

COMMENT ON "FOURIER TRANSFORM FILTERING: A CAUTIONARY NOTE" BY A. M. G. FORBES

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Although the nine-point cosine-tapered transition band suggested by Forbes [1988] produces filters whose frequency response has side lobes smaller (thereby reducing ringing in the time domain) than those of filters with an abrupt transition between pass and stop bands, it is not the optimal design for the transition band, nor does it minimize the undesirable effects of filters designed using frequency sampling. In fact, better side lobe attenuation can be obtained with a transition band substantially narrower than Forbes' suggested nine-point cosine taper. Gold and Jordan [1969], Rabiner et al. [1970], and Rabiner and Schafer [1971] develop an optimization technique that minimizes the maximum deviation between an actual filter and an ideal filter over a specified frequency range. Typically, for a given transition bandwidth (i.e., a specified number of transition band samples), optimal values for the transition band samples are found that minimize the filter's side lobes. Rabiner et al. [1970], Rabiner and Gold [1975], and Oppenheim and Schafer [1975] present tables of the optimal transition band values for transition bands with one to four samples and discuss the effects of pass band width, total number of samples used to represent the filter, relative location of the cutoff frequency, and quantization of the filter coefficients on the optimal transition band values for a variety of filter types. For example, if the low-pass discrete Fourier transform filter discussed by Forbes is designed with three transition band samples, the optimal values of these samples [c.f. Rabiner et al., 1970, Table X] are approximately 0.7089, 0.2363, and 0.0225. The resulting filter is compared to both a filter with no transition band samples and a filter with a nine-point cosine-tapered transition band in Figures 1 and 2. There is slightly less ringing in the impulse response of the three-point optimal filter than in the nine-point cosine-tapered filter (Figure 1), and the peak stop band side lobe power of the optimal filter is almost 2 orders of magnitude less than the corresponding peak side lobe of the nine-point cosine-tapered filter (Figure 2). Not only is the Gibbs phenomenon of the optimal filter reduced in relation to that of the nine-point cosine-tapered filter, but the transition bandwidth is one-third that of the nine-point filter. Similar improvements in filter design occur for other types of filters (e.g., band pass, band stop), as is discussed in the references.

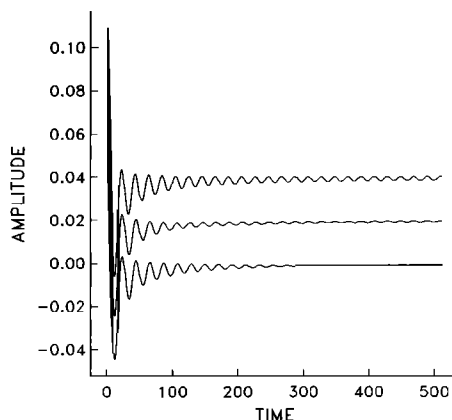


Fig. 1. Impulse response of low-pass filters: top line, filter with no transition band samples; middle line, nine-point cosine-tapered transition band; bottom line, three-point optimally designed transition band. Each curve has been vertically offset by 0.02 for display.

In conclusion, the optimization techniques of Gold and Jordan [1969] produce filters with substantially smaller side lobes and bandwidths than filters designed with the cosine tapering suggested by Forbes [1988].

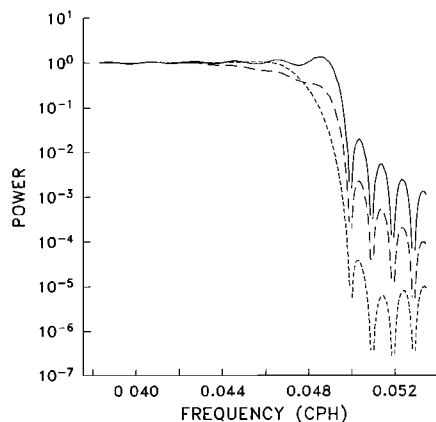


Fig. 2. Frequency response of low-pass filters: solid line, filter with no transition band samples; long-dashed line, nine-point cosine-tapered transition band; short-dashed line, three-point optimally designed transition band.

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