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Shock Waves in Stellar Atmospheres and Breaking Waves on an Ocean Beach

George Wallerstein* and Steve Elgar

The phenomenon of ocean waves breaking on a beach is analogous to shock waves in the atmosphere of a pulsating star. In both cases a velocity discontinuity is clearly present. In stars the upper, expanding layer halts and falls back so as to interact with the rising gas at a shock. Similarly, a bore on a beach reaches its maximum extension before sliding back onto the next incoming wave. Analogous quantities such as the surface gravity of the star and the beach gradient in the ocean have similar effects on the flows and the nature of the discontinuity between them. Phenomena that are not analogous include the thermodynamic properties of the two media. Ocean observations may help solve some problems in shock phenomena associated with stellar pulsation.

At the meeting of the International Astronomical Union in 1952, Sanford (*1*) reported an entirely new phenomenon in stellar spectroscopy. The 17-day variable star W Vir showed emission lines of hydrogen and doubled absorption lines during rising brightness. At the same meeting, Schwarzschild (*2*) pointed out that Sanford's observations could best be understood in terms of a shock wave separating the rising and falling gas layers in the star's atmosphere. Owing to the Doppler shift, the stellar absorption lines were separated in wavelength by an amount corresponding to about 55 km s^{-1}, which is Mach 8 for the largely hydrogen gas of the infalling layer whose temperature is near 5000 K. The hydrogen emission lines were emitted by the shock-heated gas that marked the boundary between the two layers.

Two flow regimes also exist ahead of and behind ocean surface gravity waves breaking on a beach (Fig. 1). The wind-generated waves arriving at the beach from the deep ocean steepen as they propagate into shallow water, and eventually break and form bores (discontinuities in water depth, distinguished by white foam from air entrainment, Fig. 1) that continue to propagate shoreward. Seaward of the bore the water travels toward the shore, whereas ahead of the bore there can, in addition, be water from the previous wave flowing seaward down the beach slope. A breaking wave is thus also a shock with a velocity discontinuity. In fact, the equations of motion describing long waves in shallow water are the same as those describing compressible gas dynamics in one-dimensional flows. What can be learned by comparing stellar shocks and ocean waves?

In addition to providing insights into the behavior of nonlinear hydrodynamic phenomena, the comparison between stellar shocks and ocean waves can be used as a study of the utility of a physical analogy (that is, of two apparently unrelated phenomena that show similar physical behavior). Such an analogy differs from the analogies derived from mechanics that were used to describe electromagnetic waves in the ether during the 19th century because the mechanical analogs were purely theoretical. In addition, a physical analogy differs from a mathematical analogy, such as that of vibrating electrical circuits and mechanical devices, in which the phenomena are not physically similar but are related only by the similarity of the differential equations that describe them (*3*).

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Equations of Motion

The equations of motion for a surface gravity wave propagating in shallow water ($kh << 1$, where k is the wave number and h is the water depth) are directly analogous to the equations governing a compressible gas. Fol-

Fig. 1. Bores on an ocean beach. The Lighter Amphibious Resupply Craft (LARC) provides a relative scale for the approximately 3-m-high bore covering its stern. Two preceding bores can be seen shoreward of the LARC. The photograph was taken by J. Smith during the *Samson/Delilah* nearshore field experiment in October 1990, conducted near Duck, North Carolina, at the Field Research Facility. The LARC is manned by (from left to right) T. Herbers, S. Elgar, and T. Lippmann, and piloted by G. Bicher (inside the cab).

lowing Stoker (*4*), the horizontal velocity u and free surface elevation η are given by

$$u_t + uu_x = -g\,\eta_x \qquad (1)$$

$$[u(\eta + h)]_x = -\eta_t \qquad (2)$$

where g is gravitational acceleration, and the subscripts x and t represent differentiation with respect to space and time, respectively. On a sloping beach, the depth is a function of the distance along the direction to and from the shore, x.

Introducing a "density" $\bar{\rho}$ as

$$\bar{\rho} = \rho(\eta + h) \qquad (3)$$

where ρ is the density of water, and a "pressure" $\bar{p}$ as

$$\bar{p} = \int_{-h}^{\eta} p\, dy \qquad (4)$$

where y is the vertical coordinate and p is the hydrostatic pressure, a direct analogy to the equations of a compressible gas can be made (*4*). Invoking hydrostatic pressure allows the "density" and "pressure" to be related as

$$\bar{p} = \frac{g}{2\rho}\bar{\rho}^2 \qquad (5)$$

which is analogous to an adiabatic law with exponent 2. Thus, the depth of water is analogous to the density in a gas. The equations of motion become (*4*)

$$\bar{\rho}(u_t + uu_x) = -\bar{p}_x + g\bar{\rho}h_x \qquad (6)$$

$$(\bar{\rho}u)_x = -\bar{\rho}_t \qquad (7)$$

which are equivalent to the equations describing a compressible gas in one-dimensional flow if $h_x = 0$ (that is, on a flat bottom).

A propagation speed c in water (analogous to the speed of sound in gas) is

$$c = \sqrt{\frac{d\bar{p}}{d\bar{\rho}}} = \sqrt{\frac{g\bar{\rho}}{\rho}} = \sqrt{g(\eta + h)} \qquad (8)$$

It can be shown that the propagation of a "hump" of water into still water develops a bore that is equivalent to the shock that is formed when a piston is pushed with increasing speed into a gas and creates a compression wave. In a stellar atmosphere, a shock develops when a running wave propagating upward from deeper levels maintains its velocity in a region of steadily decreasing temperature; hence, the wave velocity exceeds the local sound velocity. Gas leaves the shock discontinuity at subsonic speed and therefore the rarefaction wave does not develop a shock. Similarly, a depression in the water surface that propagates into still water will not form a bore.

The equations relating the water depths and water velocities in front of the bore to those behind the bore are completely analogous to the equations relating the velocity, density, and pressure change across a shock in a compressible flow. If we denote the velocity relative to the moving bore as v, the conditions on either side of the bore (subscripts 0, 1) are

$$\bar{\rho}_1 v_1 = \bar{\rho}_0 v_0 \qquad (9)$$

$$\bar{\rho}_1 u_1 v_1 - \bar{\rho}_0 u_0 v_0 = \bar{p}_0 - \bar{p}_1 \qquad (10)$$

These are identical to the mechanical conditions across a shock wave in a gas (*4*). Across a bore in water, a mechanical loss of energy results owing to the production of heat from turbulence (Fig. 1). In a compressible gas, energy is conserved across the shock (mechanical energy is converted to heat, which, except for radiation, is not lost to the system), and the analog to energy loss in a bore is the increase in entropy across the shock. In both cases, the discontinuous changes are proportional to the cube of the "density" differences across the shock (*4*).

Bores in water propagate in such a way that the particle velocities relative to the bore on the front side are greater than the propagation speed corresponding to the depth (termed supercritical flow), whereas the velocities on the rear side are less than the propagation speed (subcritical flow). In gas dynamics, the flow in front of the shock is supersonic, whereas that behind the shock is subsonic.

Another similarity between long waves and compressible gases is the change in flow conditions that results from reflection. Specifically, the reflection of a bore from a vertical wall results in an increase in water depth, which is analogous to the increase in density that results when a shock in a compressible gas reflects from the closed end of a tube.

Models for Bores in the Surf Zone

The shallow water wave equations (Eqs. 1 and 2) (*5*) have been used in many studies of the propagation of bores in the surf zone (that is, within and shoreward of the region of wave breaking). The prebreaking steepening of the bore face has been investigated by the method of characteristics (*6*). In the surf zone, the steepening of the front results either in undular (*7*) or turbulent (*8, 9*) bores. Energy propagates away from undular bores by means of a trailing wave system (*10*). Although early studies of bores in the surf zone (*8, 11, 12*) did not directly include the dissipation of energy that occurs when the bores propagate, the Lax-Wendroff (*13*) method of numerically solving the equations introduces a numerical dissipation (*8, 14*). The solution procedure is known as a shock capturing method, which fixes the shape of the bore over a small number of computation points. Thus, unlike shock fitting methods, this method does not require a separate treatment of the bore. A recent review of numerical methods for flows with shocks is given in (15).

Direct effects of dissipation have been included in other models of bores in the surf zone. These effects include viscosity (16), dissipation owing to percolation through a porous bed (17), and friction as a result of bottom drag on steep rough beaches (18) and gently sloping beaches (14). The crude treatment of the bore front in the shock capturing models neglects the effects of turbulence on bore dynamics. Turbulent dissipation has been included for cases of uniform and nonuniform (9) bores guided by the descriptive account given in (19). These models for bore propagation based on the shallow water equations are in good agreement with both laboratory experiments (14, 16, 18) and field observations of wave height decay across the surf zone (20).

Analogous Phenomena

The behavior of progressive waves in the atmosphere of a star seems to be determined by the velocity of the wave when it reaches the region of the star from which radiation escapes (roughly above $\tau_{5000} = 3$, where radiation of 5000 Å is reduced by $e^{-\tau}$) and by the surface gravity. The oceanographic analog to the stellar surface gravity is the beach slope. When the stellar gravity is very low, and hence the wave is short compared to the scale height [as in long-period stars whose masses are roughly 1.5 $M_\odot$ (the mass of the sun) and whose radii are a few hundred $R_\odot$ (the radius of the sun)], the shock waves are of low amplitude and extend far out into the atmosphere of the star. In fact, the calculations of Bowen (21) show that successive waves can gradually lift matter off the star. Similarly, the waves on a gently sloping beach break very gradually, and turbulent water spills down the front face.

For moderate stellar gravities such as those of the W Vir and RV Tau stars, which have periods of 15 to 75 days and radii of 20 to 80 $R_\odot$, shocks exist, along with absorption lines, that appear doubled because of the velocity jump. In the ocean, waves propagating shoreward on moderately sloping beaches steepen sharply, becoming sawtooth-like in shape. Eventually the upper part of the wave curls over and plunges down the front face. These are stronger shocks than the low stellar gravity or low beach slope shocks and have higher dissipation, which is shown by hydrogen and helium recombination lines after ionization in the stars and by substantial turbulence generation in the plunging surf. There is no oceanographic analog to the radiative losses from a stellar shock.

For stars of high surface gravity, such as the sun (log g = 4.4), and for very steep beaches, the physics is different. Of course, the sun does not pulsate with much of its energy in a single mode but does have waves of a broad frequency spectrum that propagate through its atmosphere. These waves dissipate over a short distance in the lower chromosphere, which causes heating (22). Ocean waves surge up very steep beaches, possibly with substantial amounts of energy reflected seaward. The reflected waves may propagate into deep water or may constructively interfere with the next shoreward-propagating wave, resulting in a large-amplitude wave that breaks rather than reflects.

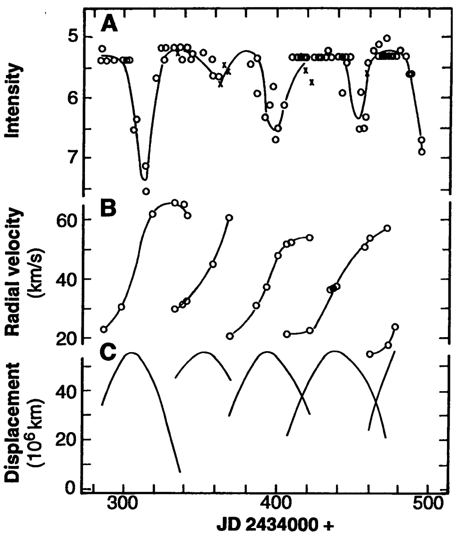

Fig. 2. **(A)** Light, **(B)** velocity, and **(C)** displacement curves for the RV Tau–type star U Mon [reprinted from H. A. Abt in (28)]. The uppermost curve shows the alternation of deep and shallow minima. The central curves show the measured Doppler velocities, with the convention that positive velocities indicate gas falling into the star. On four occasions two Doppler shifts are seen simultaneously, indicating two gas layers separated by a shock. The lower curves are integrations of the velocity curves to yield displacements of the atmospheric layers.

Shock Waves in Pulsating Stars

As Schwarzschild (2) recognized in Sanford's (1) data, stellar shock waves can be identified by the presence of two layers that differ in velocity by an amount that exceeds the local sound speed and by emission from the shock-heated gas that is superimposed on the otherwise undisturbed stellar radiation. Doubled absorption lines do not necessarily come from colliding gas layers. Gas layers moving in opposite directions might pass through each other, but in the case of W Vir, Abt (23) showed that the mean free path of the atoms was much less than the thickness of the absorbing layers, and hence collision and thermalization of the kinetic energy must follow.

The second form of evidence for shocks is the presence of emission lines of abundant elements, hydrogen being the most obvious example. If the shock has sufficient energy, helium may be ionized to yield recombination lines of He I (or even He II, if helium is doubly ionized). Most of the emission lines are attributable to recombination and cascade after the atoms have been ionized in the immediate post-shock gas. In addition, emission lines can be caused by collisional excitation of low-lying states as well as by resonance pumping of specific upper atomic levels followed by radiative de-excitation. There are, of course, many other forms of ionization and excitation in stars, but the combination of doubled absorption lines and recombination emission lines provides the strongest evidence of shocks in pulsating stars.

Several types of pulsating stars show evidence of shocks. The type II Cepheids, of which W Vir is the prototype, are the most obvious example (24). At the shortest periods, the RR Lyrae stars that pulsate in their fundamental mode (0.45 to 0.8 days) show highly asymmetric light and velocity curves accompanied by hydrogen emission and very brief line doubling (25). Because of the brevity of the phenomena and the absence of any very bright RR Lyrae variables, it is difficult to obtain time-resolved data with high spectral resolution. The type II Cepheids of short period, 1 to 10 days, do not show double lines, and their hydrogen emission is very weak if present at all. The classical Cepheids, whose periods are similar to the type II Cepheids, almost never show doubled absorption lines in the visual region, although recent observations in the near infrared do indeed show line doubling, the origin of which is still uncertain (26). Absorption components with unexpected velocity shifts are seen at Hα in classical Cepheids (27), but the anomalous velocities are probably the result of chromospheric effects, rather than of shocks in the photosphere. Stars of the RV Tau type, which have periods of 30 to 75 days and alternating depths of minimum brightness, show clear evidence of shocks that are similar to those in W Vir stars (28). The final, and probably the most complicated, type of stellar atmosphere that shows evidence of shocks is that of the Mira stars, which have periods of 200 to 500 days. Their velocity curves, when observed in the near infrared where the continuous opacity is low, show line doubling that is similar to that of the W Vir stars (29). Several overlying layers associated with the extended, mass-losing atmospheres of the Mira stars obscured this interpretation for many years.

Alternating Behavior

Stars of the RV Tau type have periods of 30 to 75 days. In addition, their light minima usually alternate between deep and shallow events, but they also interchange deep minima at times so that what was previously a deep minimum becomes a shallow minimum (Fig. 2) (*28*). An integration of the velocity curve during a low- and high-amplitude cycle yields a total displacement of the pulsating layer of 1.0×10^7 and 5.0×10^7 km. A similar alternation is sometimes seen along a beach. If an incoming wave has a larger amplitude than usual, it propagates up the beach as a large bore. The corresponding backwash (seaward return flow) has large momentum and retards the next wave, which dissipates into a bore of small amplitude. The backwash from the small-amplitude bore provides little resistance to the next incoming wave, which then creates a large bore. The alternating behavior may continue until a random fluctuation in the amplitude of the incoming wave from deep water overcomes the alternation induced by the backflowing bore. A slightly different mechanism causes alternations on steeper beaches (*30*). In this case, a large plunging breaker results in a large bore. The next wave propagates further shoreward through the relatively deeper water of the bore of the preceding wave and finally collapses very close to the shoreline, in a way that is similar to a surging breaker. The strong backwash that results interacts with the third wave to produce another plunging breaker, and the cycle is repeated. Thus, the time series of wave heights is modulated with a period corresponding to the first subharmonic of the incident wave period (Fig. 3).

Double Shocks

Observations of waves on gently sloping beaches often show two or more waves breaking simultaneously (Fig. 1). Stars behave similarly (*31*). The pulsating stars of longest periods (250 to 500 days), typified by Mira, the first pulsating star to be discovered, show subtle behavior that can be recognized as a result of the simultaneous visibility of two shocks. The basic pulsation of the atmospheres of the Mira stars is shown by their radial velocity variations observed in the near infrared (1.6 and 2.2 μm) regions (*29*). Like the W Vir variables, the Mira double lines are seen during rising light, and their amplitude indicates a shock of sufficiently high velocity to ionize hydrogen but not helium, which is in agreement with the observations of emission lines of hydrogen only. Evidence that indicates two shocks are present includes spectra in the blue region, where the continuum is relatively weak owing to the low temperatures (about 3000 to 3500 K) of these stars. These spectra show numerous emission lines from low-lying levels (about 3 eV) of neutral atoms such as iron and magnesium. These atoms can be excited by collisions in gas of moderate temperatures (6000 to 8000 K) behind a weak shock where the excitation is insufficient to ionize hydrogen. Two phenomena that indicate an additional, hotter, and higher velocity shock include the presence and width of hydrogen lines in emission and the resonant excitation of certain specific atomic levels. The latter excitation is by strong ultraviolet lines such as those of Mg II at 2795.5 and 2802.7 Å, whose wavelengths coincide with those of the iron lines only if the emitting Mg II atoms are approaching the absorbing iron atoms at about 40 km s^{-1}. The faster, and necessarily deeper, shock also provides the hydrogen emission, which often shows absorption by spectral lines formed in higher layers. The fast shock is also responsible for the Mg II resonance lines, which require ionization of magnesium (8 eV) and either the second ionization of magnesium (16 eV) or, more likely, the collisional excitation of the resonance lines (4 eV).

Nonanalogous Phenomena

In comparing analogous physical phenomena, we must also recognize aspects that are not analogous. The most obvious example is that stars are gases and hence are highly compressible, whereas compressibility is not an important consideration in the case of ocean surface gravity waves. However, by considering the surface density (proportional to the water depth) rather than the volume density of ocean waves, similarities to stellar shocks are seen. In particular, the density as a function of distance from the

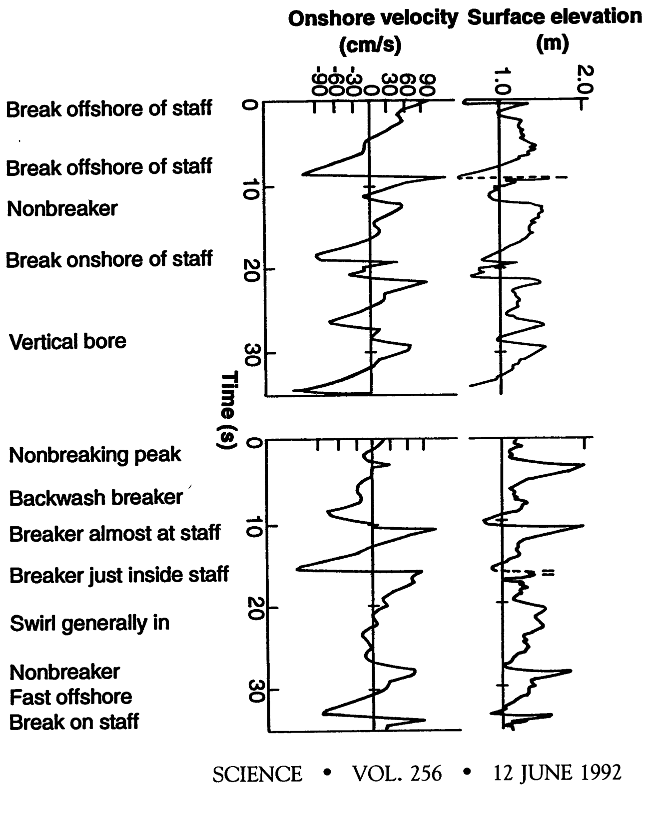

Fig. 3. Two simultaneous time series of onshore velocity (*u*) and surface elevation (η) (arbitrary zero for the elevation scale). The period of the waves was approximately 5 s, and a strong modulation with period about 10 s can be seen in the time series of both velocity and elevation. The breaker positions and other comments are from the film taken concurrently with these measurements (*30*).

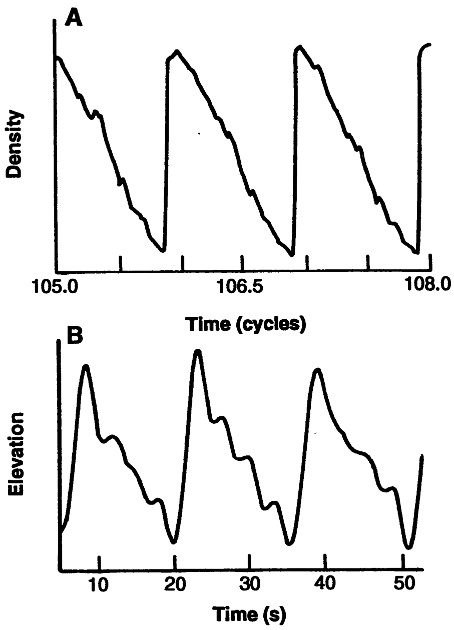

Fig. 4. (**A**) Density versus time for a single zone in a star [adapted from (*21*)] and (**B**) sea-surface elevation (analogous to surface density) versus time (time and space are approximately equivalent in the water depth of 1 m where these observations were made). The units of density and elevation are arbitrary.

center of the star is similar to the surface density as a function of distance from the rear face of an ocean wave through its crest toward the beach (Fig. 4).

The thermodynamics of stars and ocean waves are grossly different because of the huge number of internal degrees of freedom in stellar gases, especially the ionization of the most abundant elements, hydrogen and helium. Water is never appreciably heated by breaking waves and, in fact, can possibly be cooled by the evaporation of spray, a phenomenon that is impossible in a stellar atmosphere. In stars, as in any shocked gas, the internal energy increases drastically across a strong shock. At first this appears as a temperature rise from about 5,000 K to near 100,000 K of the neutral (or perhaps molecular) gas. Slowly the neutral-neutral collisions cause a small amount of ionization (or the trace elements of low ionization potential may already be ionized) and the released electrons rapidly ionize the hydrogen and helium if the temperature is high enough. Thus, an equilibrium is established as described by the Saha equation or by the ionization rate equaling the recombination rate (*32*). Energy is then lost by the shocked gas when recombination and line radiation escape.

Radiative losses reduce the temperature and pressure behind the shock and hence the overpressure that drives it forward. At the same time, the density gradient in the stellar atmosphere acts to enhance the shock velocity. Thus, shocks that maintain their velocity must be riding a density gradient that balances the radiative energy losses. For W Vir stars, the shock velocity, as indicated by the measured difference in velocity between the two gas layers, remains constant from about phase 0.8 to 0.1 (where phase 0.0 is defined as maximum light), or about 6 days. For long-period variables, the shock velocity may be constant for months (*29*). This required balance may explain why the low-amplitude semi-regular variables, whose gross properties are similar to those of Mira stars, do not develop large amplitudes of variation and the attendant complex emission line spectra. The density gradients in their atmospheres may be too low to sustain a shock against the radiative losses (which are sometimes visible in the form of Balmer line emission).

There are two extremes of density gradients in normal stars (that is, excluding white dwarfs and neutron stars). In the sun, the high density gradient permits sound waves to accelerate rapidly into small-scale (namely, supergranulation, not stellar, size) shocks that peak rapidly and deposit their energy in the chromosphere (*22*). At the other extreme, the coolest supergiant stars must have very low atmospheric density gradients so that shocks will not accelerate but perhaps dissipate their energy gradually, thus gently depositing their energy throughout the atmosphere. If properly distributed, the deposited energy could be sufficient to overcome the weak gravitational field and allow continuous mass loss to occur.

Because the stellar material is always partially ionized to at least a small degree, magnetic fields may influence its motions. There are a few measurements of magnetic fields in pulsating stars (*33*), but theorists usually ignore them because the field configuration is unknown and the magnetic pressures are much less than the gas pressures. The complications introduced by a magnetic field are so great as to make it almost impossible to compare theory with the integral observations of the whole star.

Cross Talking

Measurements. One of the values of recognizing a scientific analogy is the opportunity for scientists investigating one phenomenon to gain insights by studying the analogous phenomenon. In comparing waves on beaches with waves in stellar atmospheres, astronomers may have more to learn from oceanographers than vice versa because the former can only observe the whole star (except for the sun), whereas the latter can place instrument packages at selected points on beaches of various gradients, thereby observing the small-scale structure of the phenomenon.

Because of the grossly different thermodynamics of ionization and radiative losses by stellar shocks and breaking ocean waves, there is no parameter of ocean waves that is analogous to the astronomical measurement of the spectrum of radiation. The stellar measurement that can best be compared with its ocean analog is that of the shock velocity, which can be inferred (by the Rankine-Hugoniot equations with the energy terms handled very carefully) from Doppler shifts that indicate the gas velocity on both sides of the shock. In water the velocity of the breaking wave relative to the water in front of it is a readily measurable parameter. Both of these velocities can be followed throughout the entire interval over which the stellar shock and breaking wave are apparent. The dependence of the shock velocity on other parameters in the ocean should reveal the relevant parameters that determine the shock velocity, and hence the degree of ionization, that are vital to the emergent radiation from stellar shocks.

Unsolved problems of variable stars. There are a number of unanswered problems in the theory of pulsating stars for which detailed observation of ocean waves might provide valuable clues. There is still no full understanding of why type II Cepheids exhibit shocks during rising light, whereas classical Cepheids of the same period, with similar effective temperature and only moderately higher surface gravity, do not. It is not fully understood why the RR Lyrae stars of period 0.45 to 0.75 days show evidence of shocks [but only very high in their atmospheres where the strongest spectral lines are formed (*34*)], whereas the short-period (1 to 5 days) Cepheids of similar chemical composition and mass but lower surface gravity do not. In this comparison, it is the lower gravity stars that do not show shocks. For the long-period variables, often called Mira stars after the first pulsating star to be discovered, many mysteries remain. These cool, pulsating stars with periods of 250 to 500 days are only a minority of the numerous stars of similar effective temperature and surface gravity. It is not really understood why the majority of such stars do not pulsate at all. Some vary with surprisingly short periods, around 50 days, and a few achieve high amplitudes and characteristically long periods of fundamental mode pulsation accompanied by the shocks mentioned above (*21, 29*). An interesting clue is provided by R Doradus, a small-amplitude star of period 338 days that shows similar emission lines (H, Si I, and Fe II) to those of the Mira stars before maximum light, but never achieves a large light amplitude. Something seems to suppress the new wave as it begins to affect the stellar atmosphere. In the ocean an offshore bar can cause dissipation of an incoming wave, but there is no known analogous phenomenon in stars, except possibly the dissociation and ionization of hydrogen. However, such phase changes do not cause dissipation but rather delay the thermal wave until the hydrogen recombines, thus introducing a phase lag.

Stellar mass loss. The relation between the parameters of the ocean wave, especially the amplitude and the return flow of the bore owing to the previous wave, determines the amplitude of the next bore. It should be possible to relate these parameters to the stellar parameters that determine the distance that the stellar gas reaches before falling back into the star. Because this distance can be compared to the radius of the star, the gravity that the expanding material feels decreases appreciably as the gas reaches maximum extension. This decreasing gravity is the analog of a beach of decreasing gradient. In the star, the next shock may arrive before the return flow has reached its previous position, so gas is gradually lifted to greater heights above the stellar surface and eventually lost (*21*). No such analogy exists on a beach unless a large bore crosses a protective structure or sandbar with little or no direct return path to the ocean. However, a careful study of wave and bore amplitudes on a very gentle beach could shed light on the small-scale dissipative effects (which are analogous to the radiation from a weak shock) on the amplitude of the bore. In this comparison, the ocean wave behaves like an isothermal shock because the shock energy is lost (in the ocean wave) by negligibly heating the water and (in the star) by radiation.

Conclusions

There are many similarities between breaking waves on an ocean beach and shock waves in stellar atmospheres, including their equations of motions. Consequently, investigations of one phenomenon can be useful for studying the analogous phenomenon. For example, when a single wave "breaks" in the atmosphere of a star, it experiences the same surface gravity everywhere, but ocean beaches with a variety of beach gradients can be found in which the changing gradient will cause variable breaking action along the same beach. Thus, the dependence of velocities, displacements, and dissipation as a function of beach gradient (the analog of stellar surface gravity) can be measured at one nearshore experimental location for the same incoming wave conditions but different tidal stages.

On a beach it is possible to observe displacement, velocity, and water depth at each point of a two-dimensional network. When observing a star, astronomers can only measure Doppler shifts and emitted radiation from the entire star. Both studies are observational rather than experimental, in that neither the oceanographer nor the astronomer can control the initial conditions. Aside from being a curiosity, the analogy of stellar shocks and ocean surf may be useful to astronomers, whose observations are limited by their inability to resolve the surface of a pulsating star.

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