oceanographic Institution, MS 11, 266 Woods Hole Road, Woods Hole, MA 02543. (elgar@whoi.edu)

(Received June 12, 2000; revised November 3, 2000; accepted November 3,2000.)