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BISPECTRAL ANALYSIS OF ORDERED AND CHAOTIC VORTEX SHEDDING FROM VIBRATING CYLINDERS AT LOW REYNOLDS NUMBERS

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1. Introduction

Recent experiments [1, 2] in low-Reynolds-number flows ($40 < \mathrm{Re} < 150$) have shown that power spectra of velocity in the wake of a cylinder can have a rich and complicated structure, loosely categorized as either ordered or chaotic. In ref. [2] ordered vortex shedding from a cylinder vibrating at a high-order harmonic of its fundamental mode of oscillation was characterized by a power spectrum dominated by a narrow primary spectral peak located at the vortex shedding frequency, and associated side-band peaks, as well as harmonics of the primary spectral peak, and low-frequency peaks. On the other hand, chaotic vortex shedding was associated with wake velocity power spectra with relatively broad peaks near the shedding frequency and at very low frequencies.

Chaotic behavior may be distinguished from more ordered motions in experimental data by observation of time series, power spectra, phase space portraits and Poincaré sections, and by the calculation of quantities such as attractor dimension and Lyapunov exponent [3]. The present study uses bispectral analysis to compare and contrast ordered and chaotic vortex shedding. Bispectral analysis has been used to investigate a wide variety of nonlinear systems [4], and recently it has been used to study the transition to chaos in numerical integrations of the sine-Gordon equation [5]. The present work reports the use of bispectra to analyze the dynamics of measurements of a chaotic fluid system. Bispectral analysis (defined in section 3) is used herein to isolate the nonlinearly interacting Fourier components in the velocity field in the wake of a cylinder in low-Reynolds-number flow for the cases of ordered and chaotic vortex shedding (section 4). Bicoherence spectra enable the comparison of nonlinear interactions on a triad by triad basis, thus providing a detailed characterization of the differences between ordered and chaotic shedding. Phase coupling between Fourier components is not detectable with power spectral analysis, but such coupling is clearly shown in the auto-bicoherences of the velocity field measured in the cylinder's wake. Auto-bicoherence spectra of wake velocities in ordered vortex shedding indicate coupling between the motions at the shedding frequency, its side bands and harmonics, and their sum and difference frequencies (section 4.1). On the other hand, chaotic vortex shedding is characterized by broad bicoherence spectra, with coupling extending over a wide range of frequencies, especially very low frequencies (section 4.2). Coupling to low frequencies has also been observed during the natural transition to turbulence in a symmetric wake [6].

2. Experimental arrangement

The experiments were carried out in the low-speed, low-turbulence (u'/U less than 0.05% at a speed of 10 m/s) wind tunnel in the Department of Applied Mechanics and Engineering Sciences at the University of California, San Diego. A detailed description of the experimental arrangement is given in Van Atta and Gharib [2], while a brief description is presented here for completeness. The test section was 76 cm × 76 cm × 10 m long. The vortex shedding cylinders were steel music wires mounted horizontally in the tunnel midplane 42 cm downstream of the end of the contraction. They passed freely through clearance holes in the tunnel walls and were supported at their ends outside the tunnel walls. One end was connected to a guitar string machine head used to adjust the tension in the wire. The other end was connected to a small cantilever beam with an attached strain gauge used to measure the tension. For the data reported here, wires of diameter $d = 0.0356$ and $d = 0.0254$ cm were used (see table 1), and the working length of the wire between the two auxiliary supports inside the tunnel was 48 cm. Values for other experimental parameters are presented in table 1.

A constant temperature hot-wire anemometer was used to measure the longitudinal velocity fluctuations in the cylinder wake. The hot-wire probe body entered the wake from below at an angle of 30°, and the hot wire was placed at the midpoint of the cylinder length, and positioned near the center of the lower row of wake vortices at a downstream location of about 5 cylinder diameters. The hot-wire probe and a small Pitot-static tube used in conjunction with a Baratron pressure gauge to measure the mean free stream velocity were both mounted on a horizontal air-foil-shaped strut attached to a probe traverse outside the tunnel.

3. Bispectral definitions

For a discretely sampled time series $\eta(t)$ with the Fourier representation

$$\eta(t) = \sum_n A(\omega_n) e^{i\omega_n t} + A^*(\omega_n) e^{-i\omega_n t}, \qquad (1)$$

the power spectrum and the auto-bispectrum are defined respectively as [7, 8]

$$P(\omega_1) = E[A(\omega_1) A^*(\omega_1)], \qquad (2)$$

$$B(\omega_1, \omega_2) = E[A(\omega_1) A(\omega_2) A^*(\omega_1 + \omega_2)], \qquad (3)$$

where ω_n is the radian frequency, the subscript n is a frequency (modal) index, the A's are complex

Table 1
Values for various experimental parameters for ordered and chaotic shedding regimes.

Shedding type	Cylinder diameter (cm)	Wire tension (N)	Free stream velocity (m/s)	Reynolds number (Re)	Natural frequency of cylinder (Hz)
ordered	0.0356	4.60	2.40	52.0	234
chaotic	0.0254	7.70	4.01	65.0	75

Fourier coefficients, an asterisk indicates complex conjugate, and $E[\ ]$ is the expected-value, or average, operator. The normalized magnitude of the bispectrum, known as the bicoherence, is given by

$$b^2(\omega_1, \omega_2) = \frac{|B(\omega_1, \omega_2)|^2}{P(\omega_1)\, P(\omega_2)\, P(\omega_1 + \omega_2)}. \tag{4}$$

For a digital time series with Nyquist frequency ω_N, the auto-bicoherence is completely described by values within a triangle with vertices at ($\omega_1 = 0$, $\omega_2 = 0$), ($\omega_1 = \omega_{N/2}$, $\omega_2 = \omega_{N/2}$), and ($\omega_1 = \omega_N$, $\omega_2 = 0$).

The data sets considered here were sampled at 30 000 Hz for 17 s for each experimental run. Each time series was subdivided into segments of 1024 data points, which were fast Fourier transformed to yield the complex Fourier coefficients, $A(\omega_n)$, with a frequency resolution of 29.3 Hz. Auto-bispectra were calculated for each 1024-point segment, and then ensemble averaged over the collection of segments. Thus, the power spectra and bicoherence spectra presented here have 1000 degrees of freedom. A bicoherence value of $b > 0.1$ is statistically significant at about the 99% level for 1000 degrees of freedom [7].

4. Results

4.1. *Ordered vortex shedding*

Fig. 1 displays the power spectrum of wake velocities for the case of ordered vortex shedding. The power spectrum is dominated by a narrow peak at the vortex shedding frequency (referred to as the Strouhal frequency) $f = 850$ Hz (where $f = \omega/2\pi$), with side-band peaks located at $f = 762$ and 938 Hz, ±88 Hz from the main peak. Note (table 1) that 938 Hz is the third harmonic of the fundamental cylinder vibration frequency. There are also peaks (and associated side bands) at $f = 1700$ Hz, the first harmonic of the primary spectral peak, and at $f = 2550$ Hz, the second harmonic. Although some of the frequencies discussed here may not be precise harmonics or exact combinations of other frequencies, they are all identifiable as such within the limits imposed by the frequency resolution of the data. The wake velocity power spectrum also contains two low-frequency peaks, located at $f = 88$ and 176 Hz (fig. 1).

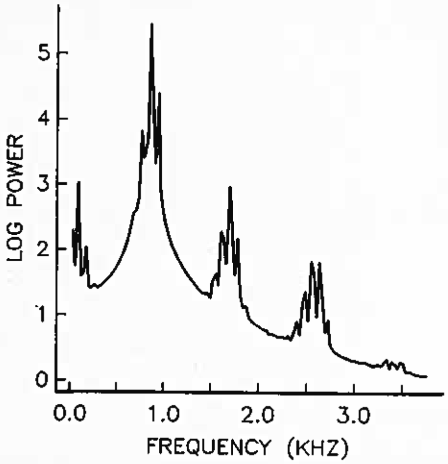

Fig. 1. Power spectrum of wake velocity for ordered shedding. The units of power are arbitrary.

The phase coupling between the wake velocity primary power spectral peak (850 Hz), its side bands, first and second harmonics, and the low frequencies is shown by the auto-bicoherence spectrum displayed in fig. 2. Several major features are evident in fig. 2, which shows f_1 and f_2, while the sum frequency $f_3 = f_1 + f_2$ is implied. First, the small circle of significant bicoherence values in the lower left-hand corner of fig. 2 (labeled "A") indicates that energy at the lowest frequency peak, $f_1 = 88$ Hz is coupled to itself, $f_2 = 88$ Hz, and its harmonic, $f_3 = 176$ Hz. Proceeding to the right in fig. 2 to the area labeled "B", the auto-bicoherence spectrum also indicates coupling between the modes located at $f_1 = 762$, $f_2 = 88$, and $f_3 = 850$ Hz. This triad consists of the power spectral peak (f_3), its lower side band (f_1), and their difference frequency (f_2). The low-frequency mode (88 Hz) is seen to be coupled to a band of modes between $f_1 = 300$ and 600 Hz,

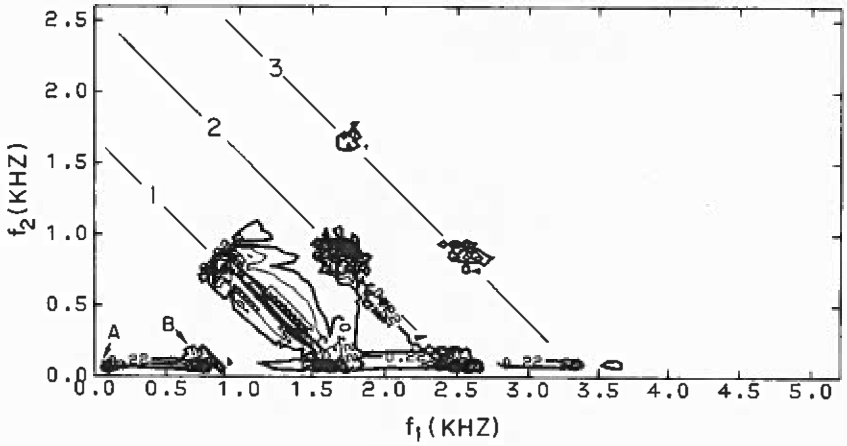

Fig. 2. Contours of wake velocity auto-bicoherence for ordered shedding. Wave triads involve frequencies f_1, f_2, and $f_1 + f_2$. The minimum value of bicoherence plotted is $b = 0.22$, with contours every 0.1. The locations labeled "A" and "B" and the numbered lines are explained in the text.

although the bicoherences are not as high as values for triads involving the power spectral peak (850 Hz) and its side bands (762 and 938 Hz). It is interesting to note that the bicoherence for the triad involving the primary spectral peak ($f_3 = 850$ Hz), its lower side band ($f_1 = 762$ Hz) and their difference frequency ($f_2 = 88$ Hz) is higher than the bicoherence value for the triad consisting of the primary spectral peak ($f_1 = 850$ Hz), its upper side band ($f_3 = 938$ Hz) and their difference frequency ($f_2 = 88$ Hz), even though the upper side band is more energetic than the lower side band (fig. 1). Thus, the bicoherence spectra indicate that nonlinear coupling between motions in the wake at the Strouhal frequency and those induced at additional frequencies not equal to the cylinder resonance appear stronger than the initial coupling with the excited cylinder vibration harmonic. This may be a consequence of the cylinder introducing velocity fluctuations into the flow at the upper side-band frequency (which corresponds to the cylinder resonance), an essentially linear flow–structure interaction.

The three numbered diagonal lines drawn in fig. 2 pass through bicoherence values for triads whose sum frequencies equal 1700, 2500, and 3400 Hz, respectively, which are harmonics of the power spectral peak (850 Hz). For example, the wide band of nonzero bicoherence values along the diagonal labeled "1" (extending approximately from the bi-frequency $f_1 = 850$, $f_2 = 850$ Hz to $f_1 = 1700$, $f_2 = 88$ Hz) indicates coupling between the three modes of those triads whose sum frequency f_3 is in the range $1500 < f_3 < 1800$ Hz, which encompasses the first harmonic of the power spectral peak (fig. 1). Along diagonal "2" there are two areas of significant coupling. The first indicates relatively strong coupling between the first harmonic ($f_1 = 1700$ Hz), the primary peak ($f_2 = 850$ Hz) and the second harmonic ($f_3 = 2550$ Hz). In addition, the second harmonic ($f_1 = 2550$ Hz) is shown to be coupled to the low-frequency peak ($f_2 = 88$ Hz) and the side band of the second harmonic ($f_3 = 2638$ Hz, almost equal in power to f_1, see fig. 1). Diagonal "3" passes through lower, but still statistically significant values of bicoherence associated with triads (from upper left along diagonal "3" to lower right) consisting of (1) the first harmonic, first harmonic, third harmonic; (2) second harmonic, primary, third harmonic; and (3) third harmonic, low-frequency, and third harmonic side band. An important aspect of fig. 2 is the coupling between the narrow low-frequency peak ($f = 88$ Hz) and almost all the energetic

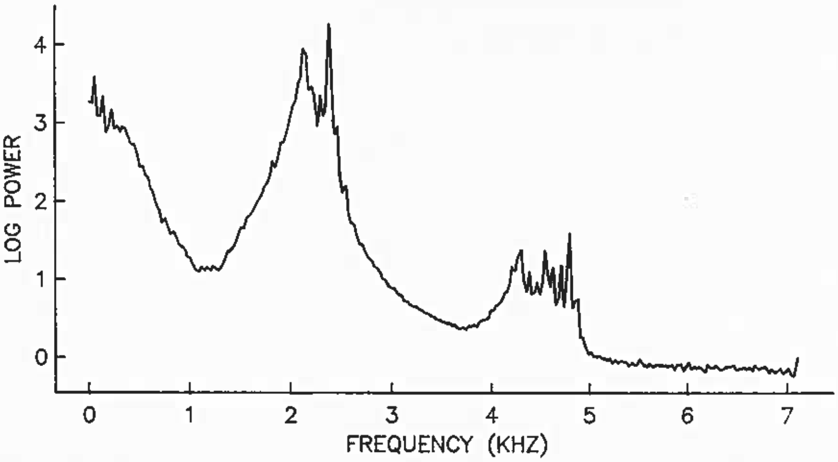

Fig. 3. Power spectrum of wake velocity for chaotic shedding. The units of power are arbitrary.

frequencies, i.e., the power spectral peak frequency and its harmonics, and the associated side bands.

Similar to the power spectrum (fig. 1), the bispectral energy of the wake velocity field in ordered vortex shedding is contained in narrow peaks (fig. 2). Thus, ordered shedding is characterized by interacting triads consisting of Fourier components located at the Strouhal frequency peak, its narrow side bands, and motions at their sum and difference frequencies.

4.2. *Chaotic vortex shedding*

The power spectrum of wake velocity in chaotic vortex shedding (fig. 3) is characterized by very broad spectral peaks and a substantial amount of low-frequency energy (compare fig. 3 with fig. 1). The Strouhal peak ($f = 2400$ Hz) in fig. 3 is located at a higher frequency than the corresponding peak in fig. 1, owing to different cylinder diameter and tension (table 1).

The auto-bicoherence spectrum of the chaotic velocity field (fig. 4) has features similar to the

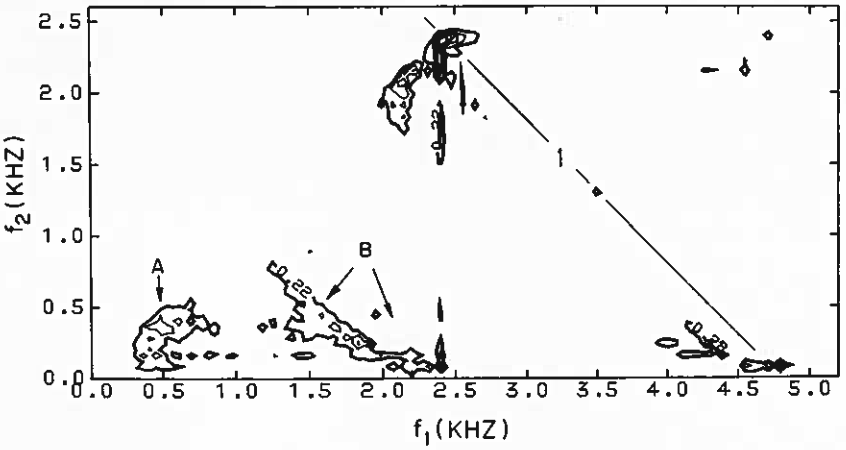

Fig. 4. Contours of wake velocity auto-bicoherence for chaotic shedding. The minimum value of bicoherence plotted is $b = 0.22$, with contours every 0.1.

ordered case (fig. 2), for example the coupling between primary and first harmonic (diagonal labelled "1" in fig. 4). However, bicoherences for the chaotic case differ from those for the ordered case, especially for triads containing a low-frequency component. Fourier components from a wide range of low frequencies interact with each other ("A" in fig. 4), and with the broad power spectral peak ("B" in fig. 4). Although some of the bicoherence values for the individual interacting triads containing a low-frequency component in chaotic shedding are lower than the corresponding values in ordered shedding, the volume under the bispectral surfaces is similar for both cases. Thus, chaotic vortex shedding is characterized by broad power spectra (fig. 3) and broad bispectra (fig. 4), especially at low frequencies.

5. Conclusions

Bispectral measurements of the velocity field in the wake of a cylinder provide quantitative information useful for comparing and contrasting ordered and chaotic vortex shedding. Autobicoherence spectra for ordered vortex shedding (fig. 2) indicate that fluid motions at frequencies with sharp peaks in the wake velocity power spectrum (fig. 1) are phase coupled to each other. This coupling is limited to Fourier components for a relatively narrow range of frequencies. On the other hand, chaotic vortex shedding is characterized by broad peaks in the velocity power spectrum at low frequencies as well as near the Strouhal frequency peak (fig. 3). Corresponding autobicoherences show coupling between a wide range of Fourier components, especially at low frequencies (fig. 4).

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