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Wave transformation across the inner surf zone

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Abstract. Sea and swell wave heights observed on transects crossing the mid and inner surf zone on three beaches (a steep concave-up beach, a gently sloped approximately planar beach, and a beach with an approximately flat terrace adjacent to a steep foreshore) were depth limited (i.e., approximately independent of the offshore wave height), consistent with previous observations. The wave evolution is well predicted by a numerical model based on the one-dimensional nonlinear shallow water equations with bore dissipation. The model is initialized with the time series of sea surface elevation and cross-shore current observed at the most offshore sensors (located about 50 to 120 m from the mean shoreline in mean water depths 0.80 to 2.10 m). The model accurately predicts the cross-shore variation of energy at both infragravity (nominally $0.004 < f \leq 0.05$ Hz) and sea swell (here $0.05 < f \leq 0.18$ Hz) frequencies. In models of surf zone hydrodynamics, wave energy dissipation is frequently parameterized in terms of γ_s, the ratio of the sea swell significant wave height to the local mean water depth. The observed and predicted values of γ_s increase with increasing beach slope β and decreasing normalized (by a characteristic wavenumber k) water depth kh and are well correlated with β/kh, a measure of the fractional change in water depth over a wavelength. Errors in the predicted individual values of γ_s are typically less than 20%. It has been suggested that infragravity motions affect waves in the sea swell band and hence γ_s, but this speculation is difficult to test with field observations. Numerical simulations suggest that for the range of conditions considered here, γ_s is insensitive to infragravity energy levels.

1. Introduction

Laboratory observations of monochromatic waves on planar beaches suggest that the heights (H) of broken waves in the surf zone are depth (h) limited,

$$H = \gamma h \tag{1}$$

with γ depending on the beach slope and the offshore wave steepness [*Iverson*, 1952; *Bowen et al.*, 1968; *Le Méhauté et al.*, 1968; *Galvin*, 1969; *Weggel*, 1972; *Battjes*, 1974; *Iwagaki et al.*, 1974; *Van Dorn*, 1978; *Sunamura*, 1980; *Hansen and Svendsen*, 1984; *Nairn*, 1990]. *Southgate* [1993] reviews observations of, and empirical models for, monochromatic wave γ. The observed values of γ ranged from about 0.7 to 1.2, similar to the theoretical values suggested for solitary [*McGowan*, 1891] and periodic [*Miche*, 1954] waves in constant depth. Many models for wave-driven currents and setup in the surf zone parameterize wave energy dissipation in terms of these semiempirical γ values [e.g., *Bowen et al.*, 1968; *Longuet-Higgins*, 1970a, b].

Field observations in the surf zone suggest that at sea swell frequencies, random wave heights are limited by the local water depth. On low-sloped approximately planar beaches, observed values of γ_s, the ratio of the significant wave height (defined as 4 times the sea surface elevation standard deviation) in the sea swell frequency band to h, were about 0.6 [*Thornton and Guza*, 1982, 1983; *Wright et al.*, 1982; *King et al.*, 1990]. Offshore of a sand bar at Duck, North Carolina, γ_s values ranged from about 0.4 to 0.8, depending on the beach slope (but independent of the offshore wave steepness) [*Sallenger and Holman*, 1985]. Combining low and high wave steepness data collected on barred and unbarred beaches in the laboratory and field, *Nairn* [1990] suggested that γ_s increases with increasing offshore wave steepness. *Vincent* [1985] presented laboratory and field observations in which the significant wave height was proportional to $h^{1/2}$ ($\gamma_s \propto h^{-1/2}$) in the outer surf zone but γ_s was approximately constant in the inner surf zone. There are many additional field and laboratory observations of γ_s [e.g., *Goda*, 1975; *Hotta and Mizuguchi*, 1980; *Mase and Iwagaki*, 1982; *Nelson and

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Gonsalves, 1992; *Nelson*, 1994; *Rakha and Kamphuis*, 1995]. Here γ_s for saturated (e.g., depth-limited) waves observed in shallow water ($0.3 \leq h \leq 2.0$ m) offshore of the swash zone on three beaches is shown to be relatively insensitive to the offshore wave steepness and to increase with increasing beach slope and decreasing water depth. These trends are combined to show that γ_s is correlated to the fractional change of water depth over a typical wavelength. In shallow water on a steep foreshore, where the fractional change in depth is large and waves sometimes plunge onto the beach face (e.g., "shore breaks"), γ_s is greater than 1.0.

Existing semiempirical models of surf zone wave transformation predict the decay of random wave heights observed in the field and laboratory across planar and barred beaches qualitatively well. In one type of model the total random wave dissipation is obtained from an energy balance that uses bore theory to calculate the dissipation rate of a single breaking wave of a given height. Assumptions are made about both the functional form of γ_s and the probability distribution of breaking-wave heights. For example, γ_s has been assumed constant [*Thornton and Guza*, 1983], a function of beach slope [*Mase and Iwagaki*, 1982], or a function of offshore wave steepness [*Battjes and Janssen*, 1978; *Battjes and Stive*, 1985]. *Whitford* [1988] discusses plausible breaking-wave probability distributions. Observed random wave heights have also been well predicted with models in which breaking-wave dissipation was assumed proportional to the difference between the wave height predicted without dissipation and a depth-dependent "stable" wave height below which breaking ceases in water of constant depth [*Dally and Dean*, 1986; *Dally*, 1990; *Larson*, 1995]. Both the "stable" wave height and the depth at which breaking begins are empirically determined. Predicted values of γ_s in the inner surf zone were shown to be dependent on the beach slope but insensitive to the offshore wave steepness. Some wave transformation models incorporate the effects of "rollers" (the region of turbulent, aerated fluid at the crest of broken waves) and are free of assumptions about γ_s [e.g., *Deigaard et al.*, 1991; *Schäffer et al.*, 1993] but contain other adjustable coefficients.

Laboratory and field observations of random wave transformation in the inner surf zone have been predicted accurately with a numerical model (hereinafter known as Rbreak [see *Wurjanto and Kobayashi*, 1991]) based on the nonlinear shallow water equations with bore dissipation [*Cox et al.*, 1992, 1994; *Raubenheimer et al.*, 1995]. Although the range of wave conditions was limited, these studies showed that Rbreak predicts qualitatively well the evolution of both surf zone wave heights and sea surface elevation spectra across the surf zone without assumptions about the form of γ_s. In the present study, Rbreak is shown to predict accurately the evolution of waves observed across the mid and inner surf zones of three beaches.

The observations and model are described in sections 2 and 3, respectively. In section 4 model predictions are compared to the observations, and in section 5 the effect of infragravity waves on γ_s is discussed.

2. Observations

Pressure fluctuations within the surf zone were measured on a steep concave-up beach, a gently sloped beach, and an approximately flat terrace adjacent to a steep foreshore. At each site, a current meter was collocated with the most offshore pressure sensor. Data from the most onshore sensors were excluded if the sensor was exposed during swash run-down (the shallowest mean depth considered is 0.3 m). The beach profiles did not change significantly.

In October 1993, nine pressure sensors were deployed on a cross-shore transect between the berm and a mean water depth of 1.75 m (about 100 m offshore of the mean shoreline) at San Onofre State Beach (Figure 1a). Beach slopes (measured daily) in the surf zone ranged from 0.01 to 0.09. A sand bar was located offshore of the deepest sensors. Deep water significant wave heights, H_s, measured in approximately 10 m depth a few kilometers southeast of the experiment site, ranged from

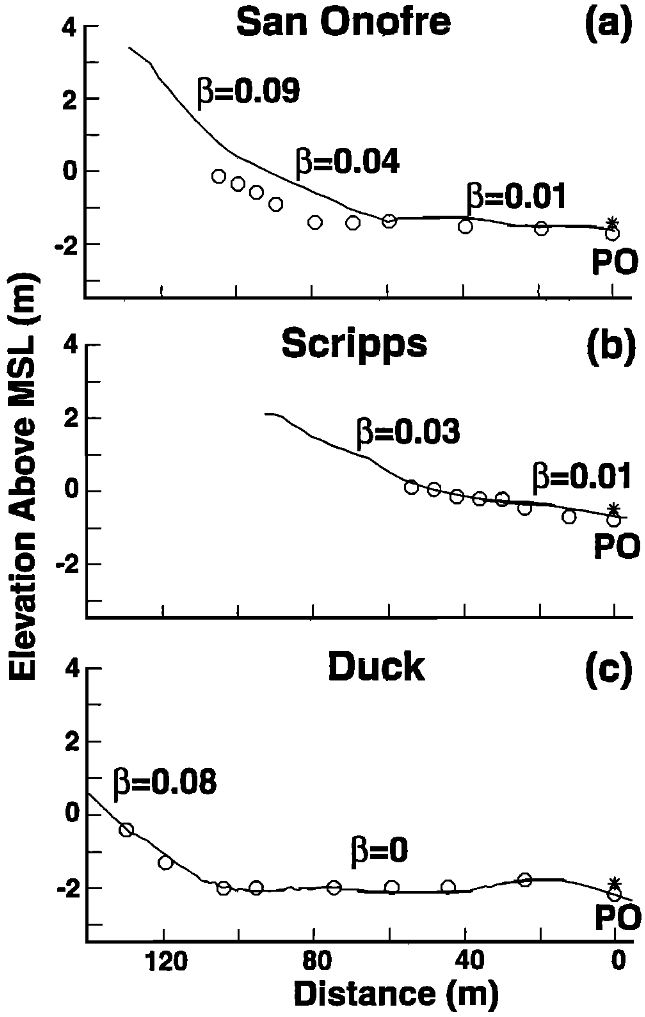

Figure 1. Locations of pressure sensors (circles) and current meters (asterisks), and approximate beach slopes, β, at (a) San Onofre, (b) Scripps, and (c) Duck. The x axis, positive onshore, is zero at the sensor location where the numerical model is initialized.

0.45 to 1.34 m and peak frequencies ranged from 0.06 to 0.09 Hz. During the 10-day experiment, two hundred forty-two 68-min long data runs were acquired at a 2-Hz sample rate.

In June 1989, eight pressure sensors were deployed across the gently sloped Scripps Beach between the shoreline and about 0.80 m mean water depth (about 50 m offshore of the mean shoreline) (Figure 1b). Beach slopes, measured daily, ranged from about 0.03 just offshore of the swash zone to about 0.01 at the most offshore sensors. Deep water significant wave heights measured in approximately 7-m depth ranged from 0.50 to 0.82 m and peak frequencies were approximately 0.10 Hz. Twenty-nine 68-min long data runs were acquired at an 8-Hz sample rate over 5 days. *Holland et al.* [1995] and *Raubenheimer et al.* [1995] describe further the Scripps and San Onofre experiments and discuss run-up and swash observations not considered here.

During two periods of large waves (September 5-6 and 22, 1994), nine pressure sensors were deployed at Duck, North Carolina, along a cross-shore transect extending from the foreshore (slope about 0.08), across a nearly flat terrace (about 2-m depth) and a small sand bar, to about 120 m offshore of the mean shoreline (about 2.10-m mean water depth) (Figure 1c). Deep water significant wave heights measured in about 8-m water depth ranged from 1.20 to 2.80 m and peak frequencies were about 0.12 Hz (runs with smaller offshore wave heights were excluded). The forty-five 68-min-long data runs selected were sampled at 2 Hz.

All data were processed similarly. Time series were quadratically detrended to remove tides and other motions with periods longer than roughly 1 hour and low-pass filtered to include only motions that are consistent with the long wave approximation ($(kh)^2 << 1$) used in the nonlinear shallow water equations. The high-frequency cutoff was 0.18 Hz for all data, and the largest value of $(kh)^2$ at the deepest surf zone sensors was 0.38 ($h/L \leq 0.1$, where L is the wavelength). Consistent with the long wave approximation, sea surface elevations were estimated assuming that the measured pressure field is hydrostatic (see also *Madsen and Svendsen* [1983]). Pressure sensors will be identified by their distance (in meters) from the most offshore pressure sensor (e.g., P0 and P105 are the most offshore and onshore gages at San Onofre (Figure 1a), respectively).

At mid surf zone sensors (P60 at San Onofre, P12 at Scripps, and P45 at Duck) and all shoreward locations, γ_s (based on integrating the energy over the sea swell frequency band, $0.05 < f \leq 0.18$ Hz) was independent of the offshore significant wave height H_s (i.e., saturated) (Figure 2). At Scripps the largest values of γ_s were observed at low tidal levels ($h < 0.8$ m), but no trend with H_s is apparent (Figure 2b).

In contrast to the suggestion of *Nairn* [1990], at sensors where sea surface fluctuations are saturated (Figure 2), γ_s does not vary systematically with the offshore wave steepness, S, the ratio of H_s to the deep water wavelength corresponding to the offshore centroidal frequency (Figure 3; the observations at Duck are representative).

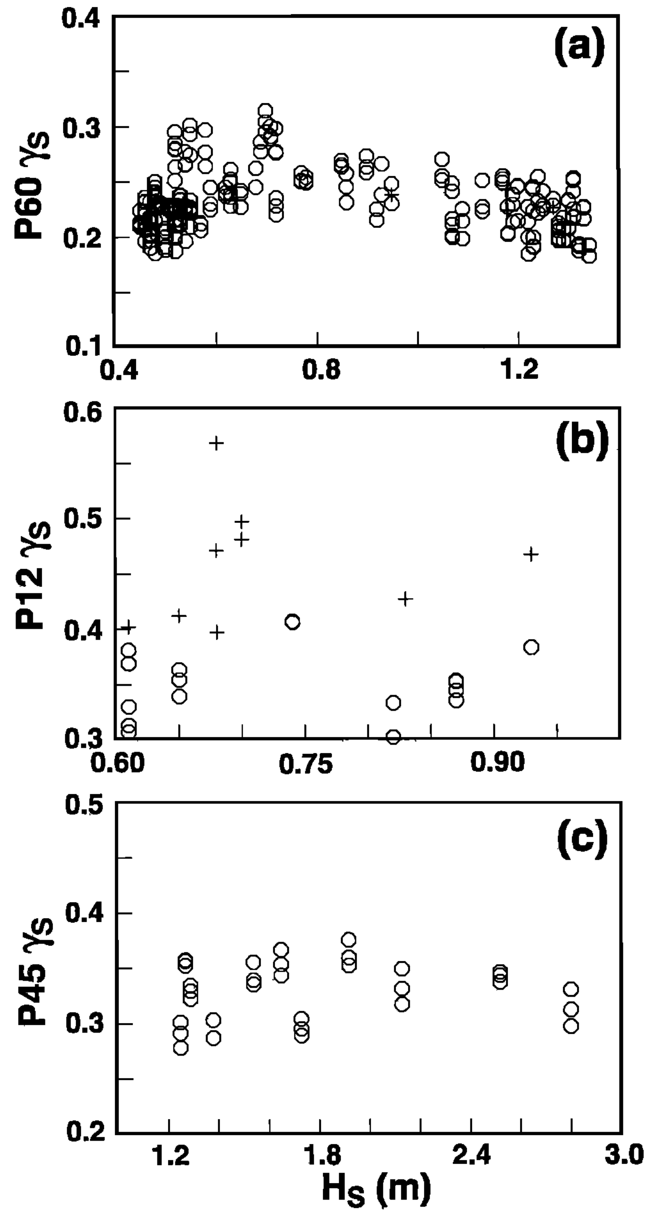

Figure 2. Ratio (γ_s) of significant sea swell wave height to water depth versus deep water significant wave height, H_s, for depths greater than (circles) and less than (pluses) 0.8 m at (a) P60 at San Onofre, (b) P12 at Scripps, and (c) P45 at Duck.

3. Model

Many analytical and numerical models of shallow water waves are based on the one-dimensional depth-averaged nonlinear shallow water equations (here with quadratic bottom friction),

$$\frac{\partial d}{\partial t} + \frac{\partial}{\partial x}(du) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(du) + \frac{\partial}{\partial x}(du^2) = -gd\frac{\partial \eta}{\partial x} - \frac{1}{2}f_c|u|u \tag{3}$$

where t is time, x is the distance onshore from the model seaward boundary, d is the total water depth, η is the deviation from the still water depth, u is the depth-

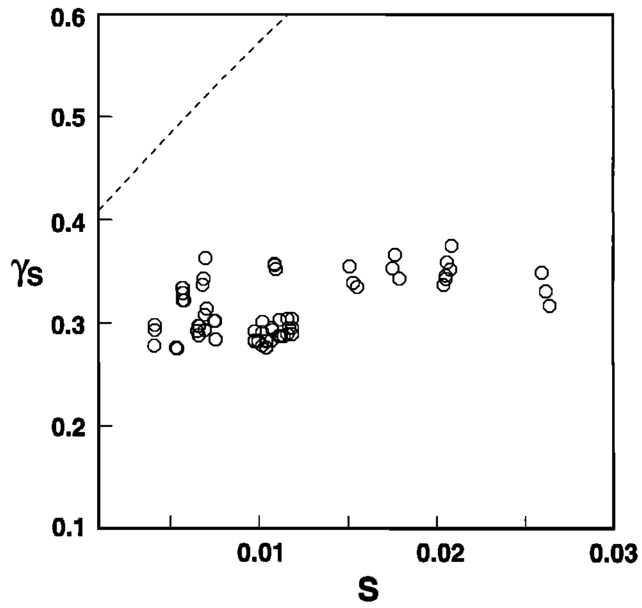

Figure 3. Observed γ_s at P45 at Duck versus offshore wave steepness, S. The dashed line represents the results of *Nairn* [1990].

averaged cross-shore velocity, f_c is a constant empirical friction coefficient, and g is gravitational acceleration [e.g., *Carrier and Greenspan*, 1958; *Shen and Meyer*, 1963]. The theoretical dissipation across a bore (or shock) front derived from (2) and (3) [e.g., *Stoker*, 1947] appears to be a good approximation to breaking-wave dissipation in the surf zone [*Thornton and Guza*, 1982; *Battjes and Stive*, 1985].

Following *Hibberd and Peregrine* [1979] and *Packwood* [1980], Kobayashi and colleagues [e.g., *Wurjanto and Kobayashi*, 1991] developed a numerical model (Rbreak) based on (2) and (3) that predicts the evolution of normally incident random waves propagating over irregular (in the cross-shore direction) bathymetry. Previous studies have shown that wave run-up is insensitive to the empirical bottom friction coefficient f_c in the range $0.01 \leq f_c \leq 0.05$ [*Cox et al.*, 1992; *Raubenheimer et al.*, 1995] and that dissipation owing to bottom friction is negligible in the surf zone [*Kobayashi and Wurjanto*, 1992]. Since waves are assumed nondispersive in (2) and (3), unbroken waves are predicted to steepen and form bores within a couple of wavelengths [*Meyer and Taylor*, 1972] and to continue breaking in the trough (e.g., deeper water) shoreward of a sand bar, contrary to the reduced breaking sometimes observed in troughs in the field. In the present study, Rbreak is initialized within the surf zone where many waves are already broken, and at Duck only runs where waves continued to break across the flat terrace are considered (e.g., $H_s >$ 1.2 m). At steep shock fronts, the bore-capturing Lax-Wendroff finite difference scheme [*Lax and Wendroff*, 1960] applies dissipation approximately equal to the theoretical bore dissipation [e.g., *Ritchmyer and Morton*, 1967; *Meyer and Taylor*, 1972]. One result of the present study is that the wave height decay predicted by Rbreak (based on the assumption of bore-like dissipation) is consistent with field observations. In the model, shoreward propagating wave energy that is not dissipated in the surf or swash zones is reflected at the shoreline (e.g., the model assumes no beach overwash or infiltration). *Kobayashi et al.* [1989] and *Raubenheimer et al.* [1995] discuss further the model assumptions.

Rbreak was initialized at the model seaward boundary ($X = 0$) with the low-pass-filtered observations from the most offshore collocated pressure sensor and current meter [*Raubenheimer and Guza*, 1996]. This method forces the predicted and observed frequency-dependent reflection coefficients to be equal at the seaward boundary. Only the final 51 min of each observed and predicted time series were analyzed to eliminate transients owing to the starting condition (at $t = 0$) of no wave motion in the model domain. On the basis of previous calibrations and model tests [e.g., *Kobayashi et al.*, 1989; *Cox et al.*, 1992; *Kobayashi and Wurjanto*, 1992; *Raubenheimer et al.*, 1995] a friction coefficient $f_c = 0.015$ and a normalized horizontal step size $\Delta x' = \Delta x / T_0\sqrt{gH_0} = 0.01$, where T_0 and H_0 are the centroidal period and significant wave height at the seaward boundary, respectively, and Δx is the dimensional step size, were used for all predictions shown here (the predictions are not sensitive to these values).

4. Model-Data Comparisons

Waves evolve across the surf zone owing to shoaling, dissipation, nonlinear transfer of energy, and reflection. At infragravity frequencies (nominally $0.004 \leq f \leq 0.05$ Hz), run-up and surf zone waves are typically unsaturated, dissipation is small, and reflection from the beach face is large [e.g., *Suhayda*, 1972, 1974; *Huntley*, 1976; *Guza and Thornton*, 1982; *Sallenger and Holman*, 1985], resulting in partial standing waves and cross-shore modulation of spectral levels owing to nodes (e.g., P36 spectral valley at $f \approx 0.025$ Hz in Figure 4b) and antinodes (e.g., P12 spectral peak at $f \approx 0.025$ Hz in Figure 4b). In contrast, at frequencies greater than about 0.05 Hz, wave-breaking-induced dissipation causes spectral levels to decrease nearly monotonically across the surf zone. Rbreak accurately predicts the observed wave spectra at infragravity and sea swell frequencies in the three example cases (Figure 4) and all other runs (not shown). At frequencies greater than 0.18 Hz, the observed wave energy is relatively small (< 20% total energy), and the predicted wave energy at these frequencies is affected by the low-pass filtering of the initial conditions. Therefore all comparisons of predicted and observed γ_s will be for the sea swell frequency band $0.05 \leq f \leq 0.18$ Hz.

The centroid of the frequency spectrum f_s ($0.004 \leq f \leq 0.18$ Hz) decreases in the onshore direction at Scripps (Figure 5a) and San Onofre (not shown) because of the shoaling and reflection of nonbreaking infragravity waves and the dissipation of sea swell waves. Across the flat (e.g., no wave shoaling) terrace at Duck, the onshore decrease of f_s is small. Although sea swell spectral levels change significantly (Figure 4), the centroidal frequency of the sea swell frequency band

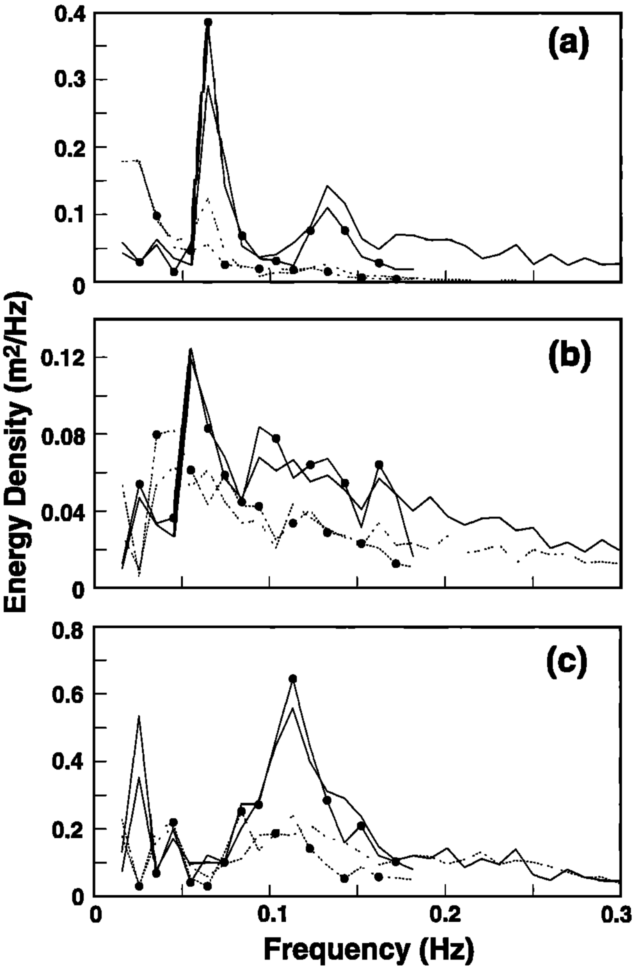

Figure 4. Observed (lines) and predicted (lines with solid circles) sea surface elevation energy density spectra at sensors (a) P60 (solid lines) and P95 (dotted lines) at San Onofre, (b) P12 (solid lines) and P36 (dotted lines) at Scripps, and (c) P45 (solid lines) and P105 (dotted lines) at Duck.

is approximately constant across the surf zone at all three sites (Figure 5c; observations and predictions at Scripps are representative). The centroidal frequency is predicted accurately with errors typically less than $0.2(f_s/f_{s0})_{obs}$ (Figures 5b and 5d). (Although not as statistically stable, cross-shore variations in peak frequency are similar (e.g., Figure 4).)

The average and standard deviation of the predicted and observed values of γ_s, based on the significant wave height and mean water depth (including setup), for all data runs at each site and cross-shore location, x, are shown in Figure 6. Rbreak predicts the individual γ_s for each data run qualitatively well, with errors usually less than 20% of the observed γ_s (Figures 6b, 6d, and 6f). At Duck, γ_s decreases slightly in the onshore direction across the constant-depth terrace (Figure 6e, 45 m $\leq x \leq$ 95 m) owing to continued wave-breaking-induced dissipation initiated over the small sand bar. At the foreshore sensors (x > 60 m at San Onofre, x > 36 m at Scripps, and x > 95 m at Duck) the observed and predicted average and range of γ_s typically increase in the onshore direction (Figures 6a, 6c, and 6e). Consistent with previous observations, the onshore increase of γ_s at sensors where wave heights are depth-limited (e.g., Figure 2) is correlated with increasing beach slope β (Figure 7, with β estimated from the observed profiles as the difference in vertical elevation over a distance equal to the shallow water wavelength at the local sea swell centroidal frequency). The observed trend of γ_s with β is similar to that observed by *Sallenger and Holman* [1985], although for the same β the average γ_s is smaller in the present data set. Differences between these data sets may be caused by different high-frequency cutoffs (e.g., *Sallenger and Holman* [1985] use a cutoff frequency of 0.33 Hz) in the estimates of sea swell wave heights. Also, the data of *Sallenger and Holman* [1985] were collected offshore of a sand bar, whereas some of the present observations (at San Onofre and Duck) were measured onshore of small bars.

The large observed and predicted ranges of γ_s at some cross-shore locations (Figure 6) or beach slopes (Figure 7) are consistent with a systematic increase of γ_s with decreasing (owing primarily to changing tidal levels) nondimensional water depth, kh (Figure 8), where k is the wavenumber at the local sea swell centroidal frequency. The slope of this trend increases as β increases (Figure 8), resulting in larger scatter in γ_s values at larger β (Figures 6 and 7). Even on the low-slope, approximately planar Scripps beach, where previous stud-

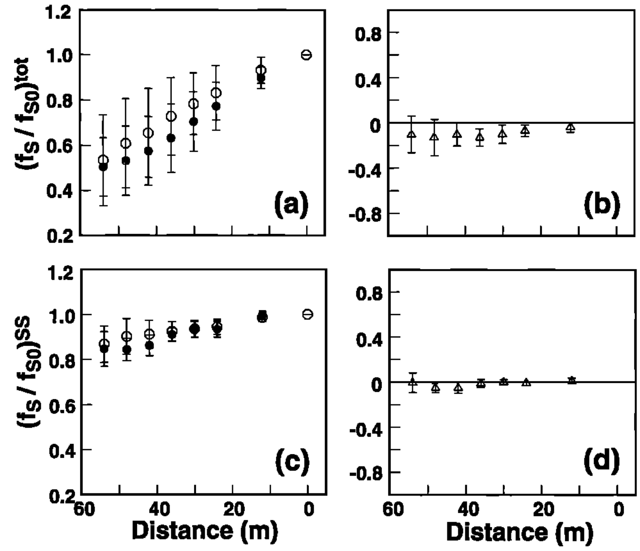

Figure 5. Variation of the observed (open circles) and predicted (solid circles) average centroidal frequency f_s versus cross-shore distance. Standard deviations are indicated by vertical bars. Centroidal frequencies are based on the total ($0.004 \leq f \leq 0.18$ Hz) (Figure 5a) and sea swell ($0.05 < f \leq 0.18$ Hz) (Figure 5c) frequency bands at each sensor and are normalized by the centroidal frequency at the model seaward boundary (f_{s0}). The average (triangles) and the standard deviation of the normalized prediction errors $\{[(f_s/f_{s0})_{pred} - (f_s/f_{s0})_{obs}]/(f_s/f_{s0})_{obs}\}$ versus cross-shore distance are shown for the total (Figure 5b) and sea swell (Figure 5d) frequency bands.

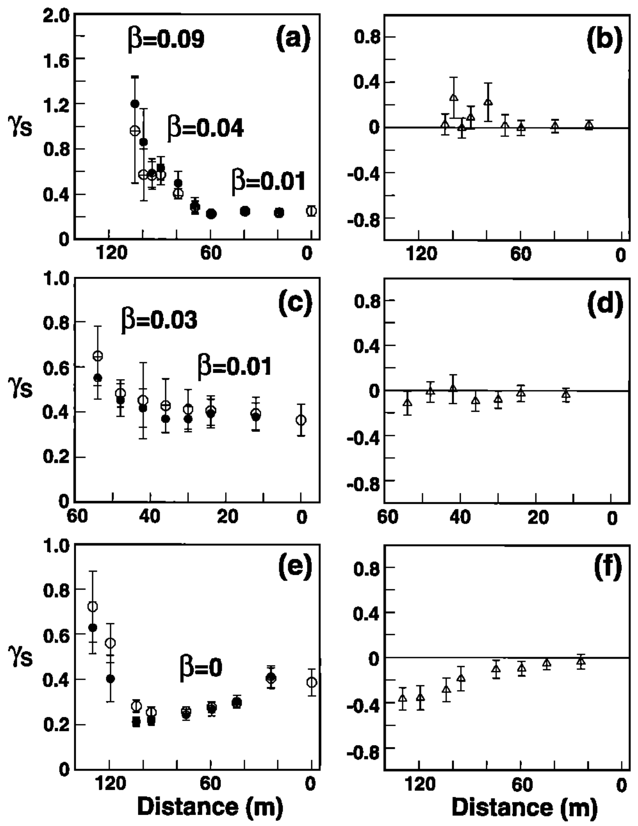

Figure 6. Observed (open circles) and predicted (solid circles) average and standard deviation (vertical bars) of γ_s at each sensor for all runs at San Onofre (Figure 6a), Scripps (Figure 6c), and Duck (Figure 6e). The average (triangles) and standard deviation of normalized prediction errors $\{[(\gamma_s)_{\text{pred}} - (\gamma_s)_{\text{obs}}]/(\gamma_s)_{\text{obs}}\}$ are shown for all runs at San Onofre (Figure 6b), Scripps (Figure 6d), and Duck (Figure 6f).

ies have suggested that γ_s is constant [*Thornton and Guza*, 1982, 1983], a dependence on $1/(kh)$ is observed and predicted (most of the data in Figure 8a are from Scripps). Observed and predicted γ_s from San Onofre and Duck, which appear in all three panels of Figure 8, have similar dependencies on $1/(kh)$.

To account for the dependence of γ_s on kh and β, the fractional change of depth over a wavelength is defined as [e.g., *Mei*, 1989],

$$\frac{\Delta h}{h} = \frac{\beta L}{h} \propto \frac{\beta}{kh} \tag{4}$$

where L is a horizontal length scale proportional to the wavelength. At sensors where wave heights are depth-limited (e.g., Figure 2), the average observed and predicted γ_s increase with increasing β/kh (Figure 9). The linear fit of the unbinned observed γ_s (2288 data points) to β/kh is statistically (at the 99% significance level) better than the fit of the observed γ_s to β and explains 12% more variance (not shown), suggesting

$$\gamma_s = C_0 + C_1\frac{\beta}{kh} \tag{5}$$

where C_0 and C_1 are constants. When β is small, $\gamma_s = C_0$, consistent with the results of *Thornton and Guza* [1982]. For the foreshore data (relatively large β), multiple linear regression of the local significant wave height, $H_{ls} = C_0 h + C_1\frac{\beta}{k}$, shows that H_{ls} is approximately equally dependent on both h and β/k (not shown). In *Sallenger and Holman* [1985], the dependence of γ_s on $(kh)^{-1}$ may not be apparent because the range of depths observed at any one beach slope was small. The dependence of γ_s on β/kh suggests that large wave height to depth ratios are associated with rapidly changing depths and the largest γ_s values (greater than 1.0) correspond to shore breaks.

Some of the scatter in Figure 9 is caused by a dependence of γ_s on the offshore bathymetry that is not accounted for by the simple β/kh parameterization (which depends only on local variables). For example, γ_s decreases across the flat terrace ($\beta/kh \approx 0$) at Duck owing to continued wave breaking initiated over the small sand bar (Figure 6). Furthermore, this parameterization is not valid for negative values of β shoreward of a bar crest.

5. Discussion

It has been suggested that infragravity frequency waves affect γ_s [*Goda*, 1975; *Mase and Iwagaki*, 1982; *Dally and Dean*, 1986; *Dally*, 1990; *Nelson and Gonsalves*, 1992], especially in the inner surf zone where fluctuations in cross-shore currents and water levels at

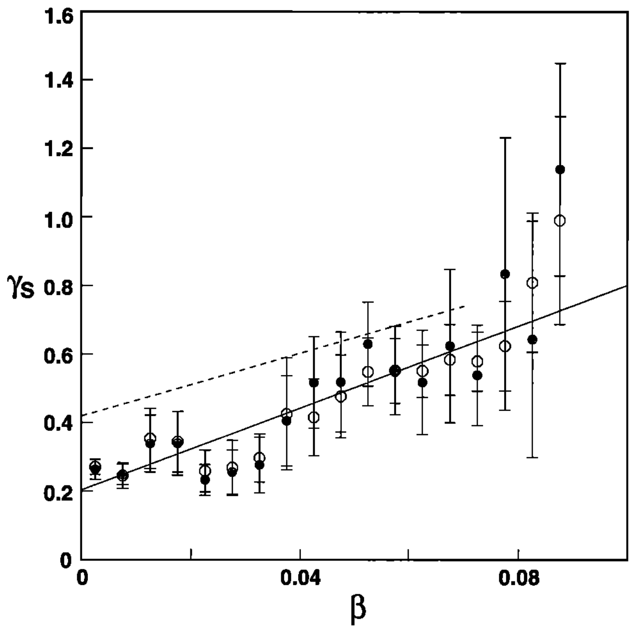

Figure 7. Observed (open circles) and predicted (solid circles) average and standard deviation (vertical bars) of γ_s versus beach slope (β) for all runs, all three sites, and all sensors where waves are saturated (e.g., Figure 2). The data are binned corresponding to $\beta\pm$ 0.0025. The least squares linear fit (solid line) and 95% confidence interval to the observed average γ_s, weighted by the number of points in each bin, are γ_s = $(0.20 \pm 0.10) + (5.98 \pm 0.79)\beta$, correlation coefficient = 0.77. The dashed line is the trend suggested by *Sallenger and Holman* [1985].

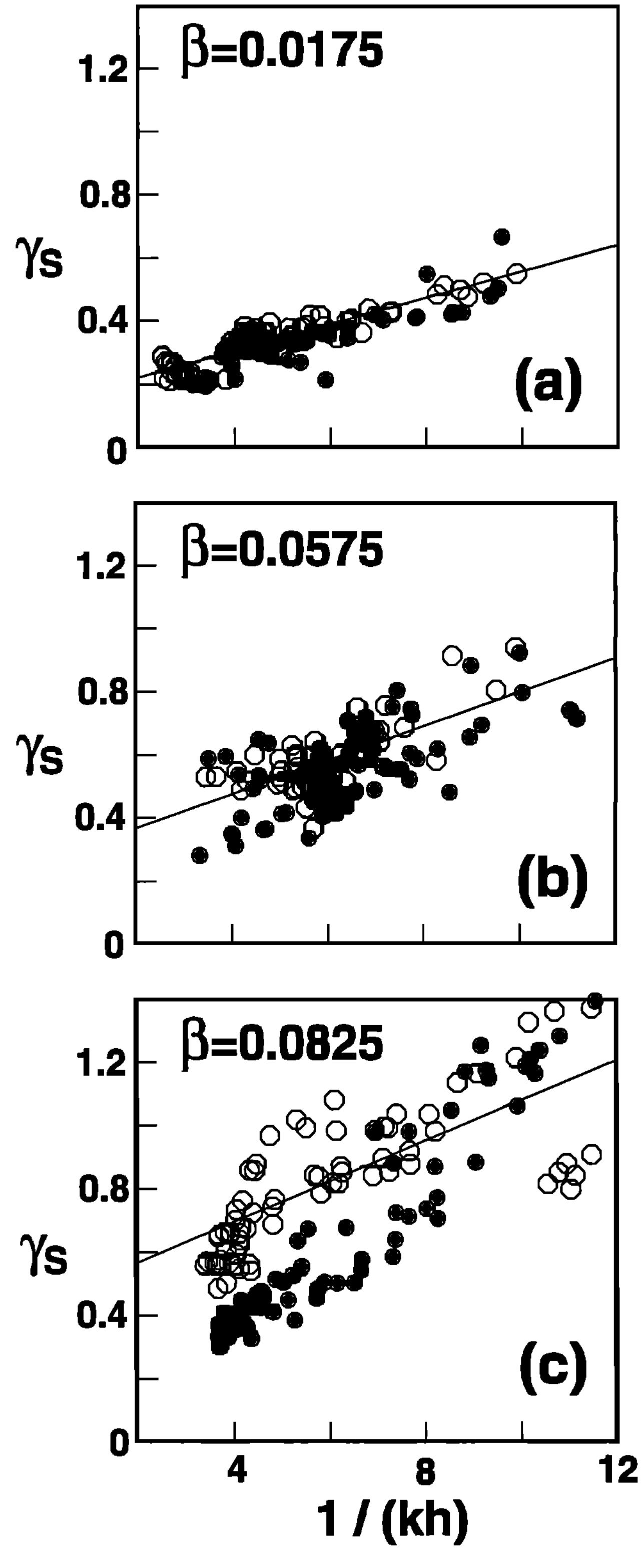

Figure 8. Observed (open circles) and predicted (solid circles) γ_s versus $1/(kh)$ (the local wavenumber, k, corresponds to the local centroidal frequency and depth) for beach slopes ($\pm$ 0.0025) (a) $\beta = 0.0175$, (b) $\beta = 0.0575$, and (c) $\beta = 0.0825$. Least squares linear fits (solid lines) and 95% confidence intervals to the observations are $\gamma_s = (0.14 \pm 0.02) + (0.04 \pm 0.003)(kh)^{-1}$, correlation coefficient r = 0.92 (Figure 8a); $\gamma_s = (0.26 \pm 0.09) + (0.05 \pm 0.01)(kh)^{-1}$, r = 0.66 (Figure 8b); and $\gamma_s = (0.44 \pm 0.08) + (0.06 \pm 0.012)(kh)^{-1}$, r = 0.73 (Figure 8c).

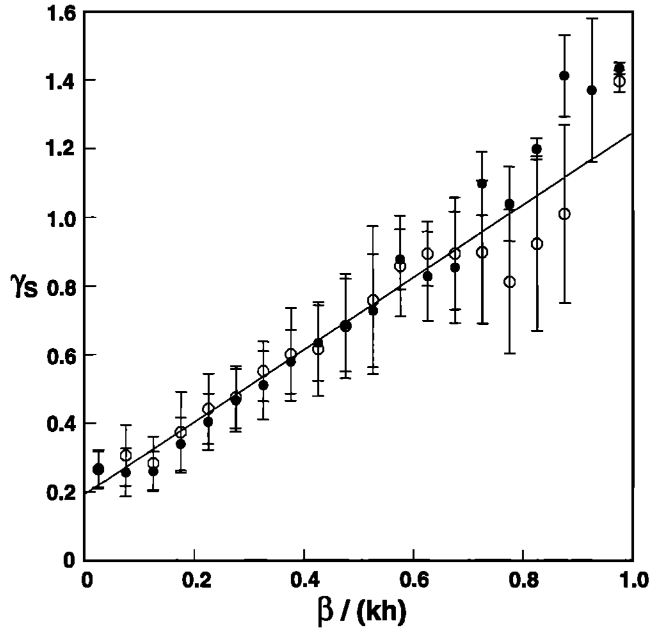

Figure 9. Observed (open circles) and predicted (solid circles) average and standard deviation (vertical bars) of γ_s versus normalized beach slopes, β/kh, for all runs, all three sites, and all sensors where waves are saturated (Figure 2). The data are binned corresponding to $\beta/kh \pm$ 0.025. The least squares linear fit (solid line) and 95% confidence interval to the observed average γ_s, weighted by the number of points in each bin, are $\gamma_s = (0.19 \pm 0.09) + (1.05 \pm 0.15)\beta(kh)^{-1}$, correlation coefficient = 0.87.

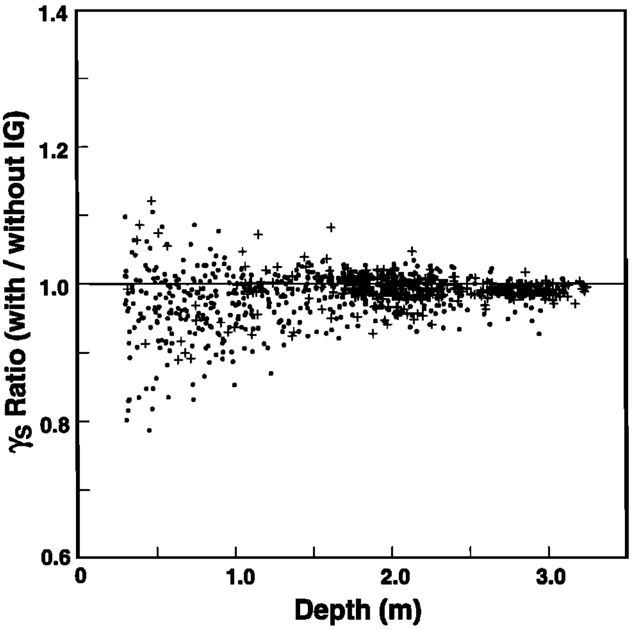

Figure 10. Ratio of γ_s predicted by the numerical model initialized with the total ($0.004 \leq f \leq 0.18$ Hz) sea surface and velocity fluctuation time series to γ_s predicted by the model initialized with band-passed ($0.05 < f \leq 0.18$ Hz) time series versus water depth for San Onofre (solid circles, $0.45 \leq H_s \leq 1.34$ m) and Duck (pluses, $1.2 \leq H_s \leq 2.8$ m) data.

infragravity frequencies may be significant fractions of the sea swell phase speed and mean water depth, respectively. The effect of infragravity waves on γ_s was explored by initializing Rbreak with time series of sea surface elevation and cross-shore currents band-pass filtered (0.05 Hz $< f \leq$ 0.18 Hz) to remove infragravity energy. Differences between the resulting predictions of γ_s and predictions of γ_s with infragravity and sea swell energy included in the model initial conditions (Figure 10) are ascribed to the influence of infragravity waves. Although the model does predict nonlinear generation of infragravity energy at shoreward locations, removing infragravity energy from the initial conditions reduces predicted infragravity wave heights at shoreward locations by at least 50% and typically by more than 70%. Predictions of γ_s are affected by changes in dissipation and nonlinear energy transfers. The absence of infragravity energy in the model initial conditions for the San Onofre and Duck data sets causes (according to the numerical model) only small changes (either positive or negative) in γ_s, with the largest changes (O(20%)) occurring in shallow water. Although infragravity energy levels in the surf zone typically increase as offshore wave heights increase, the predicted effect of infragravity energy on γ_s is no larger at Duck (1.20 m $\leq H_s \leq$ 2.80 m) than it is at San Onofre (0.45 $\leq H_s \leq$ 1.34 m) (Figure 10; compare pluses and solid circles). Thus, at least for the wave conditions considered here, the simulations suggest γ_s is not greatly affected by infragravity waves.

6. Conclusions

Wave evolution observed along transects spanning the mid and inner surf zone on three beaches with different bathymetries is accurately predicted by a numerical model based on the one-dimensional nonlinear shallow water equations with bore dissipation. In particular, the model predicts accurately the cross-shore standing wave energy structure at infragravity frequencies and the rate at which breaking-induced dissipation reduces energy across the surf zone at sea swell frequencies (Figure 4). The observed ratio, γ_s, of sea swell significant wave height to the mean water depth is predicted accurately with errors typically less than 20% (Figure 6). The observed and predicted γ_s increase with increasing beach slope β (Figures 6 and 7) and decreasing normalized water depth kh (Figure 8) and are well correlated with β/kh (Figure 9), a measure of the fractional change in water depth over a wavelength. Although potentially useful, this (and other) parameterization of γ_s must be applied carefully. For example, in constant water depth, $\beta/kh = 0$, so the parameterization suggests a constant γ_s. However, continued breaking across a flat terrace reduces the observed γ_s, as predicted by the numerical model (Figure 6e).

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