Estimating Wave Heights from Pressure Measured in Sand Bed

By B. Raubenheimer,[1] Steve Elgar,[2] and R. T. Guza[3]

Abstract: Comparison of predicted with observed attenuation of pressure fluctuations shows that wave heights can be estimated with observations from a pressure sensor that is buried a known depth in fine sand. The attenuation of pressure fluctuations within the sand bed under unbroken shoaling waves, bores in the surf zone, and swash near the shoreline was measured with vertical stacks of buried pressure sensors. The attenuation increased with increasing frequency and depth below the bed surface, consistent with previous observations under nonbreaking waves in deeper water and with model predictions based on poro-elastic theory. In the limit of an infinitely deep soil skeleton that is much more compressible than the pore fluid, the predicted pressure fluctuations decrease exponentially with increasing burial depth, and the attenuation is independent of the sediment properties. For the fine-grained sand beds considered here, this exponential limit accurately predicts the observed attenuation.

INTRODUCTION

Pressure sensors deployed near the seafloor in the surf zone to measure surface waves frequently become buried as the morphology changes (e.g., owing to migration of a sand bar). The attenuation and phase lag caused by the bed may be significant and must be included when estimating surface wave properties from the pressure data. Field observations reported here suggest that analytical theories [e.g., Madsen (1978); Yamamoto et al. (1978); Mei and Foda (1981); Hsu et al. (1993); and many others] for surface-wave-driven pressure fluctuations in sand beds apply to waves in the swash and surf zones (mean water depth 30–525 cm), as well as to unbroken waves measured outside the surf zone (water depths greater than 600 cm) [e.g., Yamamoto (1981); Maeno and Hasegawa (1987); Massel and Kaczmarek (1988); Thomas (1989); Sakai et al. (1992)]. In these studies the model sediment parameters were determined by fitting to the observed vertical attenuation profile, an approach not possible when using a single buried sensor. The attenuation is independent of soil properties in the limit of a soil skeleton that is much more compressible than the pore fluid and, thus, as is shown here, sea-surface elevation fluctuations can be estimated from a single pressure sensor buried a known depth.

OBSERVATIONS

Pore-pressure fluctuations were measured in two fine-grained sand beds ($d_{50} \sim 0.2$ mm) with vertical stacks of buried pressure sensors. Measurement locations are identified by the sensor burial depth (e.g., P^2_{20} and P^2_{220} refer to pressure variance measured with sensors buried 20 and 220 cm, respectively).

A vertical stack of five pressure sensors (each separated by 50 cm, with the upper sensor buried approximately 20 cm) was deployed near the Scripps Institution of Oceanography pier. Eighty-eight 3-h-long records were collected at 2 Hz over 12 d. The mean water depth ranged from 275 to 525 cm owing to tidal fluctuations. Pre- and postcalibrations suggest that differences (<10 cm) in mean water depths calculated using individual buried sensors resulted from drift in the sensor offsets, and not from excess pore pressures caused by cyclic loading. Errors in instrument gain were approximately 1%. Total pressure variance (integrated over the frequency range $0.001 \le f \le 0.25$ Hz) calculated every 1,024 s from measurements at the upper sensor (P_{20}) ranged from 100 to 800 cm^2 and mean periods ranged from 6 to 11 s. The wave field above the sensors sometimes consisted primarily of unbroken waves, and at other times consisted primarily of broken waves.

Additional pore-pressure observations were collected in the surf and swash zones at Torrey Pines State Beach with two buried sensors (P_0 and P_{97}) vertically separated by 97 cm. The burial depth of the upper sensor, determined from daily beach surveys, ranged from 4 to 32 cm during the 2-month-long deployment. During spring low tides, the sand surface above the sensors was exposed, allowing air to enter the interstitial pores. Only data for which the water table was at, or above, sand level throughout each 3-h record are considered here. The mean water depth in the retained 273 3-h records ranged from 30 to 160 cm. Total pressure variance (calculated every 1,024 s) at the upper sensor ranged from 40 to 600 cm^2. Mean periods ranged from 8 to 75 s [infragravity waves were relatively energetic in the shallowest water depths, Fig. 1(b)].

Pressure spectra and phase lags between sensors were computed for each 3-h record by averaging 10 1,024-s sections and merging over five frequency bands (100 degrees of freedom). Coherence between pressure fluctuations measured by sensors at different burial depths was greater than 0.99 over the frequency range $0.001 \le f \le 0.25$ Hz. Similar to previous observations in deeper water, the attenuation increases with increasing burial depth and frequency (Fig. 1). At a given frequency and burial depth, as the water depth (and wavelength) increases the pressure variance ratio increases (equivalently the variance attenuation, defined as 1.0 minus the variance ratio, decreases, Fig. 2). At infragravity frequencies (nominally $0.001 \le f < 0.05$ Hz) the observed attenuation is small at both sites (Figs. 1 and 2), in contrast to the relatively large (and variable) infragravity attenuation observed by Massel and Kaczmarek (1988). Pressure variance attenuation owing to sand can be significant (e.g., the variance attenuation through approximately 100 cm of sand is as large as 70% at 0.2 Hz in the shallowest water depths, Fig. 2). However, the observed phase lags between the upper and more deeply buried sensors are typically less than 3° (not shown), consistent with the theoretical attenuation in saturated sand (described next).

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[1]Postdoctoral Res. Assoc., Scripps Inst. of Oceanography, La Jolla, CA 92093.

[2]Prof., School of Electr. Engrg. and Comp. Sci., Washington State Univ., Pullman, WA 99164.

[3]Prof., Scripps Inst. of Oceanography, La Jolla, CA.

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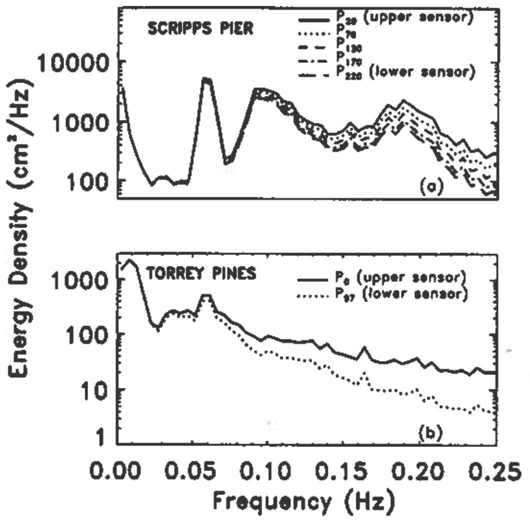

FIG. 1. Pressure Spectra Measured with Sensors Buried: (a) 20, 70, 120, 170, and 220 cm at Scripps Pier (Mean Water Depth 355 cm); (b) Approximately 0 and 97 cm at Torrey Pines (Mean Water Depth 30 cm)

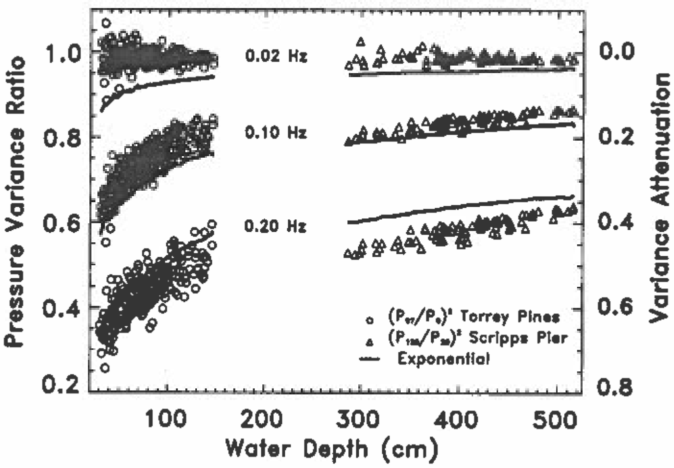

FIG. 2. Ratio of Observed Pressure Variance (Symbols) versus Water Depth for Sensors Vertically Separated Approximately 100 cm. Solid Curves Represent Averaged (Running Mean over 11 Points) Exponential Predictions (3)

THEORY

Assuming Darcian fluid flow and a perfectly elastic isotropic soil skeleton obeying linear and reversible stress-strain relationships, Biot (1941, 1956) derived linear two-phase poroelastic equations describing fluid flow in a porous soil and elastic deformation of the soil skeleton. Several recent models for attenuation through the seabed of sinusoidal water-wave-induced pore-pressure fluctuations are based on these equations and the assumption that the dissipation of surface-wave energy over a wavelength is relatively small. These linear theories may be inaccurate in soft clay or mud beds because nonlinear Coulomb damping owing to internal losses within the soil may be large [e.g., Hsiao and Shemdin (1980); and others]. However, Coulomb damping in sand beds is small and thus does not significantly affect the surface wave dispersion relationship in constant water depth h [e.g., Yamamoto, (1983)] given by

$$\omega^2 = g\lambda \tanh \lambda h \tag{1}$$

where λ = wave number and ω = radian frequency.

In an infinitely deep homogeneous seabed the ratio of the pressure at depth z below the bed surface (positive downward) (P_z) to that at the bed surface (P_0) is [Yamamoto et al., (1978)]

$$\frac{P_z}{P_0} = \left[1 - \frac{im\omega''}{-\lambda'' + i(1+m)\omega''}\right] e^{-\lambda z} + \frac{im\omega''}{-\lambda'' + i(1+m)\omega''} e^{-\lambda' z} \tag{2}$$

The parameters m, ω'', λ', and λ'' [given explicitly in Yamamoto et al. (1978)] are functions of the sand porosity n, Poisson's ratio ν, permeability k, shear modulus G, and the effective compressibility β. The effect of gases trapped in the soil pores is included by using an empirical formula for β (Verruijt 1969) that depends on the degree on saturation S.

In the limit of $\beta \ll G$ (e.g., for partially saturated dense sands or saturated sandstone) pore-pressure attenuation is rapid (owing primarily to compressibility of the fluid) and the phase lag between pore pressures and bottom pressure increases linearly with depth (Madsen 1978; Yamamoto et al. 1978; Mei and Foda 1981). In contrast, in the limit of $\beta \gg G$ (e.g., completely saturated sands and soils), pore-pressure attenuation with depth is weaker and independent of the soil type, and pore and bottom pressures are in phase. In this limit, the pore-pressure ratio in an infinitely deep bed is described by an exponential

$$\frac{P_z}{P_0} = e^{-\lambda z} \tag{3}$$

In a bed of finite thickness D [e.g., Massel and Kaczmarek (1988)]

$$\frac{P_z}{P_0} = \frac{\cosh \lambda(z - D)}{\cosh \lambda D} \tag{4}$$

Differences in pressure variance ratios predicted for beds of infinite and finite depths are small ($\leq$15%) for $z < 0.25D$. The largest relative difference between finite and infinite thickness beds occurs near the surface of the impermeable layer (e.g., $z = D$). In the present study sensor burial depths were less than about 2 m, so in the limit of $\beta \gg G$ predictions are not sensitive to an impermeable layer located more than a few meters below the instruments. The simplifying assumption of an infinitely deep bed of saturated sand yields a predicted pressure attenuation that is independent of soil parameters and depends only on the depth below the seabed and the wave number of the surface wave (1).

MODEL-DATA COMPARISONS

The variance ratio $[(P_z/P_0)^2]$ of pore pressure observed in the Scripps pier experiment was compared with predictions from both the general theory (2) and the exponential limit (3) for $\beta \gg G$ and an infinitely deep bed. Soil parameters were selected to minimize the least-squares error between predicted [with (2)] and observed frequency spectra for all sensors for the first 24 h of data. Following Mei and Foda (1981) the shear modulus G was assumed equal to 2.0×10^8 N/m^2, and other parameters were varied over values typical for fine-grained sand. The best fit values for S, ν, k, and n are 1.0, 0.3, 0.0001 m/s, and 0.25, respectively. The predicted attenuation is sensitive to variation of S, and decreasing the saturation from 1.000 to 0.995 increases the error (using best-fit parameter values) by a factor of 10. The predictions are insensitive (attenuation changes by less than 1%) to variations of ν, k, and n.

Similar to previous observations in deeper water with nonbreaking waves, the model (2) and the exponential limit (3) accurately predict the increase of the average (for 88 3-h runs) observed attenuation with increasing burial depth at a fixed frequency [Fig. 3(a) and 3(b)] and with increasing frequency at a fixed burial depth [Fig. 3(c)]. Predicted and observed attenuation at infragravity frequencies is weak [Figs. 2 and 3(c)]. At Torrey Pines, the linear dispersion relationship (1) may not

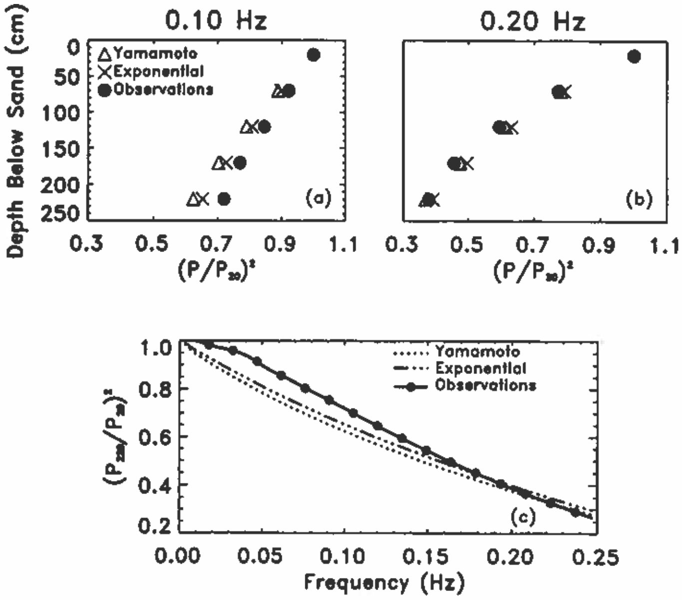

FIG. 3. Average Ratio of Observed and Predicted Pressure Variances (Relative to P_{20}^2) at Frequencies: (a) 0.10; (b) 0.20 Hz versus Sensor Burial Depth; (c) Pressure Variance Ratio $(P_{220}/P_{20})^2$ (at Fixed Sensor Burial Depth) versus Frequency

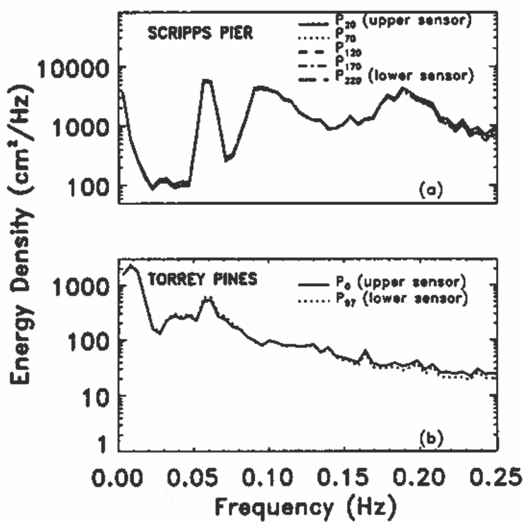

FIG. 4. Sea-Surface Elevation Spectra Predicted Assuming Exponential Attenuation Through Sand Bed for Pressure Observations Shown in Fig. 1

be valid at infragravity frequencies, both because the water depth is changing relatively rapidly over a wavelength and because low-mode edge waves [with dispersion relationships different from (1)] may be significant. Despite the possible errors in the dispersion relationship, average prediction errors are less than 15% (Figs. 2 and 3). Solutions of the boundary-layer model (Mei and Foda 1981) for finite- and infinite-depth beds are similarly accurate (not shown).

The model-data comparisons at Scripps pier (Fig. 3) suggest that predictions of pressure-variance attenuation in a sand bed are not degraded significantly by using the exponential limit that is independent of soil characteristics. The agreement between the observed and exponential-predicted pressure ratios at Torrey Pines is similar to that in deeper water at Scripps pier (Fig. 2), suggesting that air entering the interstitial pores during low tides was replaced with seawater on the rising tide.

Sea-surface elevation fluctuations (P_{sse}) were calculated using the exponential limit (3) and linear finite-depth theory

$$\frac{P_{sse}}{P_z} = e^{\lambda z} \cosh \lambda h \quad (5)$$

Although spectra of pressure observed at sensors buried at different depths are different (Fig. 1), sea-surface elevation spectra calculated using (5) are similar (Fig. 4). Significant pressure and wave height (defined as four times the standard deviation of the pressure and sea-surface elevation fluctuations, respectively) were calculated for 2,730 and 880 1,024-s data records at Torrey Pines and Scripps pier, respectively. The mean (±1 standard deviation) difference in significant pressure is 16% (±5%) for sensors vertically separated 97 cm (at Torrey Pines) and 22% (±4%) for sensors vertically separated 200 cm (at Scripps pier). In contrast, the average differences in significant wave height for the same sensor pairs are 2% (±3%) at Torrey Pines and 1% (±2%) at Scripps pier.

CONCLUSIONS

Attenuation by a sand bed of surface-gravity-wave induced pressure fluctuations was measured with vertical stacks of five buried pressure sensors deployed in the shoaling and surf zone, and two sensors buried in the swash zone. Even at modest burial depths (e.g., 1–2 m) the attenuation becomes significant as the wave frequency increases and the water depth decreases. For example, in 50-cm water depth, burial by 1 m reduces the variance of swell (0.1 Hz) and sea (0.2 Hz) by about 30 and 70%, respectively, in relation to the variances near the seafloor.

Using best-fit soil parameters, a model based on two-phase poro-elastic theory predicts well the observed increase of attenuation with increasing burial depth and wave frequency. The observed attenuation is approximately exponential with depth and observed phase lags between pressure fluctuations at different burial depths are small, consistent with the theoretical limit for an infinitely thick, saturated, sand bed. In this limit, the exponential attenuation is independent of the soil characteristics and depends only on the burial depth and wave number of the surface waves. Sea-surface elevation spectra estimated using the exponential limit and observations from sensors buried at different depths are similar, suggesting that wave heights can be estimated with observations from a single pressure sensor buried a known depth in a sand bed even when the sand parameters are unknown.

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