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PALEOCLIMATIC ATTRACTORS: NEW DATA, FURTHER ANALYSIS

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Analysis of a high resolution record of climate change spanning the past 2.6 million years suggests that the dynamics of the climate is not high dimensional, similar to previous results using much shorter records [Nicolis & Nicolis 1984, Vautard & Ghil 1989]. The correlation dimension is significantly less than the dimension of a linear, Gaussian process with the same power spectrum (i.e., phase-randomized surrogate data) as the climate record. The climatic correlation dimension is similar to the dimension of a model of the solar forcing of climate, suggesting the possibility of a linear climate-system response to orbital forcing, consistent with frequency-domain analysis results [Hays *et al.*, 1976, Imbrie & Imbrie, 1980]. Consequently, inferring an inherently nonlinear (possibly chaotic) climate system without considering the nonlinear (possibly chaotic) insolation forcing dynamics could be misleading.

1. Introduction

Since the seminal work of Hays *et al.* [1976], statistical analysis of Pleistocene sedimentary records has confirmed a strong link between motions of the Earth's orbit and glacial-interglacial climatic fluctuations [Imbrie & Imbrie, 1980; Imbrie *et al.*, 1984, Imbrie *et al.*, 1989, and references therein]. Although most of these studies have suggested that many aspects of Pleistocene climate are consistent with a linear response to orbitally-induced changes in radiation, the complexity of proxy climate records (e.g., deep-sea cores) has led others to investigate the nonlinear dynamics of the paleoclimatic record [Nicolis & Nicolis, 1984; Grassberger, 1986; Vautard & Ghil, 1989]. These studies have sought to explain climate change on millenial time scales as the result of chaos in the climate system, and thus have attempted to determine features of a "paleoclimatic attractor," in particular the attractor's dimension. The present study investigates recently obtained, high resolution records of climate change spanning the past 2.6 million years [Shackleton & Hall, 1989; Shackleton *et al.*, 1990], which provide better estimates of the climatic correlation dimension than previously available. None of the previous investigations of the paleoclimatic dimension compared the dynamics of the observed climate records (i.e., the earth's response) to those of the theoretical solar insolation forcing. The climatic dimension obtained here is approximately the same as that obtained from theoretical time series of solar insolation [Berger, 1978, 1988, and references therein]. Thus, the climate record is not necessarily inconsistent with a linear response to nonlinear solar insolation forcing.

Knowledge of the climatic dimension can be useful in the development of climate models because it provides constraints on the number of independent variables. Nicolis & Nicolis [1984] found the dimension, $D \sim 3.1$. However, further analysis of the same paleoclimate records ($\delta^{18}O$) suggested that the data were insufficient to draw such conclusions [Grassberger, 1986; Vautard & Ghil, 1989]. Grassberger [1986] found no sign of a low dimensional attractor, that D was not statistically different from the dimension of a purely random time series, and concluded that the low dimension estimated was an artifact of the sparseness of the data (only 184 raw data points). Vautard & Ghil [1989] analyzed the same data and showed that about 10 degrees of freedom were significant, but owing to the short record length and disturbances (noise) in the data, reliable estimates of D were not possible.

The present study differs from previous investigations in several respects. First, by analyzing the longer (1298 points), higher resolution record of $\delta^{18}O$ from ODP Site 677 [Fig. 1(a)] [Shackleton & Hall, 1989; Shackleton *et al.*, 1990], problems associated with extremely short and/or noisy time series are reduced. Although the longer record analyzed here is still too short to precisely estimate the dimension [Lorenz, 1991, references therein, and many others] comparison with dimension estimates of phase randomized versions of the original data (surrogate data) imply that the observed low dimensionality is not an artifact. Second, comparison of the correlation dimension of the $\delta^{18}O$ record to the dimension of the solar insolation record provides the opportunity to determine if the earth's climate is inherently nonlinear (possibly chaotic) or is a linear response to nonlinear (possibly chaotic) insolation forcing. Although previous studies of time series of paleoclimate have compared changes in the earth's insolation to glacial-interglacial climate change [Imbrie & Imbrie, 1980; Imbrie *et al.*, 1984; Imbrie *et al.*, 1989, and references therein], no studies of climatic dimension have compared the earth's climatic response to insolation forcing. Conclusions about the earth's climate (and the corresponding

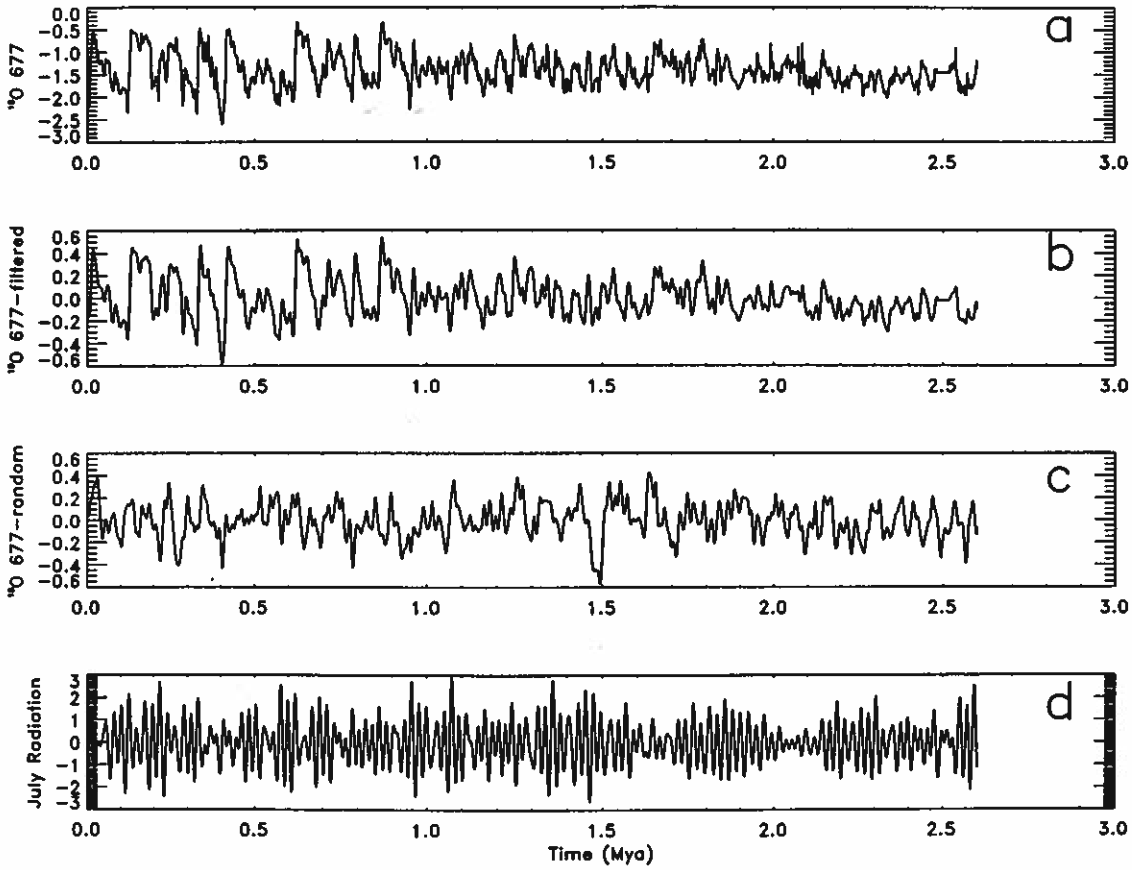

Fig. 1. (a)–(c) $\delta^{18}O$ (arbitrary units) versus time. (a) ODP site 677 data; (b) low pass filtered ODP Site 677 data; (c) a linear, Gaussian time series constructed by combining the power spectrum of ODP Site 677 data with random Fourier phases; (d) July solar insolation from Berger [1988] (arbitrary units).

implications for climate models) based solely on the earth's response without consideration of the forcing may be misleading.

2. Results

The data examined in the present study are $\delta^{18}O$ from planktic foraminifera from eastern equatorial Pacific ODP Site 677 [Shackleton & Hall, 1989; Shackleton *et al.*, 1990] [Fig. 1(a)]. These data approximate global ice volume. The record is 2.6 Myr long, and has an average sampling interval of 2000 years. The combination of approximately four times higher sedimentation rates at site 677 [Shackleton & Hall, 1989] and advanced coring technology reduces noise in the record relative to core V28-239, the data examined by Nicolis & Nicolis [1984], Grassberger [1986], and Vautard & Ghil [1989]. The solar insolation record [Fig. 1(d)] consists of insolation values for July at 65°N [Berger, 1978] sampled identically to the Site 677 isotope data.

The Grassberger–Procaccia correlation dimension [Grassberger & Procaccia, 1983] was calculated from state space dynamics geometrically reconstructed from the time series using the method of time delays [Packard *et al.*, 1980]. A time delay of 16 was selected using standard techniques, and the reconstructed phase spaces had dimensions ("embedding dimension") ranging from 2 to 12. The dimension estimates presented here were not sensitive to changes in the details of the algorithm parameters.

The correlation dimension of the $\delta^{18}O$ record as a function of embedding dimension is shown in Fig. 2. Also shown in Fig. 2 are the dimensions for a low-pass filtered $\delta^{18}O$ record, obtained by applying an acausal 3rd-order Butterworth filter to the $\delta^{18}O$ record, effectively removing all energy with periods shorter than 10,000 years. The filtered time series is shown in Fig. 1(b). The acausal filter was chosen such that it did not effect the underlying dimension of the time series [Mitschke, 1990]. Although precise determination of D for dimensions close to 6 requires many more data points than available, in both cases the correlation dimension appears to be approaching a value of about 6 (i.e., $D \sim 6$ within the confidence limits of the calculations).

The slow increase of D with embedding dimension for embedding dimensions greater than about 8 (Fig. 2) is significantly different from that which occurs for a linear, white noise process, which is characterized by a D that linearly increases with embedding dimension (Fig. 2). To verify that the observed low dimensionality is different from a linear, Gaussian process with the same power spectral shape as the data, surrogate data methods were used. Specifically, correlation dimensions for 30 phase-randomized time series (with power spectra identical to the spectrum of the site 677 data) were calculated [one random phase time series is shown in Fig. 1(c)]. For higher embedding dimensions (> 8) the observed dimensions from the site 677 data are significantly below those of the random phase simulations (Fig. 2).

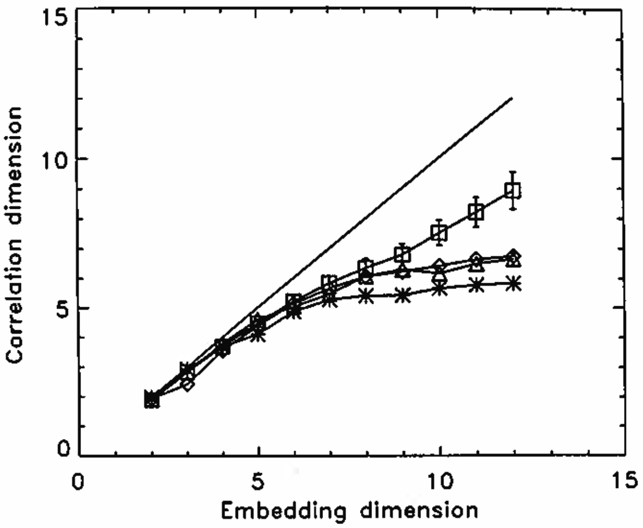

Fig. 2. Grassberger–Procaccia dimension verus embedding dimension. Diamonds, ODP Site 677 data; triangles, low-pass filtered ODP Site 677 data; squares, mean dimension from the 30 random phase data sets (the bars indicate ±1 standard deviation); stars, July insolation; solid line without symbols, theoretical dimension for an infinitely long, perfectly sampled, purely random white noise process.

Previous studies have suggested that the Plio-Pleistocene climate is consistent with a linear response to long term, orbitally induced changes in solar insolation [Hays *et al.*, 1976; Imbrie & Imbrie, 1980; Imbrie *et al.*, 1984, Imbrie *et al.*, 1989]. For example, the energetic 100 kyr fluctuations in the climate record may be the result of a resonant (e.g., linear) response to the solar forcing [Hagelberg *et al.*, 1991]. Although the 677 isotope record shows evidence of nonlinear coupling between oscillations corresponding to the periods of Milankovitch cycles [Hagelberg *et al.*, 1991], the orbital forcing itself contains similar nonlinear coupling. Thus, previous studies have suggested that the 677 record is not inconsistent with a linear response to nonlinear insolation forcing. To investigate this further, the correlation dimension of a time series of July insolation forcing [Berger, 1978] [Fig. 1(d)] was also calculated. The July insolation time series was low pass filtered and processed in the same manner as the 677 data, and the corresponding D is very similar to the D for the 677 data (Fig. 2). The correlation dimension was also calculated for a 20,000 point time series of July insolation obtained by extending the algorithm that generates the insolation time series further back in time. Correlation dimensions calculated from this much longer record are less susceptible to possible artifacts associated with short records. The curves of D versus embedding dimension for the long record of July insolation (not shown) were not significantly different from those obtained from the 2.6 Myr record shown in Fig. 2. Since the insolation forcing is the result of the interactions of six orbital parameters [Berger 1988], $D \sim 6$ is not surprising. Moreover, the similarity of the correlation dimensions of the 677 data and the July insolation could be consistent with a linear response of the climate to the nonlinear insolation forcing.

3. Conclusions

Analysis of a high resolution, 2.6 Myr record of climate ($\delta^{18}O$ from planktic foraminifera from eastern equatorial Pacific ODP Site 677), including comparison to random phase versions of the observations, suggest the Pleistocene climate has low dimensionality. Moreover, although not necessarily precise owing to the relatively short data record, the estimated dimension is similar to that from a model of the solar forcing of climate. This result is not inconsistent with a linear response of climate to the insolation forcing, similar to previous findings [Hays *et al.*, 1976; Imbrie & Imbrie, 1980; Imbrie *et al.*, 1984; Imbrie *et al.*, 1989; Hagelberg *et al.*, 1991]. One implication is that studies which draw conclusions about the earth's climate system (e.g., it may be inherently chaotic) without simultaneously considering the forcing of the climate system may be misleading.

Acknowledgments

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