JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 97, NO. C10, PAGES 15,573–15,577, OCTOBER 15, 1992

Observations of Infragravity Waves

STEVE ELGAR[1], T.H.C. HERBERS[2], MICHELE OKIHIRO[2],
JOAN OLTMAN-SHAY[3], and R. T. GUZA[2]

Infragravity-wave (periods of one-half to a few minutes) energy levels observed for about 1 year in 8-m water depth in the Pacific and in 8- and 13-m depths in the Atlantic are highly correlated with energy in the swell-frequency band (7- to 20-s periods), suggesting the infragravity waves were generated locally by the swell. The amplification of infragravity-wave energy between 13- and 8-m depth (separated by 1 km in the cross shore) is about 2, indicating that the observed infragravity motions are dominated by free waves, not by group-forced bound waves, which in theory are amplified by an order of magnitude in energy between the two locations. However, bound waves are more important for the relatively few cases with very energetic swell, when the observed amplification between 13- and 8-m depth of infragravity-wave energy was sometimes 3 times greater than expected for free waves. Bispectra are consistent with increased coupling between infragravity waves and groups of swell and sea for high-energy incident waves.

1. INTRODUCTION

Motions in the infragravity-frequency band (frequencies lower than the incident sea and swell) are important for many nearshore processes. Previous studies have shown that infragravity- and incident-wave energy levels are correlated, and that waves in the infragravity band may be either freely propagating (leaky waves radiating to or from deep water and edge waves) or bound (forced secondary waves nonlinearly coupled to groups of incident waves [*Hasselmann*, 1962; *Longuet-Higgins and Stewart*, 1962]). Very close to shore, within the surf zone, low-mode edge waves dominate the longshore velocity field [*Huntley et al.*, 1981], sometimes contributing well over half the total infragravity energy [*Oltman-Shay and Guza*, 1987]. Although low-mode edge waves are also detectable in the cross-shore velocity and pressure fields in the surf zone, other motions contribute substantially [*Suhayda*, 1974; *Huntley*, 1976; *Holman*, 1981; *Howd et al.*, 1991; and others]. Phase coupling between infragravity and incident waves suggest some local forcing of infragravity waves in the surf zone [*Guza et al.*, 1984; *Huntley and Kim*, 1984; *Elgar and Guza*, 1985; *List*, 1986]. Less comprehensive observations well seaward of the surf zone show that the relative contributions to the total energy by different types of infragravity motions vary with offshore distance. In particular, low-mode edge waves may become less important with increasing offshore distance because they are trapped close to shore. In 40-m water depth, a few hundred kilometers from shore in the North Sea, analysis of a few hours of high-energy sea-surface elevation data suggested that bound waves can dominate the infragravity band [*Sand*, 1982]. Bound waves also have been shown to contribute as much as 50% of the infragravity energy in pressure measurements made close to shore (within 1 km) in the Pacific Ocean in mean depths of about 10 m [*Okihiro et al.*, 1992]. Bound-wave predictions in both these studies were qualitative since detailed measurements of incident wave directional properties were not available. Preliminary results from measurements made with an array of pressure sensors in 8-m depth suggest that high-mode edge waves sometimes dominate the infragravity-band energy [*Elgar et al.*, 1989; *Oltman-Shay et al.*, 1989]. Thus, some previous studies outside the surf zone have concluded that high-mode edge waves contribute the majority of the infragravity energy, while others suggest that bound waves are dominant. The causes of this apparent variability in the relative contributions of different types of infragravity wave motions are unknown.

In this study, long-term observations of infragravity waves in 8- and 13-m depths, offshore of Duck, North Carolina, are presented. As in previous studies, the total infragravity and incident-swell energy are strongly correlated. The amplification of infragravity-wave energy between 13- and 8-m depth is compared to the theoretical amplification for free leaky surface-gravity waves and group-forced bound waves. The results show that bound-wave contributions to the infragravity band are small except for the relatively few cases with very energetic swell. These observations may provide useful constraints on models of infragravity wave generation now under development.

2. OBSERVATIONS

Data were collected with bottom-mounted pressure sensors 24 hours/day for 3 months and 12 hours/day for 6 months in 8- and 13-m water depth (about 1 and 2 km from shore, respectively) between September 1990 and June 1991 at the U.S. Army Corps of Engineers Field Research Facility, Duck, North Carolina. The field site is located near a sandy beach along a barrier island with no nearby headlands or inlets [*Birkemeier et al.*, 1981]. Additional data were obtained from a bottom-mounted pressure sensor in 8-m depth near the mouth of a small harbor close to Barber's Point, Hawaii. The Barber's Point data were collected for 9 hours/day for more than 1 year [*Okihiro et al.*, 1992]. All three data sets were subdivided into 85-min records, detrended to remove tides, tapered, and Fourier transformed to produce spectral estimates, $E(f)$, with a final frequency resolution of 0.0078 Hz and 80 degrees of freedom. The pressure spectra were converted to sea-surface elevation spectra in the swell and sea frequency range ($0.04 < f < 0.30$ Hz, where f is the frequency) using linear theory. The 0.04 Hz division between swell and infragravity energy is intended to insure infragravity wave estimates were not contaminated by long-period swell (for further discussion, see *Okihiro et al.* [1992]).

Mean frequencies, corresponding to the centroid of the power spectrum,

$$\int_{0.04Hz}^{0.3Hz} fE(f)df \Big/ \int_{0.04Hz}^{0.3Hz} E(f)df$$

---

[1] Electrical Engineering & Computer Science, Washington State University, Pullman.

[2] Scripps Institution of Oceanography, La Jolla, California.

[3] QUEST Integrated Inc., Kent, Washington.

Paper number 92JC01316.
0148-0227/92/92JC-01316$05.00

and total energies of the incident waves at Duck ranged from 0.08 to 0.24 Hz, and from 25 to 11,000 cm^2, respectively (Figure 1*a*). This is somewhat larger than the range at Barber's Point, Hawaii, where low-frequency swell was predominant (Figure 1*b*). The gauges were well outside the surf zone except for a few occasions at Duck with very energetic waves, when the gauge in 8-m depth was within the surf zone. The maximum significant wave height in 13-m depth at Duck was 4.2 m. The ratio of wave height to water depth indicates that wave breaking also occurred at the 13-m depth gauge during the largest wave events.

Infragravity energy (E_{ig}, defined as the energy in the range $0.004 < f < 0.04$ Hz) and total incident-wave energy (E_{tot}, defined as the energy in the range $0.04 < f < 0.30$ Hz) are strongly correlated at both field sites (Figure 2*a*), suggesting that the low-frequency waves are primarily locally driven (as opposed to arriving from remote locations with different incident-wave energy). As shown in Table 1, the correlation of E_{ig} with swell energy (E_{swell}, defined as the energy in the range $0.04 < f < 0.14$ Hz, Figure 2*b*) is significantly higher than the correlation with sea energy (E_{sea}, defined as the energy in the range $0.14 < f < 0.3$ Hz, Figure 2*c*) or with E_{tot} (Figure 2*a*). A stronger infragravity response to swell than to sea is consistent with bound-wave theory [*Hasselmann*, 1962; *Longuet-Higgins* and *Stewart*, 1962], and has been observed previously [*Middleton et al.*, 1987; *Nelson et al.*, 1988; *Okihiro et al.*, 1992].

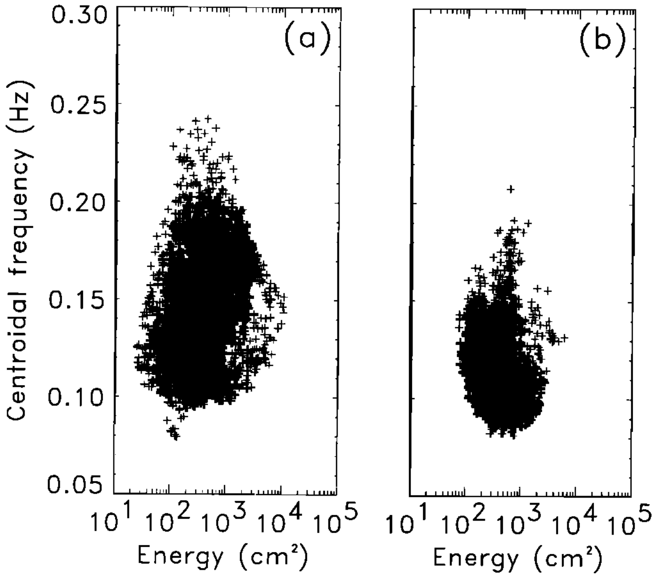

Fig. 1. Centroidal frequency versus total sea-surface elevation energy for the frequency range 0.04 to 0.3 Hz. (*a*) Duck, North Carolina; (*b*) Barber's Point, Hawaii. The depth at both locations was approximately 8 m.

If the infragravity motions were bound waves, nonlinearly driven by groups of swell, then $E_{ig} \propto E^2_{swell}$ [*Hasselmann*, 1962; *Longuet-Higgins* and *Stewart*, 1962]. The constant of proportionality depends on the water depth and the frequency-directional spectrum of the incident waves, $E(f,\theta)$ (θ is the direction relative to the beach normal), but bound-wave energy is expected to increase nonlinearly with swell energy. However, fitting power laws to the observed relationship between E_{ig} and E_{swell} yields $E_{ig} \propto E^{1.4}_{swell}$ at Barber's Point, while the exponents are 1.0 and 0.9 in 8- and 13-m depths, respectively, at Duck (Figure 2*b*). The observed large deviations from a quadratic dependence of E_{ig} on E_{swell} suggest that motions other than bound waves contribute significantly to infragravity-band energy at both sites.

As shown in Figure 3, the total infragravity energy in 8- (E_{ig_8}) and 13-m ($E_{ig_{13}}$) depths are highly correlated. A least squares fit between log E_{ig_8} and log $E_{ig_{13}}$ yields $E_{ig_8} = 1.7\, E^{1.1}_{ig_{13}}$, a nearly linear dependence. The theoretical ratio, R, between E_{ig} in 8- and 13-m depths for bound waves is markedly different from the observed ratio. Neglecting alongshore depth variations, bound-wave energy forced by unidirectional, normally-incident long waves is proportional to h^{-5} [*Longuet-Higgins* and *Stewart*, 1962], which for the depths of 8 and 13 m yields $R = 11$ (upper dashed line in Figure 3). On the other hand, for normally incident free (leaky) surface-gravity waves the amplification in shallow water is [e.g., *Eckart*, 1951] $h^{-1/2} = 1.3$ (lower dashed line in Figure 3). Directional and finite-depth effects change these limiting values of R only slightly, as demonstrated by calculations based

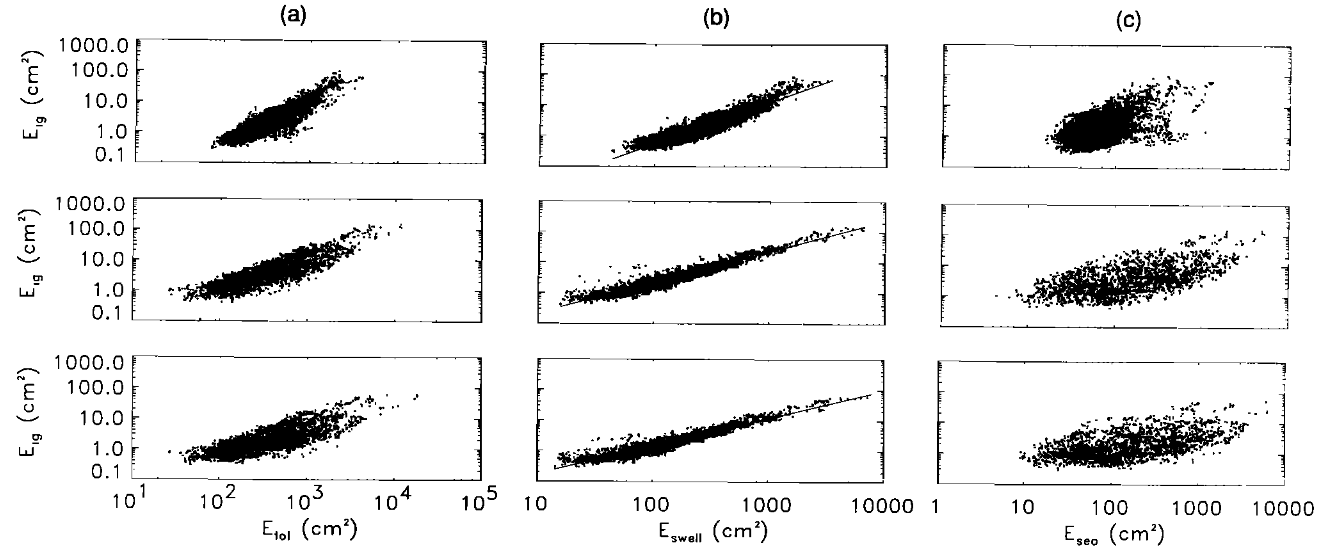

Fig. 2. Infragravity-band energy, E_{ig} ($0.004 < f < 0.04$ Hz) versus (*a*) total incident-wave energy E_{tot} ($0.04 < f < 0.3$ Hz); (*b*) incident-swell energy E_{swell} ($0.04 < f < 0.14$ Hz); and (*c*) incident-sea energy, E_{sea} ($0.14 < f < 0.3$ Hz). The top panels are Barber's Point; the middle panels are Duck, 8-m depth; and the lower panels are Duck, 13-m depth. The lines through the data on panels *b* are least squares fits to the logarithm of the data. The correlation coefficients for each panel are listed in Table 1.

TABLE 1. Number of Data Runs and Correlation Coefficients for the Three Data Sets

Data Set	Number of Runs	E_{ig} Versus E_{tot}	E_{ig} Versus E_{swell}	E_{ig} Versus E_{sea}
Barber's Point	3644	0.90	0.93	0.60
Duck, 8 m	2154	0.87	0.96	0.63
Duck, 13 m	2154	0.82	0.95	0.58

on more general nonlinear [*Hasselmann*, 1962] and linear [e.g., *Collins*, 1972] theories (Figures 4*a* and 4*b*, respectively). For incident-wave directional spreads typical of those observed at Duck, the values of R predicted for bound waves ($5 < R < 10$, Figure 4*a*) are much larger than the values of R for leaky waves ($1.0 < R < 1.3$, Figure 4*b*). For edge waves with a turning point offshore of both measurement locations, $R \sim 1$. However, seaward of their turning point, edge-wave amplitudes decay exponentially [*Eckart*, 1951]. Consequently, if the edge-wave turning point occurs between 8- and 13-m depth, then the edge-wave energy will be much greater in 8- than in 13-m depth, with a correspondingly large value of R. Thus, small values of R ($R = 0(1)$) suggest that bound waves are not important, whereas large values of R ($R = 0(10)$) suggest that either bound wave contributions are important or that a significant fraction of the infragravity-wave energy observed in 8-m depth is trapped between 8- and 13-m depth and does not contribute to E_{ig} in 13-m depth.

The overall average value of R, defined as the mean value over all data runs of the observed ratio between E_{ig} in 8- and 13-m depth is approximately 1.8, suggesting E_{ig} is not generally dominated by bound waves. On the other hand, for data sets with energetic infragravity waves, R is higher (Figure 3). Moreover, the energy ratios for individual frequency bands, $R(f)$, are often larger than 1.8. To investigate the variation of $R(f)$, the data were subdivided into five groups according to the swell energy. At all frequencies below 0.06 Hz, $R(f)$ systematically decreases with decreasing swell energy, as shown in Figures 5 and 6. Additionally, $R(f)$ is largest in the 0.03 to 0.05-Hz range (Figures 5 and 6). The average ratio of 1.8 heavily weights both the frequently occurring low-energy incident waves at Duck (Figures 2*b*, 2*c*), and the generally more-energetic infragravity motions with frequencies ~ 0.02 Hz (Figure 7, upper panels). Higher values of R (~ 3-4) observed at higher infragravity frequencies (~ 0.04 Hz) during cases of energetic swell (Figures 5*b*, 6) suggest that significant contributions of bound waves sometimes occur. How-

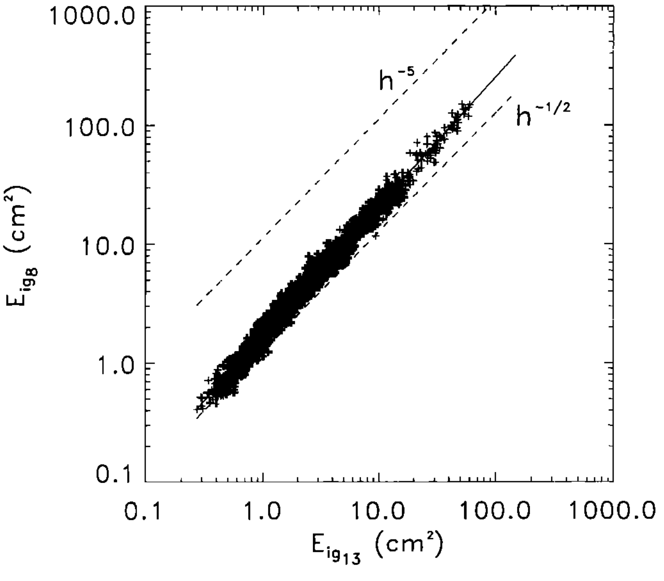

Fig. 3. Infragravity energy observed in 8-m depth versus infragravity energy observed in 13-m depth at Duck. The solid line is a least squares fit to the logarithm of the data, $E_{ig_8} = 1.7\, E_{ig_{13}}^{1.1}$, with correlation coefficient 0.99. The dashed lines indicate the theoretical shallow water relationships between the energies in 8- and 13-m depth for normally incident free waves (energy ~ $h^{-1/2}$, $E_{ig_8} = 1.3\ E_{ig_{13}}$) and bound waves (energy ~ h^{-5}, $E_{ig_8} = 11\ E_{ig_{13}}$).

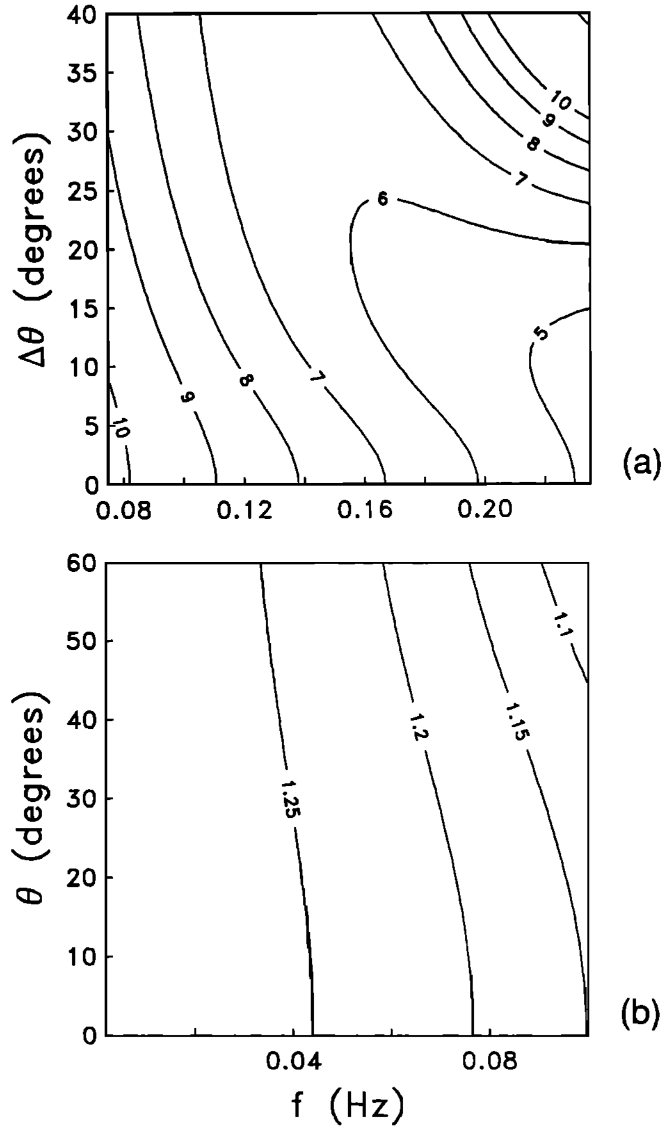

Fig. 4. Contours of the theoretical ratio of infragravity energy in 8-m depth to that in 13-m depth. (*a*) Bound waves with frequency 0.03 Hz forced by the nonlinear difference-frequency interaction of two swell components with frequencies $f - \Delta f/2$ and $f + \Delta f/2$ (where $\Delta f = 0.03$ Hz) and propagation directions $-\theta/2$ and $+\theta/2$ relative to normal incidence. (*b*) Leaky waves of frequency f and deep-water direction θ.

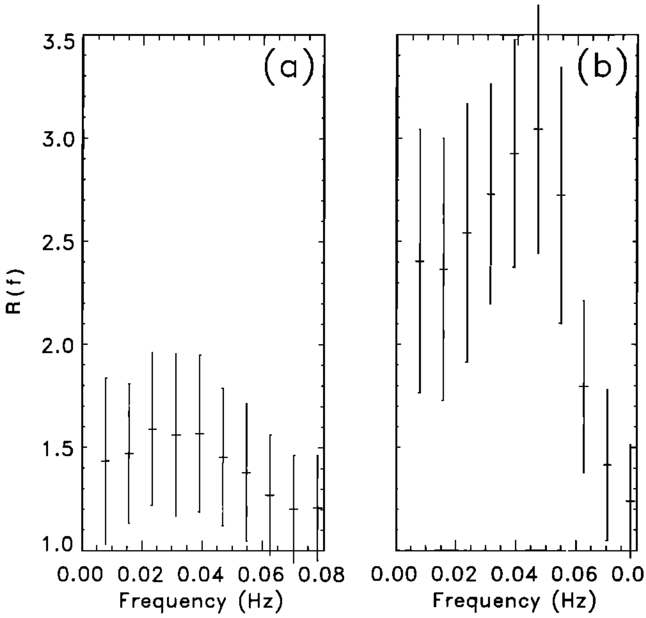

Fig. 5. The ratio ($R(f)$) of infragravity-wave energy observed in 8-m depth to that observed in 13-m depth versus frequency. (Left) Small incident waves, 240 observations with swell energy ranging from 25 to 50 cm^2, (Right) Energetic incident waves, 18 observations with swell energy ranging from 3200 to 11,000 cm^2. The bars indicate ± 1 standard deviation of the observed ratios.

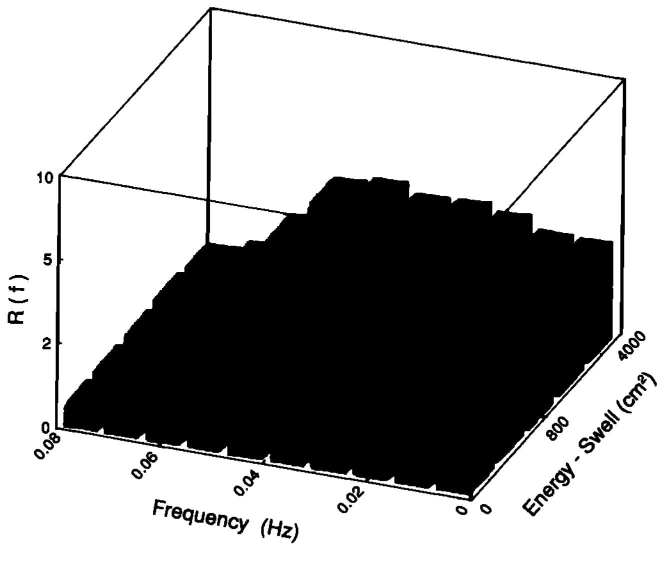

Fig. 6. Histogram of the ratio ($R(f)$) of infragravity energy in 8-m depth to that in 13-m depth as a function of frequency and swell energy. The vertical axis (ratio) is logarithmic, as is the spacing of the swell-energy bins. The frequency bins are 0.0078 Hz wide, and spaced linearly. The range of swell energy (and the number of data runs for each bin) are 25 - 50 cm^2 (240 runs), 50 - 200 (1043), 200 - 800 (691), 800 - 3200 (163), and 3200 - 11,000 (18).

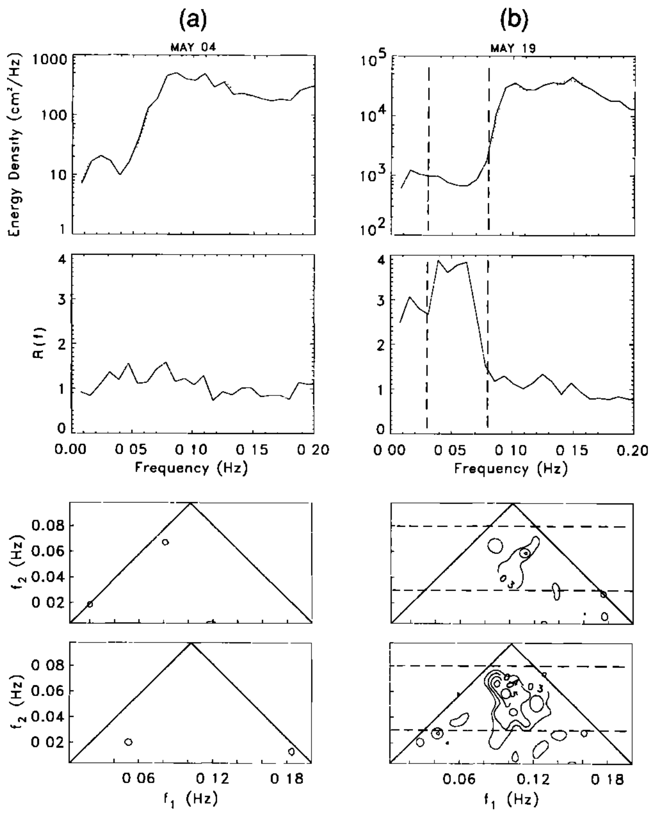

Fig. 7. (From top to bottom) Energy spectra, ratio ($R(f)$) of infragravity energy in 8-m depth to that in 13-m depth, bicoherence in 13-m depth, bicoherence in 8-m depth. The solid and dotted lines in the top panels are the 8- and 13-m depth spectra, respectively. The minimum value of bicoherence plotted is 0.3 (99% significance level), with contours every 0.1. Owing to symmetries (i.e., redundancies) of the bicoherence [*Hasselmann et al.*, 1963], only values in the indicated triangle are shown. (Left) Small incident waves (total energy = 75 cm^2, May 4, 1991). (Right) Large incident waves (total energy = 4000 cm^2, May 19, 1991). (*b*) The vertical (upper two panels) and horizontal (lower two panels) dashed lines at f = 0.03 and f = 0.08 Hz for May 19 delineate the range of infragravity frequencies most strongly phase coupled to swell and sea.

ever, these higher values of $R(f)$ are still not as large as would be expected if the infragravity motions consisted entirely of bound waves (Figure 4*a*), implying that free- (leaky or edge) infragravity waves are rarely negligible in these observations.

There is considerable variability in the ratios shown in Figure 5, as indicated by the bars. This scatter is partially owing to statistical uncertainty of the spectral estimates, but may also be the result of different incident-wave $E(f,\theta)$ or beach morphology. The theoretical bound-wave energy certainly depends on $E(f,\theta)$ (e.g., Figure 4*a*), but the dependence of free-infragravity energy on $E(f,\theta)$ is unknown.

If the high R values are indeed caused by bound-wave contributions, and not by trapping of edge-wave energy between 8- and 13-m depth, then non-Gaussian statistics are expected owing to phase coupling between free swell and bound-infragravity waves. Nonlinear coupling between wave triads with frequencies f_1, f_2 and f_1+f_2 can be detected with the bicoherence [*Hasselmann et al.*, 1963], defined as

$$b^2(f_1,f_2)=\frac{|\langle A(f_1)A(f_2)A^*(f_1+f_2)\rangle|^2}{\langle|A(f_1)A(f_2)|^2\rangle\langle|A(f_1+f_2)|^2\rangle} \qquad (1)$$

where A is the complex Fourier coefficient at frequency f, the asterisk denotes complex conjugate, and angle brackets denote the expected value. Phase coupling between bound low-frequency waves (frequency f_2) and higher-frequency incident waves (frequencies $f_1 > f_2$ and f_1+f_2) results in nonzero $b(f_1,f_2)$, whereas $b = 0$ for statistically independent free waves. Spectra, ratios between infragravity-energy spectral densities in 8- and 13-m depth ($R(f)$), and bicoherences for two representative example cases of low- and high-energy swell are shown in Figures 7*a* and 7*b*, respectively. When the swell energy is low (Figure 7*a*), both $R(f) \sim 1$ and the bicoherences in 8- and 13-m depth are small, consistent with infragravity motions dominated by free waves. On the other hand, with energetic swell (Figure 7*b*), the bicoherence is nonzero in 8- and 13-m depth, and $R(f)$ is larger. In this case, the bicoherence in 13-m depth is maximum for the frequencies $f_2 = 0.06$, $f_1 = 0.11$, $f_1+f_2 = 0.17$ Hz, indicating that 0.06-Hz bound waves are forced by the difference interaction between 0.11-Hz swell and 0.17-Hz sea. Note that 0.06 Hz is in a spectral valley (Figure 7*b*, 13-m depth, upper panel), and the bicoherence is small at frequencies f_2 corresponding to the 0.01- to 0.04-Hz infragravity peak. In 8-m depth, the bicoherence is even larger for the (0.17, 0.11 Hz) sea-swell difference interaction, and is statistically significant for a wider frequency range ($0.01 < f_2 < 0.07$ Hz, $0.08 < f_1 < 0.14$ Hz), corresponding to difference-frequency interactions of the entire sea-swell spectrum (0.08 - 0.21 Hz). In both 8- and 13-m depth, the range of f_2 with large bicoherence values (0.04 - 0.06 Hz) corresponds to the frequency range of largest $R(f)$. The biphases (approximately 180°, not shown) are consistent with second-order theory for weakly nonlinear waves in finite-water depth [*Hasselmann et al.*, 1963].

An estimate of the fraction of energy at a particular infragravity-frequency band that is locally forced by nonlinear difference-frequency interactions of directionally narrow surface waves can be obtained by summing the bicoherence values (b^2) along lines of constant f_2 in f_1, f_2 space (after accounting for statistical bias [*Elgar* and *Sebert*, 1989]). Thus, for example, the fraction of energy at 0.03 Hz excited by difference interactions of 0.08- to 0.15-Hz swell and sea waves is roughly determined by summing the debiased $b^2(f_1, f_2)$ for constant $f_2 = 0.03$ Hz and for $0.08 \le f_1 \le 0.12$ Hz. For the low-energy incident waves shown in Figure 7*a*, the contribution of bound waves to the infragravity wave spectrum is less than 10% in both 8- and 13-m depths at all frequencies. On the other hand, for the high-energy incident waves (Figure 7*b*), bound waves account for approximately 30% to 50% of the infragravity energy in 13-m depth, while in 8-m depth, this fraction is about 70% to 100%. Thus, the bispectral estimates suggest that with large incident waves the infragravity energy in 13-m depth still contains a significant amount of free waves, whereas in 8-m depth, bound waves are the primary source of infragravity energy. These bispectral results showing negligible bound-wave contributions with low-energy incident waves, and significant bound-wave contributions with energetic swell are qualitatively consistent with the observed energy ratios, $R(f)$ (Figures 5-7). Detailed comparisons of theoretically predicted and observed bound-wave energies, and bispectral analysis of this entire data set (and several others) will be presented elsewhere.

3. CONCLUSIONS

Wave data from two approximately 1-year long deployments in the Atlantic and Pacific oceans indicate that infragravity-wave energy (0.004 - 0.04 Hz) is well correlated with energy levels of swell, suggesting the infragravity waves were locally generated. The average (over all data sets) amplification of infragravity energy between 13- and 8-m depth is much less than would be expected if the infragravity waves consisted entirely of bound waves, nonlinearly forced by groups of swell and sea. On the other hand, with energetic swell the amplification is sometimes a factor of 3 greater than would occur for shoaling leaky waves or edge waves not near a turning point. The amplification is usually largest at the high-frequency end of the infragravity band. Bispectral analysis shows that this amplification is associated with significant bound-wave contributions. Thus, for the data presented here, bound-wave contributions are significant for energetic incident waves (significant wave heights greater than about 2 m), but for more commonly observed moderate conditions (significant wave heights between 0.5 and 1.5 m), free waves dominate infragravity energy in 8- and 13-m depth.

Acknowledgments. Support was provided by the Office of Naval Research (Coastal Sciences and Geology & Geophysics) and the National Science Foundation (Physical Oceanography). W. Birkemeier, C. Long, K. Hathaway, and all the staff of the Field Research Facility (FRF), Coastal Engineering Research Center, Duck, North Carolina, contributed greatly to this effort.

REFERENCES

Birkemeier, W., A. DeWall, C. Gorbics, and H. Miller, A user's guide to CERC's Field Research Facility, *Rep. CERC MR 81-7*, U.S. Army Corps of Eng., WES, Vicksburg, Miss., 1981.

Collins, J.I., Prediction of shallow water spectra, *J. Geophys. Res.*, *77*, 2693–2707, 1972.

Eckart, C., Surface waves on water of variable depth, *Wave Report 100*, 99 pp., Scripps Institution of Oceanography, Univ. of California, La Jolla, 1951.

Elgar, S., and R. T. Guza, Observations of bispectra of shoaling surface gravity waves, *J. Fluid Mech.*, *161*, 425–448, 1985.

Elgar, S., and G. Sebert, Statistics of bicoherence and biphase, *J. Geophysical Res.*, *94*, 10,993–10,998, 1989.

Elgar, S., J. Oltman-Shay, and P. Howd, Observations of infragravity-frequency long waves. I, Coupling to wind waves, *Eos Trans. AGU*, *70*, 1133, 1989.

Guza, R. T., E. B. Thornton, and R. A. Holman, Swash on steep and shallow beaches, *Proc. Int. Coastal Eng. Conf.*, *19*, 708–723, 1984.

Hasselmann, K., On the non-linear energy transfer in a gravity-wave spectrum, Part 1, General theory, *J. Fluid Mech. 12*, 481–500, 1962.

Hasselmann, K., W. Munk, and G. MacDonald, Bispectra of ocean waves, in *Time Series Analysis*, edited by M. Rosenblatt, pp. 125–139, Wiley, New York, 1963.

Holman, R. A., Infragravity energy in the surf zone, *J. Geophys. Res.*, *86*, 6442–6450, 1981.

Howd, P., J. Oltman-Shay, and R. A. Holman, Wave variance partitioning in the trough of a barred beach, *J. Geophys. Res.*, *96*, 12,781–12,795, 1991.

Huntley, D. A., Long period waves on a natural beach, *J. Geophys. Res.*, *81*, 6441–6449, 1976.

Huntley, D. A., and C. S. Kim, Is surf beat forced or free?, *Proc. Int. Coastal Eng. Conf.*, *19*, 871–885, 1984.

Huntley, D. A., R. T. Guza, and E. B. Thornton, Field observations of surf beat, 1, Progressive edge waves, *J. Geophys. Res.*, *86*, 6451–6466, 1981.

List, J. H., Wave groupiness as a source of nearshore long waves, *Proc. Int. Coastal Eng. Conf.*, *20*, 497–511, 1986.

Longuet-Higgins, M., and R. Stewart, Radiation stress and mass transport in gravity waves, with application to "surf beats," *J. Fluid Mech.*, *13*, 481–504, 1962.

Middleton, J., M. Cahill, and W. Hsieh, Edge waves on the Sydney coast, *J. Geophys. Res.*, *92*, 9487–9493, 1987.

Nelson, R., P. Treloar, and N. Lawson, The dependency of longshore long waves on the characteristics of offshore short waves, *Coastal Eng.*, *12*, 213–232, 1988.

Okihiro, M., R. T. Guza, and R. J. Seymour, Bound infragravity waves, *J. Geophys. Res.*, *97*, 11,453–11,469, 1992.

Oltman-Shay, J., and R. T. Guza, Infragravity edge wave observations on two California beaches, *J. Phys. Oceanogr.*, *17*, 644–663, 1987.

Oltman-Shay, J., S. Elgar, and P. Howd, Observations of infragravity-frequency long waves, II, Comparisons with a 2-D wave group generation model, *Eos Trans. AGU*, *70*, 1133, 1989.

Sand, S. E., Wave grouping described by bounded long waves, *Ocean Eng.*, *9*, 567–580, 1982.

Suhayda, J. N., Standing waves on beaches, *J. Geophys. Res.*, *79*, 3065–3071, 1974.

S. Elgar, Electrical Engineering & Computer Science, Washington State University, Pullman, WA 99164-2752.

R. T. Guza, T. H. C. Herbers, and M. Okihiro, Scripps Institution of Oceanography, 0209, La Jolla, CA 92093-0209.

J. Oltman-Shay, QUEST Integrated Inc., 21414 68th Avenue South, Kent, WA 98032.

(Received January 6, 1992;
revised June 8, 1992;
accepted June 8, 1992.)