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BRIEF NOTE

BISPECTRAL ANALYSIS OF A FLUID ELASTIC SYSTEM: THE CANTILEVERED PIPE

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Bispectra of the vibrations of a cantilevered pipe conveying fluid are calculated from numerical integrations of the equations of motion. Previous experimental and theoretical work has shown that for certain parameter ranges the vibrations follow a period-doubling route to chaos. Bispectral analysis isolates the nonlinear phase coupling and energy exchange between Fourier components of the vibrating pipe. As flow through the nonlinear stiffened pipe increases, phase-coupled harmonics of the power-spectral primary frequency grow. For parameters close to those associated with chaotic vibrations, subharmonics become phase-coupled to the primary frequency. Cross-bispectra indicate that both sum and difference interactions result in nonlinear phase coupling and energy exchange between the modes of pipe vibration.

1. INTRODUCTION

PAÏDOUSSIS & MOON (1988) RECENTLY PRESENTED EXPERIMENTAL and theoretical evidence of a period doubling route to chaos in vibrations of a cantilevered pipe conveying fluid. In particular, the general equation of motion [equation (1) of Païdoussis & Moon's] was discretized in accordance with Galerkin's technique, truncated to two modes of vibration (i.e., two eigenfunctions), and recast in first-order, state variable form [equation (8) of Païdoussis & Moon's]. Numerical integrations of the equations of motion produced power spectra, phase plane portraits, Lyapunov exponents and bifurcation diagrams of pipe displacement that were remarkably similar to experimental observations.

Power spectra of a system undergoing a period-doubling route to chaos typically develop super- and sub-harmonics of the power-spectral primary peak frequency as the effects of the nonlinearity of the system are increased, corresponding to period one, two, four, . . . motion. When chaos is reached, the power spectrum becomes broad, and individual harmonic peaks are less easily distinguished. Since they are produced by nonlinearities, the Fourier components of the system are not independent of one another, but are phase-coupled. This phase coupling can be isolated and quantified using higher-order spectral analysis [see Nikias & Raghuveer (1987) for a recent review]. Quadratic nonlinear interactions are detected by the bispectrum.

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In this study, displacements of cantilevered pipe conveying fluid as described by the equations of motion presented by Païdoussis & Moon (1988) are analysed with bispectral techniques. As the flow through the nonlinear piping system is increased, causing period doubling, phase coupled harmonics are produced. Cross-bispectra indicate that the two modes of pipe displacement are coupled to each other and exchange energy.

In Section 2 bispectral quantities are defined, and details of the numerical integrations of the equations of motion are presented. Results are described in Section 3, followed by conclusions in Section 4.

2. DEFINITIONS AND NUMERICS

Details and applications of bispectral analysis are given by Nikias & Raghuveer (1987) and Kim & Powers (1979). A brief description is presented here for completeness.

For a discretely sampled time series $h(t)$ with the Fourier representation

$$v(t) = \sum_n C(\omega_n)e^{i\omega_n t} + C^*(\omega_n)e^{-i\omega_n t}, \tag{1}$$

the power, auto- and cross-bicoherence spectra (normalized bispectra) are defined, respectively, as

$$P(\omega) = \mathrm{E}[C(\omega)C^*(\omega)], \tag{2}$$

$$B(\omega_1, \omega_2) = \frac{\mathrm{E}[C(\omega_1)C(\omega_2)C^*(\omega_1 + \omega_2)]}{P(\omega_1)P(\omega_2)P(\omega_1 + \omega_2)}, \tag{3}$$

$$XB_{j,k}(\omega_1, \omega_2) = \frac{\mathrm{E}[C_j(\beta_1)C_j(\omega_2)C_k^*(\omega_1 + \omega_2)]}{P_j(\omega_1)P_j(\omega_2)P_k(\omega_1 + \omega_2)}, \tag{4}$$

where ω_n is the radian frequency, the subscript n is a frequency index, the C's are the complex Fourier coefficients of the time series, an asterisk indicates complex conjugate, and E [ ] is the expected-value, or average, operator. The subscripts j, k in (4) indicate separate time series; in the cases considered here, input and output, respectively.

It is illustrative to consider the bispectrum to understand the function forms discussed above. Rewriting the complex Fourier coefficients

$$C(\omega_n) = c_n e^{i\Phi_n},$$

the bispectrum becomes

$$B(\omega_1, \omega_2) = \mathrm{E}[c_1 c_2 c_{1+2} e^{i\Phi_1 + \Phi_2 - \Phi_{1+2}}]. \tag{5}$$

If the three modes of the triad are independent of each other (i.e., Φ_1, Φ_2, Φ_{1+2} are random phases), then when averaged over many realizations the triple products in (5) will be zero. On the other hand, if the modes at frequencies ω_1, ω_2, and $\omega_1 + \omega_2$ are quadratically coupled, the biphase $(\Phi_1 + \Phi_2 - \Phi_{1+2})$ will be non-random even if Φ_1 and Φ_2 are randomly varying. Thus,the bispectrum will be non-zero. Consequently, the bicoherence indicates the relative amount of quadratic phase coupling between the three modes in a triad.

For a digital time series with Nyquist frequency ω_n, the auto bicoherence is completely described by values within a triangle with vertices at $(\omega_1 = 0,\ \omega_2 = 0)$, $(\omega_1 = \omega_{N/2},\ \omega_2 = \omega_{N/2})$ and $(\omega_1 = \omega_N,\ \chi_2 = \omega_0)$ (Kim & Powers 1979). The resulting plot is a contour plot, with the abscissa value for the frequency adding to the ordinate value to give the sum frequency. The contour represents the energy at the sum frequency due to the nonlinear quadratic interaction between the abscissa frequency value and the ordinate frequency value.

Because of the existence of both sum and difference interactions due to input and output trajectories, the cross-bispectrum requires the additional region bounded by $(\omega_1 = 0,\ \omega_2 = 0)$, $(\omega_1 = \omega_N,\ \omega_2 = 0)$ and $(\omega_1 = \omega_N,\ \omega_2 = -\omega_N)$.

The equations of motion that describe the displacement of the free end of the cantilevered pipe are given by Païdoussis & Moon (1988), equations (1)–(11), and will not be repeated here. Similar to that work, the equations here were integrated with a fourth-order Runge-Kutta scheme on an IBM 3090. For each set of parameters, an 8192-point time series (corresponding to 4096 s in dimensional units) was produced, with $\omega_N = 20\pi$ rad/s. The time series were subdivided into 64 sections, and auto- and cross-bispectra were calculated for each section and ensemble averaged for equation (1) over the collection of 64 sections. Thus, the power and bispectra presented here have 128 degrees-of-freedom and the final frequency resolutions of 0·49 rad/s.

Nondimensional constants identical to those given by Païdoussis & Moon's (1988) equation (10) were used here, and the dimensionless flow velocity, u, was used as a variable parameter. The displacement and velocity at the free end of the pipe were calculated from equation (11) of the same reference, with the mode-one and -two eigenfunctions having values of 2·0000 and −2·0000, respectively (Thomson 1988).

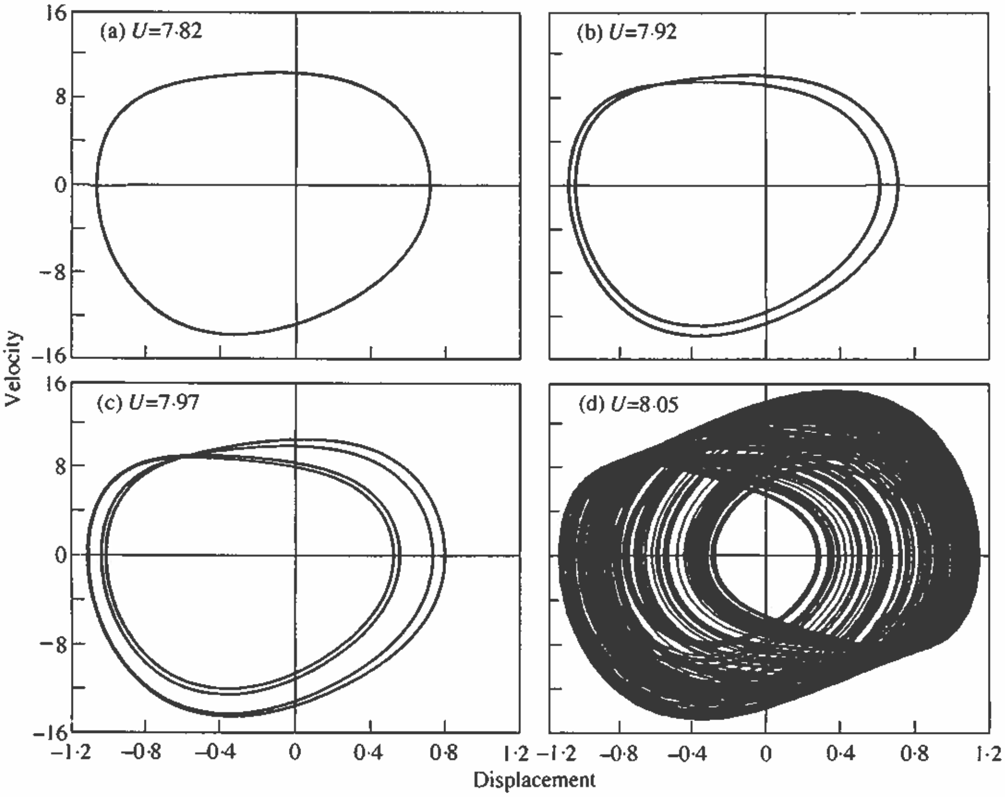

Figure 1. Phase portraits of the vibrations of the free end of the pipe for four different values of u. Phase portraits are of total displacement.

3. RESULTS

The power spectra and auto-bispectra of displacement at the end of the pipe for period-1 motion ($u = 7\cdot82$) are shown in Figure 2. The corresponding phase portrait is Figure 1(a). Even for period-1 motion, there are superharmonics of the power spectral primary peak frequency ($\omega = 14$ rad/s). These harmonics are phase-coupled to the primary, as indicated by the auto-bicoherence (Figure 2). In particular, the first harmonic ($\omega = 28$) of displacement is coupled to the primary [$b^2(14, 14) = 0\cdot5$], as well as to a range of lower frequencies [the diagonal line labelled "A" in Figure 2(b)]. The second harmonic ($\omega = 42$) is coupled to the primary and first harmonic [$b^2(28, 14) = 0\cdot6$].

As the flow through the nonlinear system is increased, the coupling between the primary, first and second harmonic also increased (Figures 3 and 4). In addition, for $u = 7\cdot92$, a subharmonic appears [Figure 3(a), $\omega = 7$] in the displacement power spectrum. Motions at this frequency are weakly coupled to motions at the primary [$b^2(7, 7) = 0\cdot4$]. For $u = 7\cdot97$, a second subharmonic appears [$\omega = 3\cdot5$, Figure 4(a)]; the subharmonic at $\omega = 7$ becomes coupled to the primary and to motions at $\omega = 21$ [$b^2(14, 7) = 0\cdot6$, Figure 4(b)].

For large values of u ($u > 8\cdot05$) the displacement time series become chaotic, and their power spectra become broad band [Figure 5(a)]. In this case, the cubic nonlinearity imposed by the nonlinear stiffness terms dominates the motion: one can

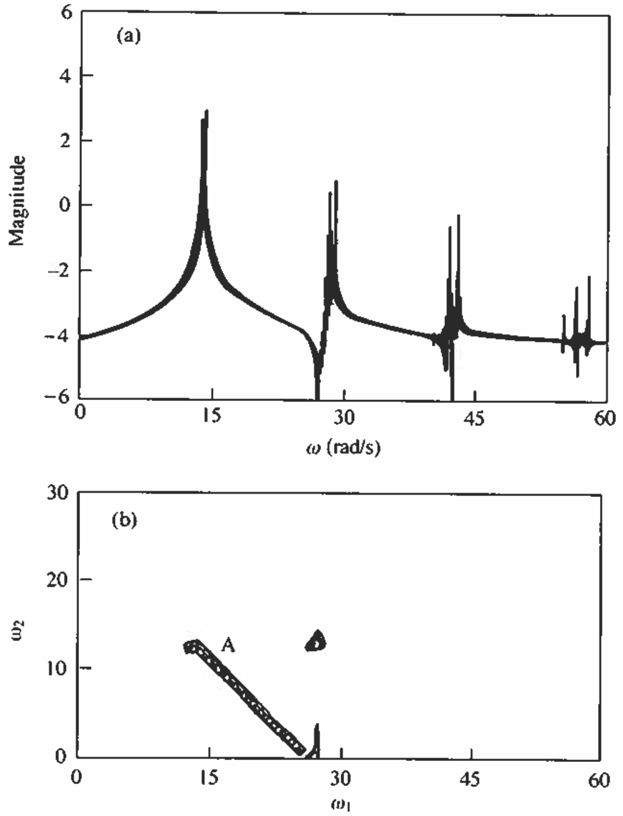

Figure 2. Power spectra (above) and contours of bicoherence (below) of pipe displacement for $u = 7\cdot82$. The units of power are arbitrary. The minimum bicoherence contour plotted is $b^2 = 0\cdot4$, with contours every $0\cdot1$.

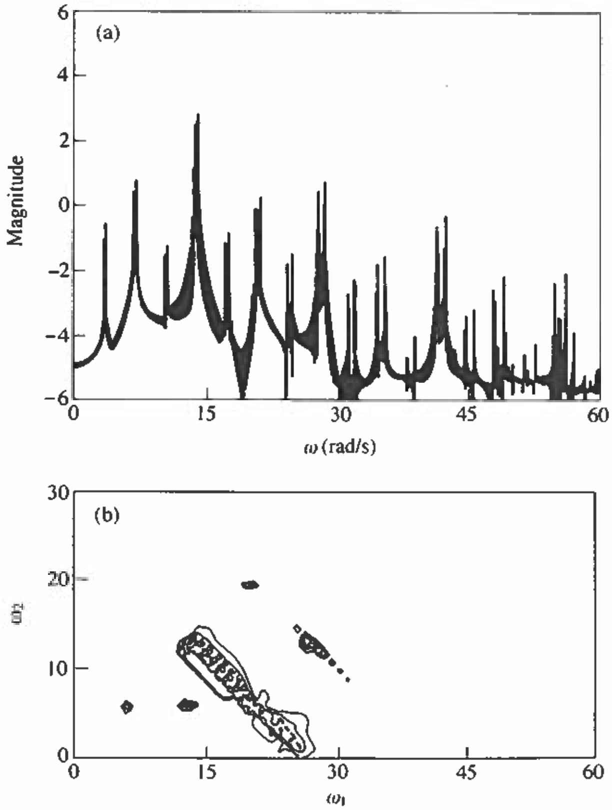

Figure 3. Power spectra (above) and contours of bicoherence (below) of pipe displacement for $u = 7{\cdot}92$. The units of power are arbitrary. The minimum bicoherence contour plotted is $b^2 = 0{\cdot}4$, with contours every 0·1.

see that the spike at $\omega = 14$ is one-third that of $\omega = 42$, the other predominant frequency spike in the power spectrum of the chaotic motion. This indicates a strong cubic nonlinearity. Trispectral analysis is required to detect these cubic nonlinearities, while the bispectrum is small [Figure 5(b)].

The coupling and energy exchange between the two modes of displacement are displayed by the cross-bicoherence between time series of each mode, as shown in Figure 6. Cross-bispectra for $u = 7{\cdot}82$ [Figure 6(a)] above the $\omega_2 = 0$ line indicate coupling between motions at the primary power spectral frequency of mode 1 and its harmonic in mode 2 [$b^2(14, 14) = 0{\cdot}7$], as well as coupling between the mode-1 primary, mode-1 first harmonic, and mode-2 second harmonic [$b^2(28, 14) = 0{\cdot}5$], similar to the auto-bispectra [Figure 2(b)]. In addition, lower frequencies of mode 1 ($\omega < 14$) are coupled to the first harmonic of mode 2. Difference interactions couple mode-1 motions at the first harmonic ($\omega_1 = 28$), a range of mode-1 motions ($-28 < \omega_2 < 0$), and a range of mode-2 motions ($0 < \omega_1 + \omega_2 < 28$), as indicated by the vertical band of high bicoherence values in Figure 6(a). As period doubling takes place, the coupling between the modes also increased [Figure 6(a–c)], and spreads to include coupling to subharmonics. Once the system becomes chaotic [$u = 8{\cdot}05$, Figure 6(d)] the cross-bicoherence becomes small, as cubic interactions dominate the displacement of the pipe.

The structure of the cross-bicoherence spectra of the displacement of the can-

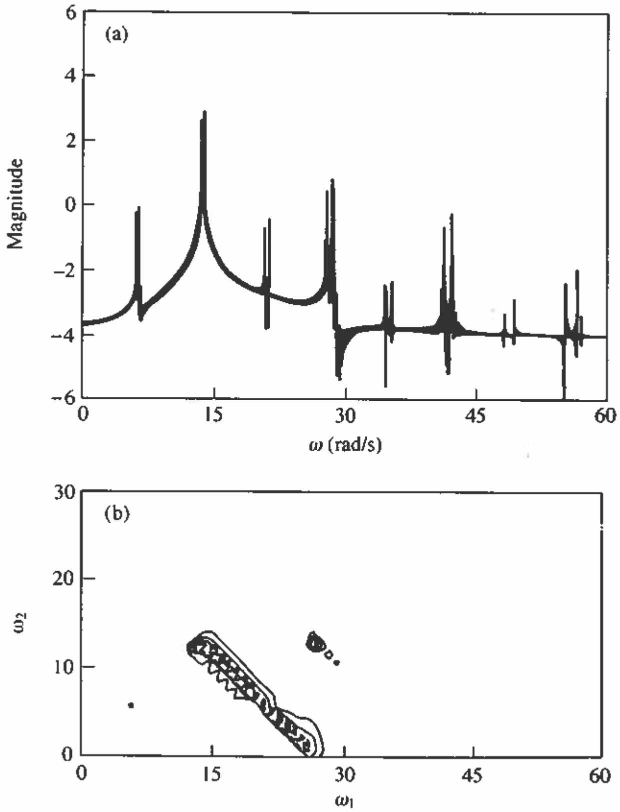

Figure 4. Power spectra (above) and contours of bicoherence (below) of pipe displacement for $u = 7{\cdot}97$.

tilevered pipe is similar to that of a multi-frequency excited buckled beam described by the Holmes-Moon Duffing equation (Pezeshki *et al.* 1991). The beam was forced by motions at frequencies corresponding to the first resonance and twice the first resonance of the system. Quadratic interactions lead to a transfer of energy from the higher frequency forcing to the lowest nonlinear natural frequency of the system. This low frequency energy was then redistributed to many frequencies via a quadratic sum interaction, eventually producing a broad power spectrum. Through a variation of the phase angle of the higher-frequency forcing term, the period doubling route to chaos would be controlled, and chaos prevented. Similar dynamics may occur in the cantilevered pipe, where difference interactions between the two modes of vibration produce motions at low frequencies. Sum interactions between the low frequencies and the harmonics redistribute energy to motions at a range of frequencies, eventually broadening the power spectrum.

4. CONCLUSIONS

Previous experimental and theoretical work demonstrated a period doubling route to chaos, as flow increases in a cantilevered pipe conveying fluid with nonlinear stiffness. In the present study, bispectra of time series of pipe displacement produced by numerical solutions to the equations of motion describing the cantilevered pipe system were calculated. Bispectra indicate nonlinear phase coupling between pipe displacements at the primary power spectral peak frequency and its super and subharmonics.

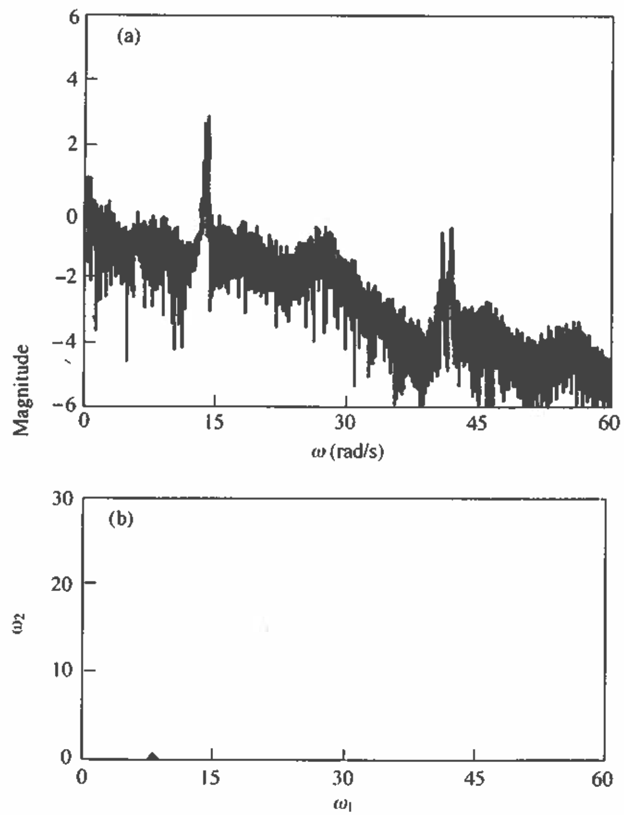

Figure 5. Power spectra (above) and contours of bicoherence (below) of pipe displacement for $u = 8{\cdot}05$.

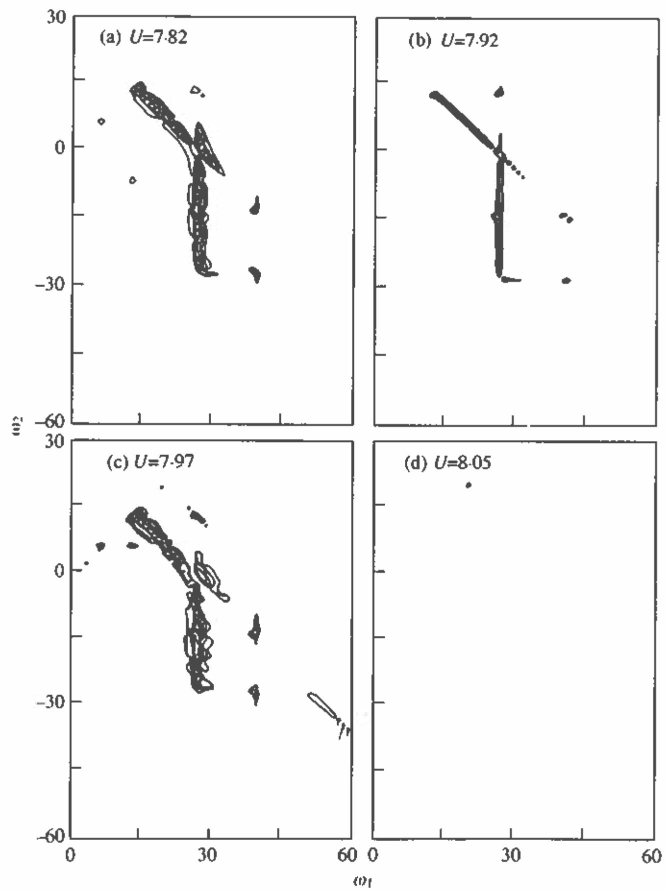

Figure 6. Contours of cross-bicoherence between mode 1 and mode 2 of pipe displacement: (a) $u = 7{\cdot}82$; (b) $u = 7{\cdot}92$; (c) $u = 7{\cdot}97$; (d) $u = 8{\cdot}05$. The minimum contour plotted is $b^2 = 0{\cdot}4$, with contours every 0·1.

The coupling increases and spreads to incorporate motions at more frequencies as the system undergoes a period doubling route to chaos. Once the pipe displacement is chaotic, the bicoherence becomes small because the system is dominated by cubic interactions. Trispectra are necessary to investigate the pipe system in the chaotic regime.

Cross-bispectra between the two modes of motion of the pipe indicate they are phase-coupled to each other, and exchange energy. In particular, low frequency motions of one mode are coupled to and exchange energy with motions at harmonic frequencies in the other mode.

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