

Observations and modeling of the tidal bottom boundary layer on the southern flank of Georges Bank

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[1] We examine the vertical structure of the tidally driven bottom boundary layer during nearly homogeneous ($N^2 < 10^{-5} \text{ s}^{-2}$) and strongly stratified ($N^2 \sim 10^{-4} \text{ s}^{-2}$) conditions in a shallow coastal region dominated by semidiurnal tides. From moored array and shipboard measurements taken at a 76-m-deep study site on the southern flank of Georges Bank, we infer tidal velocity profiles, bottom stress estimates, Richardson numbers, and turbulent dissipation rates. On the basis of our measurements, we discuss changes in tidal boundary-layer dynamics in the presence of weak and strong stratification. We compare observations to results from two different one-dimensional numerical circulation models: a two-layer eddy viscosity model with linear eddy viscosity distribution in the lower layer and constant eddy viscosity in the upper layer (2LK), and a continuously varying eddy viscosity model (both in time and in the vertical) with Mellor-Yamada level 2.5 closure (MY2.5). Both models compare favorably with observations during nearly homogeneous conditions, but show disagreement with data when the water column is strongly stratified. In the case of 2LK, the model overestimates the bottom stress and does not reproduce the observed velocity maximum at mid-depth. This behavior is clearly related to the absence of buoyancy effects in the simplistic turbulence closure scheme. The advanced MY2.5 scheme, on the other hand, reproduces the observed velocity distribution and bottom stress well. However, the model also predicts an abrupt adjustment from the turbulent bottom boundary layer to a nearly nonturbulent region above which is not supported by our Richardson number estimates and observed turbulent dissipation rates. Potential reasons explaining the discrepancies between observations and MY2.5 include high-frequency internal-wave mixing and underestimation of the critical Richardson number used by the model to describe the transition from active to decaying turbulence.

INDEX TERMS: 4211 Oceanography: General: Benthic boundary layers; 4219 Oceanography: General: Continental shelf processes; 4560 Oceanography: Physical: Surface waves and tides (1255); 4512 Oceanography: Physical: Currents; **KEYWORDS:** bottom stress, tidal boundary layer, tidal currents

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1. Introduction

[2] In many coastal and estuarine regions, the interaction of tidal flow with rough bottom topography accounts for the major part of turbulence production at the lower boundary. The result is the formation of a tidally driven bottom boundary layer, i.e., a region of strong mixing where turbulence production is sustained by flow shear. The

bottom boundary-layer thickness varies depending on current strength, bottom roughness, and vertical stratification.

[3] In steady geophysical flows, bottom friction together with the Earth's rotation result in cyclonic veering of the velocity vector toward the bottom [Ekman, 1905]. Similar physics apply to oscillating flows in the absence of rotation, where veering of the velocity vector is replaced by phase lead of the near-bottom currents with respect to the surface [Grant and Madsen, 1986]. In the case of tidal currents, both the Earth's rotation and tidal acceleration affect the velocity vector. The result is a tidal ellipse with eccentricity

between zero (rectilinear) and one (circular). Analytical solutions predict increasing phase-advance of the tidal velocity vector with depth, which is accompanied by changes in ellipse orientation and eccentricity [Prandle, 1982a].

[4] Previous observations of tidal velocity distribution support analytical predictions of ellipse geometry. Such observations frequently focus on estuarine environments and river plumes, where tidal currents tend to be nearly rectilinear throughout at least part of the water column. For example, Visser *et al.* [1994] observed semidiurnal currents in the Rhine region of fresh water influence, which changed from nearly rectilinear at the surface to weakly anticlockwise at depth during well-mixed conditions. At the same location during stratified times, decoupling of the surface and bottom layers caused anticlockwise and clockwise rotation above and below the pycnocline, respectively, in agreement with Souza and Simpson [1996]. Also in concordance with theory, Maas and van Haren [1987] report changes in ellipse inclination and eccentricity with depth as well as phase-advance of the near-bottom velocity vector for nearly rectilinear tidal currents in the central North Sea. Maas and van Haren's [1987] study site is more typical of shelf conditions than the Rhine region of freshwater influence, where vertical stratification is greatly enhanced by riverine freshwater input. Observations of rotating semidiurnal currents have been presented by Soulsby [1983] for the Celtic Sea, also displaying qualitative agreement with analytical predictions.

[5] Analytical investigation of oscillating and tidal flows requires the a priori specification of eddy viscosity profiles $K_m(z)$ to parameterize the stress vector $\tau(z) = K_m(z) \frac{\partial \mathbf{u}}{\partial z}$. Solutions have been derived using vertically constant [Sverdrup, 1927; Prandle, 1982a, 1982b], linear [Prandle, 1982a; Soulsby, 1983], and linear-constant K_m distributions [Kagan, 1966; Trowbridge and Madsen, 1984]. Although some analytical models perform well for homogeneous conditions [e.g., Trowbridge and Madsen, 1984], they are of limited use during stratified conditions when meaningful parameterization of K_m requires consideration of buoyancy effects. An attempt to incorporate buoyant destruction of turbulence was made by Maas and van Haren [1987], who used a three-layer K_m model with K_m fitted such that model predictions best matched observed current profiles.

[6] Prognostic estimation of K_m frequently involves application of numerical models with advanced turbulence closure schemes which are based on the turbulent kinetic energy equation and prognostic or diagnostic expressions for a turbulent mixing length [e.g., Davies and Jones, 1990; Chen, 1992; Simpson and Sharples, 1994; Naimie, 1995; Simpson *et al.*, 1996]. These models rely on K_m parameterizations that attempt to account for shear production and buoyant destruction of turbulence in hydrostatic flows. Other, nonhydrostatic sources of turbulence such as high-frequency internal waves and internal-wave breaking do not form part of present models, although their effects are known to be significant [MacKinnon and Gregg, 2003].

[7] We examine the vertical structure of the tidally driven bottom boundary layer in a shallow coastal region dominated by the semidiurnal M_2 tide, i.e., the southern flank of Georges Bank. The use of an extensive observational data set with velocity measurements in and above the bed shear

layer distinguishes our study from previous work. Observations were made continuously from winter to summer at a 76-m-deep study site and combine moored temperature, conductivity, and vector-measuring current meter data with measurements from bottom tripod-mounted acoustic travel-time current meters. Our data provide tidal velocity profiles as well as estimates of bottom roughness and bed shear stress. We discuss variations of velocity distribution between nearly homogeneous and stratified conditions and derive Richardson number estimates to examine when and where turbulent mixing occurred.

[8] Using realistic bottom roughness estimates, tidal forcing, surface winds, and density distributions derived from data, we investigate the performance of two different one-dimensional numerical models with vertical resolution: a linear-constant eddy viscosity model and a turbulent energy model with Mellor-Yamada level 2.5 closure [Mellor and Yamada, 1974, 1982]. Emphasis is placed on the models' ability to accurately predict observed velocity profiles and bed shear stresses during nearly homogeneous and stratified conditions. In this context, we also discuss the role of internal waves on mixing and compare predictions from the turbulent energy model to microstructure estimates of turbulent dissipation rate made using data collected on the southern flank of Georges Bank. Comparison of modeled and measured turbulent dissipation rates on Georges Bank was first done by Horne *et al.* [1996], who compared results from Naimie's [1996] three-dimensional numerical circulation model featuring Mellor-Yamada turbulence closure to dissipation rates estimated from microstructure measurements made with the EPSON profiler at a vertically well-mixed site and a second site in the northern frontal zone. However, their comparison only partly addressed the question of model performance during stratified conditions, since (1) EPSON measurements did not cover the 10 m closest to the surface where vertical stratification is largest in summer and (2) model results were taken from model runs that used climatological average density fields rather than the exact density profile at the time when the microstructure measurements were taken. Here we run our one-dimensional numerical model assuming a density profile that matches the measured profile at the time of the measurements. Results of our model-data comparison allow for detailed evaluation of model performance, particularly the ability of the Mellor-Yamada level 2.5 turbulence closure scheme to incorporate the effects of stratification.

2. Physical Setting and Moored Array

2.1. Physical Setting

[9] Georges Bank is a shallow submarine bank located between the deeper Gulf of Maine and the continental slope (Figure 1). The basin geometry of the Gulf of Maine and adjacent Bay of Fundy is near quarter-wave resonance with the oceanic semidiurnal tidal forcing, resulting in large tidal transports across the southern flank of the bank and the shallow bank plateau [Garrett, 1972; Brown, 1984]. Depth-averaged M_2 velocities on the 76-m isobath are 40 cm s^{-1} (26 cm s^{-1}) along the major (minor) axis of the current ellipse, with amplitude modulation of 35% and 17% during the large (27.3 days) and small (14.8 days) spring-neap cycle, respectively. The orientation of the tidal ellipse is

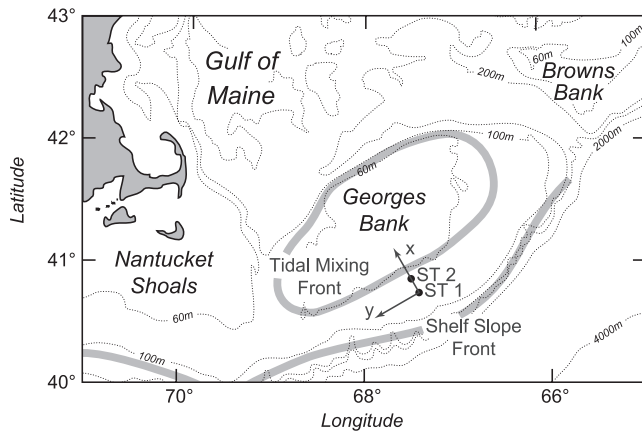


Figure 1. Bathymetry (in meters) of Georges Bank and adjacent region, approximate location of the Tidal Mixing Front (TMF) and Shelf-Slope Front (SSF), and the GLOBEC Stratification Study mooring sites ST1 and ST2. The $+x$ direction is on-bank (330°T).

approximately across-bank ($+x$) and perpendicular to the local isobath, indicative of a Sverdrup plane wave propagating on-bank from the open ocean.

[10] Temperature, salinity, and density fields on Georges Bank are subject to large seasonal variation caused by changes in meteorological forcing and horizontal gradients between adjoining water masses. In winter, convective overturning and mixing by tides and winter storms act to homogenize the water column. Over wide parts of the southern flank, weak vertical stratification is maintained in the upper water column by buoyant water from the Scotian

Shelf, which originates in the Labrador Current and the Gulf of St. Lawrence [Chapman and Beardsley, 1989]. In late spring and summer, the competing effects of bottom-friction-induced turbulent mixing and surface heating manifest as a tidal mixing front (TMF) which surrounds the bank near the 60-m isobath (Figure 1). Inside the TMF lies Georges Bank water, which is distinguished in all seasons by its vertical and horizontal homogeneity [Hopkins and Garfield, 1981]. Characteristic temperatures and salinities of Georges Bank water range from 3° to 16°C and 33.0 to 32.2 psu between winter and summer [Flagg, 1987]. Outside the 60-m isobath, a seasonal thermocline develops.

[11] Approximately 50 km to the south of the TMF, a second front, the shelf-slope front (SSF), marks the boundary from fresh shelf water to saline upper slope water with salinities 35–36 psu. The base of the SSF is located at the shelfbreak near the 100-m isobath, but may at times move into shallower water, for example, as a result of SSF intrusions caused by Gulf Stream warm core rings [Churchill *et al.*, 2003]. Density gradients across the SSF are weak in winter, when temperature and salinity tend to compensate for one another. In spring and summer, the offsetting effects of temperature diminish, and the SSF coincides with a density front. Previous investigators have shown that seasonal intensification of the SSF is in geostrophic balance with vertical shear in the along-bank direction, resulting in a gradual increase of the thermal wind currents from roughly 2 cm s^{-1} to 10 cm s^{-1} (depth-average) between winter and summer [Butman *et al.*, 1987].

2.2. Moored Array

[12] As part of the U.S. GLOBEC Northwest Atlantic/Georges Bank 1995 Stratification Study, moored current,

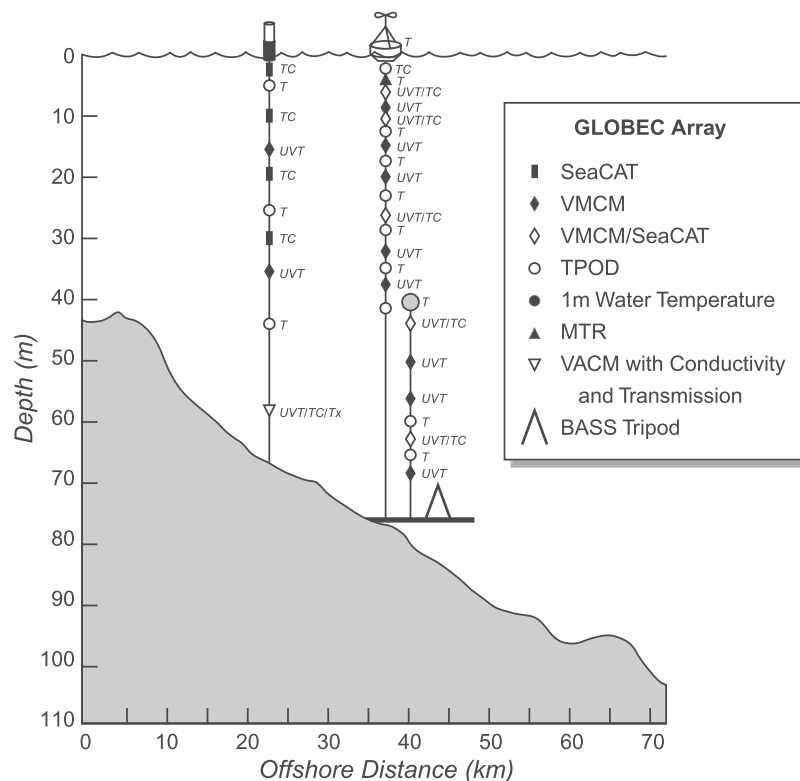


Figure 2. Schematic of the GLOBEC array at ST1 and ST2.

Table 1. Sample Rates and Elevation Above Bottom of the Moored Instrumentation at ST1

	Mooring ^a			Tripod	
	VMCM	TPOD	SeaCat	Current Meter	Thermistor
Type of measurement	velocity, temperature	temperature	temperature, conductivity	velocity	temperature
Sample rate	7.5 min	30 min	1.5 min	30 min (burst-averaged)	30 min (burst-averaged)
Sensor height (meters above bottom)	SS: 6 ^b , 12, 18 ^c , (24), 31 S: 39, 45, (51), 57 62, 66, 69, 71	SS: 9, 15 S: 35, (42), 47 54, (59), 63	SS: 11, 29 S: 50, 65, 70, 75	0.22, 0.58, 1.18, 2.53, 4.43	(0.24), 0.62, 1.22 1.90, 2.53, 3.24 4.43, 5.72

^aSS: subsurface, S: surface.^bNo temperature.^cVelocity record ends May 21, 1995.

temperature, and conductivity data were taken on the southern flank of Georges Bank [Alessi *et al.*, 2001]. Measurements were made from February 1 to August 23, 1995, on the 76-m isobath at Stratification Site 1 (ST1) located at 40°51.8'N, 67°33.5'W, approximately mid-distance between the SSF and TMF (Figure 1). The bottom slope in this region is about 8×10^{-4} . ST1 consisted of a surface and a subsurface mooring separated by 260 m, with the surface mooring occupying the northernmost position (Figure 2). The surface mooring was equipped with eight Vector Measuring Current Meter/temperature units (VMCMs, sample rate 7.5 min), four internally recording conductivity/temperature instruments (SeaCATs, sample rate 1.5 min), six temperature loggers (TPODS, sample rate 30 min), and one Miniature Temperature Recorder (MTR, sample rate 30 min) at elevations listed in Table 1. Meteorological data were taken by a Vector Averaging Wind Recorder (VAWR, sample rate 15 min) and an Improved Meteorological Recorder (IMET, sample rate 1.0 min), both mounted on the 3-m discus buoy supporting the subsurface instruments. The instrumentation of the subsurface mooring consisted of five VMCMs, two SeaCATs, and two TPODs (Figure 2 and Table 1). Measurement uncertainties were $\pm 2 \text{ cm s}^{-1}$ for VMCM velocity, 5° for the VMCM compass, $\pm 0.005^\circ\text{C}$ for TPOD, SeaCAT, and VMCM temperature, and $\pm 0.005 \text{ S m}^{-1}$ for SeaCAT conductivity (R. Limeburner, personal communication, 1996), where 0.1 S m^{-1} translates roughly into 1 psu. Consistent offsets due to calibration error were found in some temperature records, and bias $< 0.015^\circ\text{C}$ was removed from VMCM and TPOD temperature data to obtain a smooth temperature profile for nearly homogeneous conditions [Lentz *et al.*, 2003]. Also corrected were conductivity data at 70 and 65 m above the bottom for bias $< 10^{-3} \text{ S m}^{-1}$ (< 0.01 psu).

[13] Also used in this study are velocity and temperature measurements from deployment of a Benthic Acoustic Stress Sensor (BASS) tripod in winter (February 4 to April 14, 1995). The deployment site was located on the 76-m isobath about 100 m to the southwest of the subsurface mooring. BASS featured five acoustic travel-time current meters [Williams *et al.*, 1987] and eight thermistors (Table 1). Data were recorded at 2 Hz during 7.5-min-long bursts which occurred every half hour. In this study, only burst-averaged data are used. The expected compass/sensor alignment uncertainty of BASS is 8° [Werner *et al.*, 2003].

[14] Additional measurements were taken February 3 to August 4, 1995, at Stratification Site 2 (ST2) located at 40°57.4'N, 67°37.6'W on the 69-m isobath, about 12 km

onbank of ST1 (Figures 1 and 2). Because of their limited vertical resolution, we use ST2 data only for verification of the across-bank temperature, salinity, and density gradients.

3. Nearly Homogeneous Conditions

3.1. Analysis Period

[15] The period 1500 GMT February 11 to 0500 GMT March 11, 1995, was chosen for analysis for the following reasons. First, the length of the analysis period (27.6 days) allows resolution of the S_2 (12.00 hours), M_2 (12.42 hours) and N_2 (12.66 hours) semidiurnal tidal constituents. Second, the weak vertical stratification at mid-depths does not influence the near-bottom waters. This follows from BASS thermistor measurements at $z = 0.6\text{--}5.7 \text{ m}$ ($z = 0$ at the bottom) indicating that temperature was vertically homogeneous to less than $\pm 0.003^\circ\text{C}$ during 94% of the investigation period. Individual thermistor uncertainties were $\pm 0.001^\circ\text{C}$, so that $\pm 0.003^\circ\text{C}$ temperature difference correspond to the expected 95% uncertainty imposed by random instrument noise. Third, vertical stratification was extremely weak during the analysis period throughout the water column, with typical buoyancy frequencies squared $N^2 = \frac{-g}{\rho_0} \frac{\partial \rho}{\partial z} < 10^{-5}$ (Figure 3d).

3.2. Temperature, Salinity, and Density Fields

3.2.1. Time-Mean Vertical Structure

[16] Averaged over the 27.6-day analysis period, temperature, salinity, and potential density stratification from surface to bottom were $< 0.1^\circ\text{C}$, $< 0.03 \text{ psu}$ and $< 0.03 \text{ kg m}^{-3}$, respectively, where density stratification was largely determined by salinity (Figures 3a–3c). Estimates of time-mean buoyancy frequency squared were $N^2 \sim 10^{-6} \text{ s}^{-2}$ in the lower third of the water column and about 5 times greater at mid-depth (Figure 3d). Stratification decreased toward the surface where the influence of wind mixing was largest, except during a few intermittent events (February 21 and 29 and March 1) that covered less than 3% of the analysis period and were associated with rainfall. N^2 estimates in the upper $\sim 10 \text{ m}$ exceeded 10^{-4} s^{-2} during each event.

3.2.2. Richardson Number and Vertical Mixing

[17] We investigate the effects of weak vertical stratification on mixing by estimating Richardson numbers $Ri = \frac{N^2}{U^2}$. Ri estimates were derived from hourly averaged current shear using VMCM measurements at $z = 18, 31, 39, 45, 57, 62, 66$, and 71 m and buoyancy-frequency estimates from SeaCAT data at the elevations shown in Figure 3d. Velocity

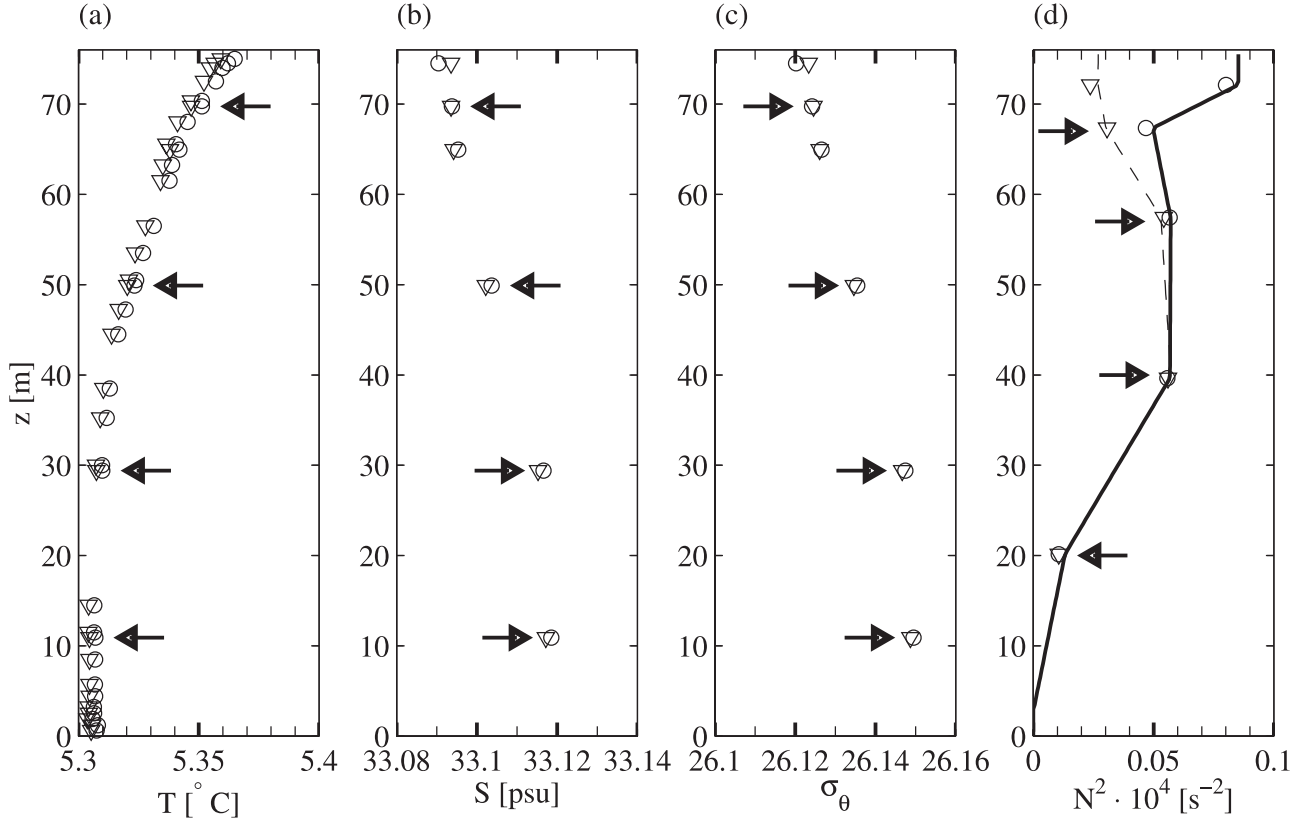


Figure 3. Vertical distribution of time-mean (a) temperature, (b) salinity, (c) potential density, and (d) buoyancy frequency squared for the winter investigation period February 11 to March 11. Diamonds and circles indicate, respectively, with and without intermittent low-salinity water intrusions excluded from the time average. The elevation above the bottom is z , given in meters. Arrows refer to elevations shown in Figure 4. Solid and dashed lines in Figure 3d are the average N^2 distribution specified in MY2.5.

shear was interpolated linearly with depth to obtain shear estimates at midpoints between the lowest five SeaCATs. Results are listed as the percentage of time that $Ri < Ri_c$ for critical values $Ri_c = (0.25, 0.5, 1.0)$ (Table 2). The lower limit of this range, $Ri_c = 0.25$, corresponds to the necessary criterion for development of growing instabilities according to linear theory [e.g., Kundu, 1990]. The upper limit, $Ri_c = 1.0$, is the necessary condition for existence of nonlinear turbulence as derived by Abarbanel *et al.* [1984] and Canuto *et al.* [2001]. Usage of $Ri < Ri_c$ to infer the onset of active turbulence assumes that measurements can resolve the vertical scale of density overturns. This was not always the case, even during the weakly stratified conditions presented here (Appendix A).

[18] Our Ri estimates suggest that $Ri < Ri_c$ for all Ri_c more than 90% of the time in the lower water column ($z = 20$ m) and near the surface ($z = 67$ m) where the respective effects of bottom friction and wind mixing were large (Table 2). At mid-depth ($z = 40$ and 57 m), $Ri < Ri_c$ about 60–90% of the time. Given the limited vertical resolution of the density estimates used to derive Ri_c (Appendix A), these results indicate that turbulent mixing took place at all depths during most, if not all, of the winter analysis period.

3.2.3. Tidal Variation

[19] The vigorous vertical mixing that occurred during the winter analysis period also contributed to weak horizontal gradients in temperature, salinity, and density at the

ST1 site. On-/off-bank advection of these gradients by the rotary tidal motion caused variations of the corresponding properties at tidal frequencies. Temperature was highest around the reversal from flood to ebb ($\alpha = 90^\circ$) when salinity and density approached their minima (Figures 4a–4c). Half a tidal cycle later, temperatures reached their lower limits, while salinity and density increased. The observed behavior indicates on-bank advection of warmer, fresher, and lighter water during flood, but off-bank advection of colder, saltier, and denser water during ebb. This implies that the temperature gradient was positive off-bank while the salinity and density gradients were positive on-bank, in agreement with

Table 2. Percent of Time That $Ri < Ri_c$ for the Nearly Homogeneous Period February 11 to March 11 at N^2 Elevations ≤ 67 m Shown in Figure 3d^a

Meters Above Bottom	$Ri < Ri_c$, %		
	$Ri_c = 0.25$	$Ri_c = 0.5$	$Ri_c = 1.0$
67	92	95	97
57	69	81	89
40	65	77	87
20	95	98	99

^aResults are from Monte Carlo simulation: Time series of N^2 and U_z were perturbed by random instrument noise, and $Ri = N^2/U_z^2$ was computed 100 times. Listed are the average percentages when $Ri < Ri_c$ for all simulations. The 95% confidence levels are $\leq 3\%$.

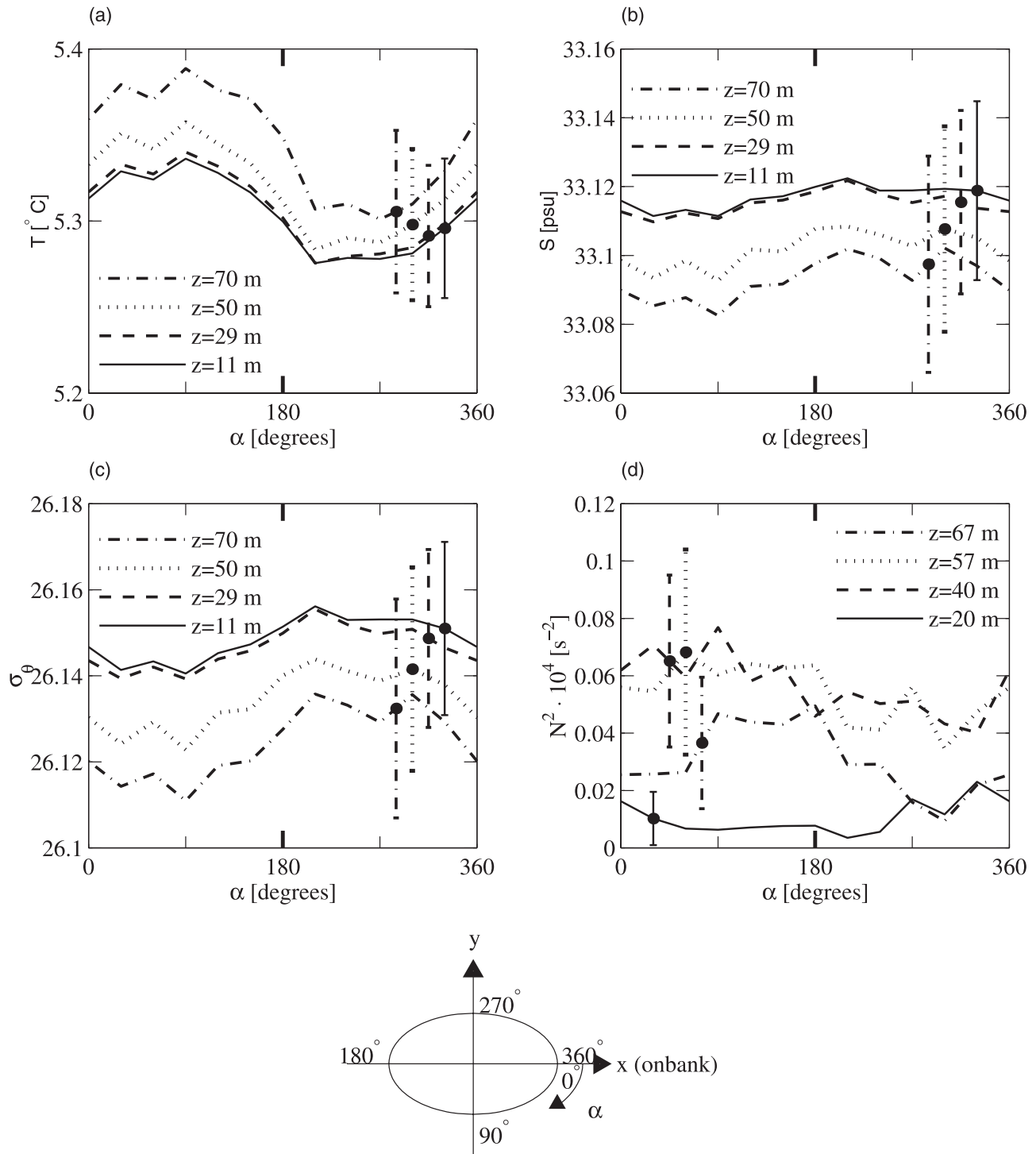


Figure 4. (a) Temperature, (b) salinity, (c) potential density, and (d) buoyancy frequency squared for February 11 to March 11 as a function of depth-averaged tidal flow direction α (see schematic at bottom of figure for definition of α) divided in bins of 30° width. Results are shown at the four elevations marked by arrows in Figure 3, with lines connecting the mean values in each direction bin. Typical error bars are shown in each panel, and denote the standard error at the 95% confidence level.

gradient estimates from ST1 and ST2 data. Colder temperatures on the crest of the bank reflect the effects of winter cooling over the shallow bank plateau [Beardsley *et al.*, 2003]. The on-bank salinity gradient reveals the influence of fresh Scotian Shelf water south of ST1. Tidal variation of

salinity was most pronounced in the upper water column (Figure 4b), suggesting that Scotian Shelf water was largely surface-trapped [Lentz *et al.*, 2003].

[20] Buoyancy frequencies also display tidal variation above the nearly well-mixed bottom waters (Figure 4d).

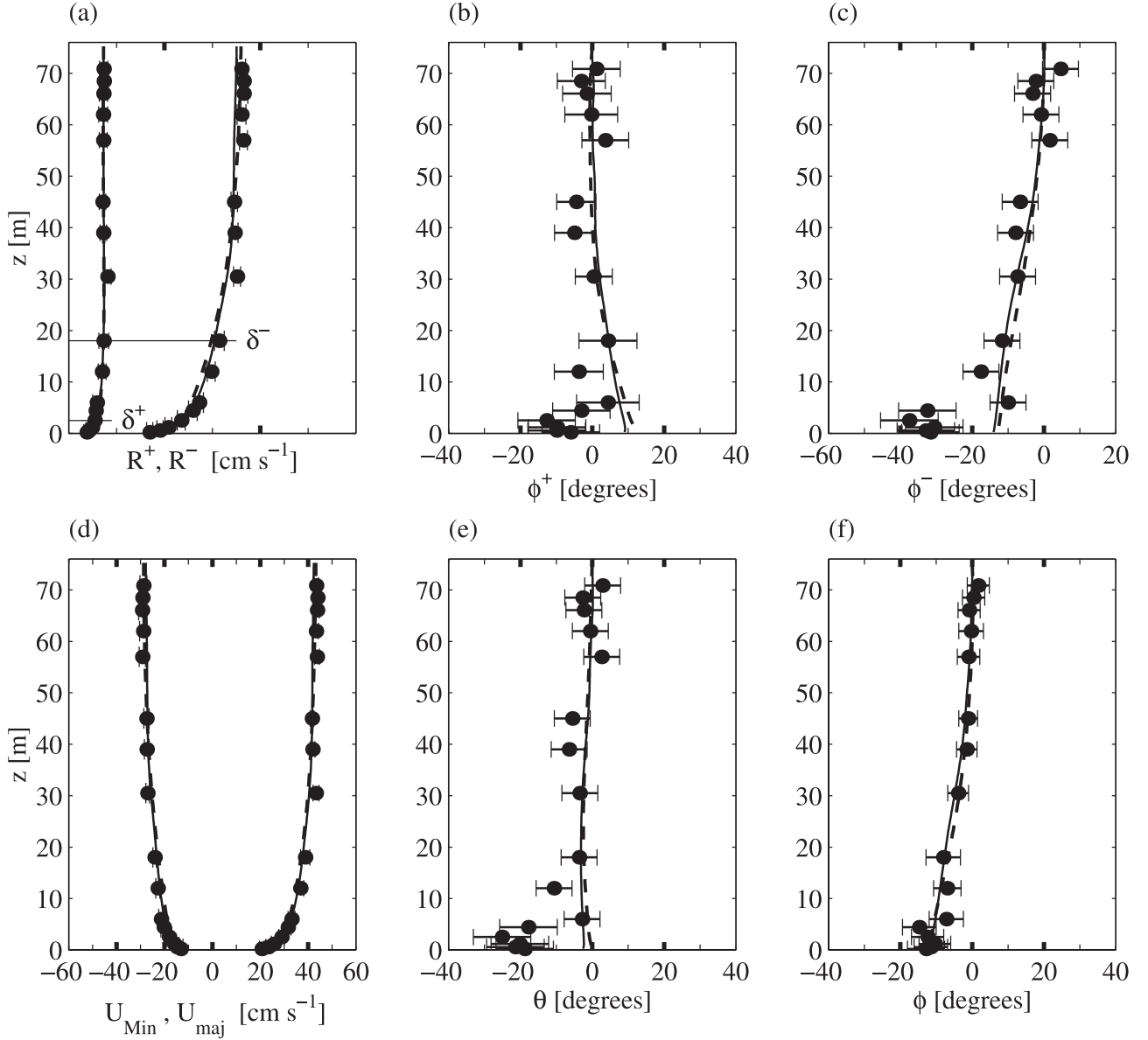


Figure 5. Profiles of M_2 current ellipse parameters for February 11 to March 11: (a) magnitudes of (smaller values) R^+ and (larger values) R^- , phase angles (b) Φ^+ and (c) Φ^- , (d) amplitudes U_{Maj} and U_{Min} , (e) inclination θ , and (f) phase of the tidal velocity vector ϕ . Phases and inclination are displayed as veering with respect to the surface (vertically averaged velocity vectors at top five VMCMs). Error bars denote the standard error of the tidal fit at the 95% confidence level, or, for $\Phi^\pm$ and θ , the compass uncertainty (depending on which is larger). Also shown are model results from 2LK (dashed line) and MY2.5 (solid line). Scale heights $\delta^\pm$ mark elevations where $R^\pm$ first exceed 90% of their near-surface values.

The general trend is smaller N^2 during the second half of ebb/first half of flood ($\alpha = 180-360^\circ$) than during the second half of flood/first half of ebb ($\alpha = 0-180^\circ$), indicating less stratified water to the north (on-bank) than to the south (off-bank) of ST1. The explanation is that bottom-friction-induced turbulent mixing is stronger in the on-bank direction as the water depth shoals toward the crest of the bank and the tidal currents increase.

3.3. Tidal Flow Dynamics

[21] The dominant tidal constituent at the ST1 mooring site is the M_2 (12.42 hours), which carries about 85% of the

total kinetic energy. Next largest are the N_2 (12.66 hours) and S_2 (12.00 hours), which contribute about 4% and 2%, respectively [Werner et al., 2003]. We present here the M_2 current ellipse parameters and their 95% confidence levels in the form of rotary components, which are reviewed in Appendix B. For easier presentation, phase and ellipse inclination are shown as veering with respect to the surface, where reference angles are defined as the vertically averaged phase and inclination between $z = 57-71$ m (the upper five VMCMs). The inclination of the surface tidal ellipse is $2 \pm 2^\circ$ counterclockwise from the across-bank axis and thus roughly perpendicular to the local isobath.

[22] Tidal decomposition was carried out on hourly averaged velocity data using Godin's harmonic method [Foreman, 1978]. Results show $R^- > R^+$ throughout the water column, implying clockwise rotation of the current ellipse (Figures 5a and 5d). Ellipse eccentricities $e = \frac{|U_{Min}|}{U_{Maj}}$ range from 0.62 to 0.67 between bottom and surface. With $\frac{f}{\omega} = 0.68$, where $f = 0.95 \times 10^{-4} \text{ s}^{-1}$ is the Coriolis parameter and $\omega = 1.41 \times 10^{-4} \text{ s}^{-1}$ is the M_2 frequency, e differs from $\frac{f}{\omega}$ by less than 9%, indicative of a Sverdrup plane wave propagating across Georges Bank. For a Sverdrup plane wave [Soulsby, 1983],

$$\mathbf{P}^\pm = -i(f \pm \omega)\mathbf{R}_S^\pm, \quad (1)$$

where $\mathbf{P}^\pm$ are the rotary components of the horizontal pressure gradient divided by density, $\mathbf{R}^\pm$ are the rotary components of tidal velocity (Appendix B), and the subscript S refers to the surface (vertically averaged currents at the upper five VMCMs). Underlying equation (1) are the assumptions that $\mathbf{R}_S^\pm$ represent flow conditions well above the bottom boundary layer and that the linear momentum equations describe the tidal flow field to lowest order [Brown, 1984]. Results from equation (1) show the M_2 pressure gradient is almost entirely across-bank (Figure 6). Alternatively, $\mathbf{P}^\pm$ may be estimated from vertical integration of the momentum equations,

$$\mathbf{P}^\pm = \frac{1}{H} \left(\tau_b^\pm + i(f \pm \omega) \int_0^H \mathbf{R}^\pm dz \right), \quad (2)$$

where $\tau_b^\pm$ are the rotary components of bottom stress divided by density (kinematic bottom stress), H is water depth, and z is elevation. Equation (2) ignores the negligible contribution of wind stress variance to the tidal-frequency band. Bottom stress estimates used in equation (2) were derived from BASS data: Logarithmic fits gave estimates of friction velocity u_* , which were fitted to current speed U to determine the quadratic drag coefficient c_D [Werner et al., 2003]. Application of the quadratic drag law at BASS sensors 1–3 and subsequent averaging of the results yields a time series of bottom stress which can be decomposed into its tidal constituents. The resulting major and minor axis of the M_2 bottom stress ellipse are $\tau_{b_{Maj}} = 1.97 \pm 0.09 \text{ cm}^2 \text{ s}^{-2}$ and $\tau_{b_{Min}} = -1.08 \pm 0.07 \text{ cm}^2 \text{ s}^{-2}$, respectively, where $\pm$ denotes the 95% confidence limits of the tidal fit obtained as described in Appendix B. Pressure estimates from equations (2) and (1) agree closely (Figure 6).

[23] Figure 5a shows that the sheared bottom boundary layer extends farther into the water column for R^- than for R^+ , in agreement with earlier observations in shelf seas and analytical predictions [e.g., Prandle, 1982a; Soulsby, 1983; Maas and van Haren, 1987]. A physical explanation is that the counterclockwise rotation of $\mathbf{R}^+$ assists the Coriolis force in balancing bottom friction, while the clockwise rotation of $\mathbf{R}^-$ has the opposite effect. For the case of a linear eddy viscosity near the bottom $K = \kappa u_* z$ consistent with BASS results, simple scaling of the momentum equation gives the boundary-layer scale height [e.g., Prandle, 1982a; Soulsby, 1983],

$$\delta^\pm \sim \frac{\kappa u_*}{|\omega_{M_2} \pm f|}, \quad (3)$$

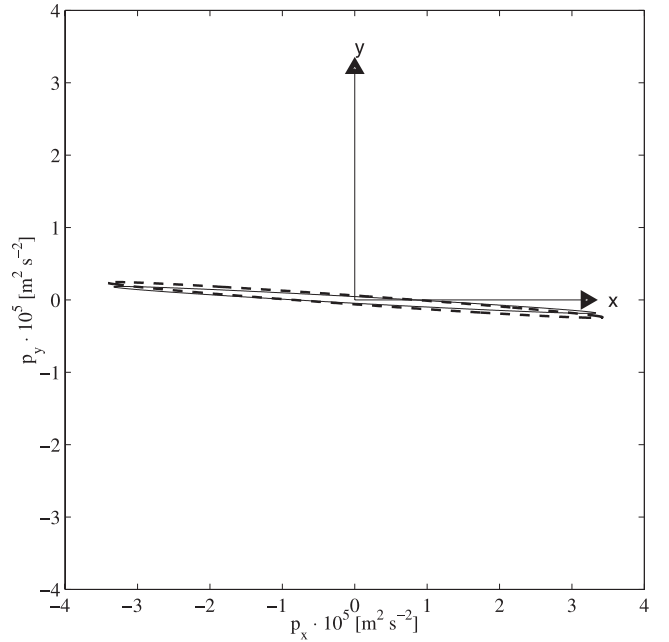


Figure 6. M_2 kinematic pressure gradient (pressure gradient divided by density) computed from the vertically averaged current vectors at the top 5 VMCMs (dashed line) (equation (1) in text) and vertical integration of the momentum equations (solid line) (equation (2) in text).

where $\kappa = 0.4$ is von Karman's constant and u_* is a characteristic bottom-friction velocity. Equation (3) gives $\frac{\delta^-}{\delta^+} = \frac{\omega+f}{\omega-f} = 5.2$. Within the confidence limits of the fits, R^+ and R^- reach more than 90% of their surface values (i.e., vertically averaged magnitudes at 57–71 m) near $z = 2.5$ and 18 m, respectively (Figure 5a). Hence $\frac{\delta^-}{\delta^+} \sim 7$, in reasonable agreement with equation (3) given the uncertainties and limited vertical resolution of the observations. Amplitudes of both rotary components display a weak maximum near 30 m, which we attribute to measurement bias. Our conclusion is based on subsequent analysis of the subtidal flow (not presented here), which indicates a 1–2 cm s^{-1} offset at the same elevation.

[24] Theory predicts phase-lead of the velocity vector near the bottom relative to the surface. This is a direct result of bottom friction, which forces velocity to be more in quadrature with the pressure gradient the smaller the distance from the bottom. With $\mathbf{R}^+$ rotating counterclockwise and $\mathbf{R}^-$ clockwise, phase-lead corresponds to $\Phi^+ > 0$, $\Phi^- < 0$, and $\Phi < 0$. Our results show that phase changes with depth are distinguishable from zero for Φ^- , but not for the much smaller Φ^+ (Figures 5b and 5c). Most of the veering of Φ^- takes place very near the bottom, i.e., between the lowest VMCM at 6 m and the BASS current meter at 4.5-m elevation. We do not believe that this behavior can be explained by tidal dynamics. Instead, we assume that the observed veering is either the consequence of spatially varying bottom topography over the 350-m distance separating the BASS and subsurface mooring sites, or that the BASS compass is subject to a 15°–20° offset in addition to the expected compass/sensor alignment uncertainty of 8°. Either reason would equally affect Φ^+ and Φ^- , which give the

Table 3. M_2 , N_2 , and S_2 Pressure Gradients Used to Force the Numerical Models^a

	M_2 (12.42 Hours)	N_2 (12.66 Hours)	S_2 (12.00 Hours)
$p_x \cdot 10^5 \text{ [m}^2 \text{ s}^{-2}\text{]}$	3.33	0.56	0.67
$p_y \cdot 10^5 \text{ [m}^2 \text{ s}^{-2}\text{]}$	0.19	0.01	0.11
ϕ_x	0.83	-2.36	-2.39
ϕ_y	-0.49	0.41	2.68

^aListed are the kinematic across- and along-bank pressure gradients p_x and p_y , respectively, and their phases ϕ_x and ϕ_y (in radians) with respect to Greenwich.

direction of $\mathbf{R}^+$ and $\mathbf{R}^-$ relative to the on-bank axis at the center of the investigation period (Appendix B). The resulting shift of Φ^+ would then propagate into ellipse inclination $\theta = 0.5 \times (\Phi^+ + \Phi^-)$, which is significantly different from the surface only at the BASS current meters (Figure 5d). On the other hand, such a shift would not affect the tidal phase $\Phi = 0.5 \times (\Phi^- - \Phi^+)$ which refers to the time of maximum current independent of orientation. Hence we deduce from Figure 5f that the near-bottom velocities measured by BASS lead the surface currents by at least 5° within confidence limits, corresponding to a phase advance of ~ 10 min.

3.4. Numerical Modeling

3.4.1. Numerical Models

[25] We compare observations of tidal currents and bottom stress to numerical results from two different one-dimensional circulation models with vertical resolution: a two-layer eddy viscosity model with linear-constant eddy viscosity distribution (hereinafter 2LK) and a continuously varying eddy viscosity model utilizing the Mellor-Yamada level 2.5 turbulence closure scheme (hereinafter MY2.5). Both models integrate the linearized along-bank and across-bank momentum equations forward in time, utilizing the parameterization $(\tau_x, \tau_y) = K_m (\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z})$ to describe the stress vector. Model time step and vertical grid spacing are 60 s and 0.5 m, respectively. Velocity and density are evaluated at mid-depth between grid points, yielding $z_b =$

0.25 m as the lowest elevation above the bottom for which velocity is computed.

[26] We implemented tidal forcing as surface pressure gradients derived from equation (2), using bottom stress estimates and velocity measurements for February 4 to April 14. To account for the spring-neap modulation of the M_2 by the N_2 and S_2 , we integrated equation (2) for all three semidiurnal constituents (Table 3). Both 2LK and MY2.5 use the bottom boundary condition,

$$K_m \left(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z} \right) = (\tau_b^x, \tau_b^y) \quad \text{at } z = z_0, \quad (4)$$

where τ_b^x and τ_b^y are the across-bank and along-bank components of bottom stress, respectively, and z_0 is apparent bottom roughness. Computation of bottom stress follows the quadratic drag law

$$(\tau_b^x, \tau_b^y) = c_D \times \sqrt{u^2 + v^2} (u, v), \quad (5)$$

where

$$c_D = \left[\frac{\kappa}{\ln \left(\frac{z_b}{z_0} \right)} \right]^2, \quad (6)$$

u and v are across-bank and along-bank velocity, respectively, and $z_b = 0.25$ m is the elevation of the first grid cell above the bottom. Using estimates of $|\tau_b|$ from logarithmic fits to BASS velocity measurements and the measured current speed at BASS sensor 3, we derived $c_D = 3.0 \pm 0.1 \times 10^{-3}$ at 1.18-m elevation above the bottom [Werner *et al.*, 2003]. Using equation (6), the central estimate of c_D translates into the apparent bottom roughness $z_0 = 0.7$ mm which enters our model.

[27] At the upper boundary, we applied the surface boundary condition,

$$K_m \left(\frac{\partial u}{\partial z}, \frac{\partial v}{\partial z} \right) = (\tau_{wx}, \tau_{wy}) \quad \text{at } z = H, \quad (7)$$

where τ_{wx} and τ_{wy} are across- and along-bank wind stress divided by density, respectively. Hourly wind stress

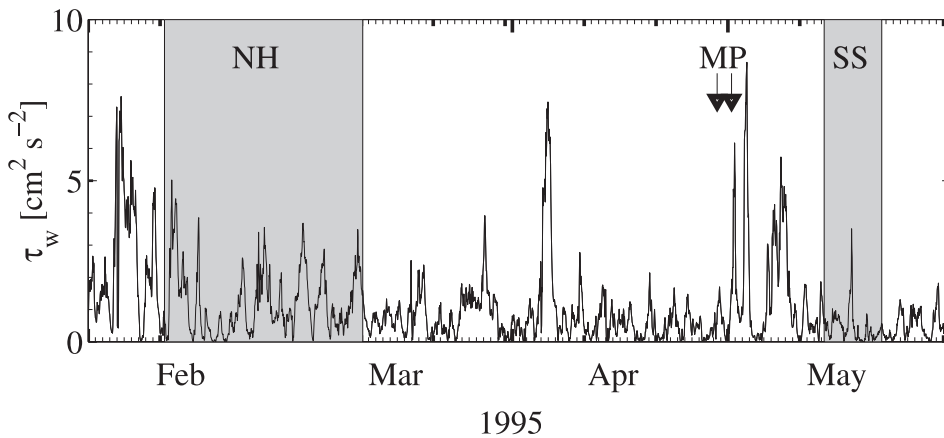


Figure 7. Time series of kinematic wind stress amplitude (wind stress divided by density) at ST1. Shaded areas indicate the nearly homogeneous (NH) and strongly stratified (SS) analysis periods February 11 to March 11 and May 14–22, respectively, and arrows mark when microstructure profiles (PM) were taken.

Table 4. M_2 Bottom Stress Estimates Along the ($\tau_{b_{\text{Major}}}$) Major and ($\tau_{b_{\text{Minor}}}$) Minor Axes of the Near-Bottom Current Ellipse From Data, and the Two Models 2LK and MY2.5^a

	$\tau_{b_{\text{Major}}} (>0)$ $\tau_{b_{\text{Minor}}} (<0)$		
	Data	2LK	MY 2.5
Feb. 11 to March 11	1.97 ± 0.09	2.04	2.14
	-1.08 ± 0.07	-1.03	-1.06
May 14 to May 22	2.38 ± 0.18	3.04	2.43
	-1.11 ± 0.11	-1.59	-1.03

^aObservational estimates are from harmonic analysis of the bottom stress vector derived from the quadratic drag law using BASS measurements [Werner et al., 2003]. Numbers denoted by $\pm$ give the 95% confidence level of the tidal decomposition.

estimates were derived from ST1-meteorological data using the *Large and Pond* [1981] neutral stability algorithm (Figure 7). Wind stress was interpolated linearly in time to match the 60-s model time step.

[28] Buoyancy forcing does not enter 2LK, since its parameterization of eddy viscosity depends only on bottom friction (Appendix C). On the other hand, MY2.5 accounts for effects of vertical stratification on turbulent mixing and flow dynamics (Appendix D). For meaningful comparison to observations, MY2.5 requires either the choice of initial density conditions such that model solutions evolve toward the observed density distribution, or the a priori specification of density both in space and time. We chose the second approach, since the physical processes sustaining vertical stratification on Georges Bank cannot be modeled with a one-dimensional model (Appendix D). A time series of density distribution was derived such that vertical stratification matched N^2 estimates from data. These estimates were interpolated linearly with depth to give N^2 (and thus density) on the model grid (Figure 3d). On the basis of tripod temperature

measurements, we assumed that the first 3 m above the bottom were well mixed ($N^2 = 0$).

[29] We applied continuous model forcing to 2LK and MY2.5 starting on February 1, 1995, averaged and stored the results at hourly intervals, and extracted those model data that correspond to the investigation periods of our observations. Tidal decomposition of the predicted currents and bottom stress followed the procedure applied to measurements.

3.4.2. Model-Data Comparison

[30] Tidal velocity distributions predicted by 2LK and MY2.5 compare well with observations (Figure 5). However, neither model reproduces the abrupt veering of $\Phi^\pm$ and θ near the bottom suggested by BASS measurements (Figures 5b, 5c, and 5e). Instead, both models propose gradually increasing (decreasing) Φ^+ (Φ^-) toward the bottom, and almost no rotation of the current ellipse with depth. Hence model results support the notion that either the ellipse orientation at the BASS deployment site differs from that at the subsurface mooring, or that the compass uncertainty of BASS is larger than expected by about 15° – 20° . Numerical results closely reproduce the observed phase-lead of the near-bottom velocity vector, which is independent of the orientation of the current ellipse (Figure 5f). M_2 bottom-stress estimates from 2LK and MY2.5 are within 5% and 9% of those derived from BASS data, respectively (Table 4).

[31] The 2LK and MY2.5 predict time-mean eddy viscosities that are on the order of 300 – $400 \text{ cm}^2 \text{ s}^{-1}$ in the depth average (Figure 8a). Hence both models suggest that K_m exceeds the molecular diffusivity of sea water ($\nu = 10^{-2} \text{ cm}^2 \text{ s}^{-1}$) by several orders of magnitude, indicating turbulent mixing happens at all depths. This is in agreement with Richardson number estimates from measurements suggesting that $Ri < Ri_c$ ($Ri_c = 0.25$ – 1.0) during more than half of

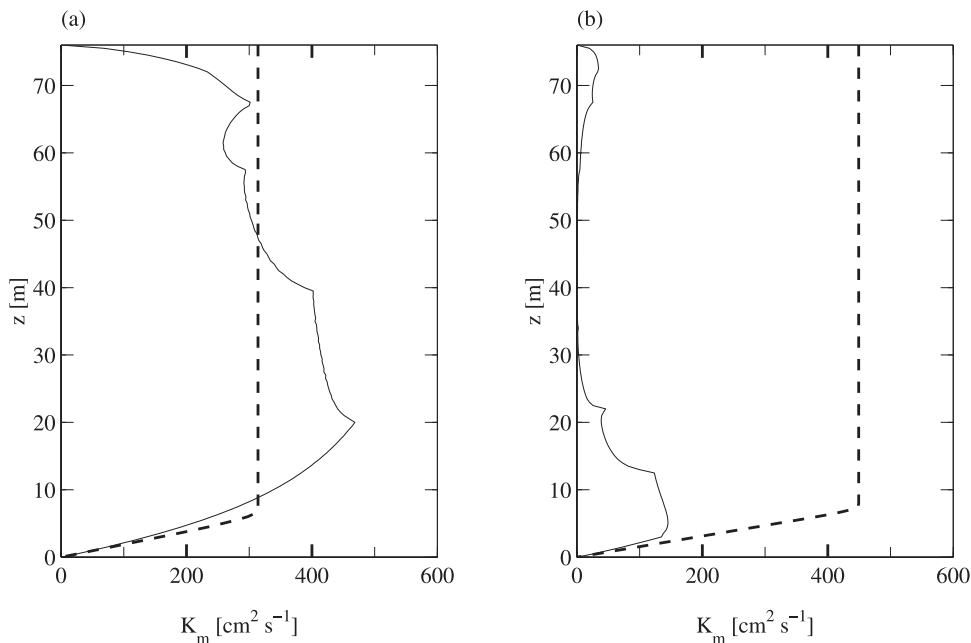


Figure 8. Time-averaged eddy viscosity profiles from 2LK (dashed line) and MY2.5 (solid line) for (a) the nearly homogeneous period February 11 to March 11 and (b) the stratified period May 14–22.

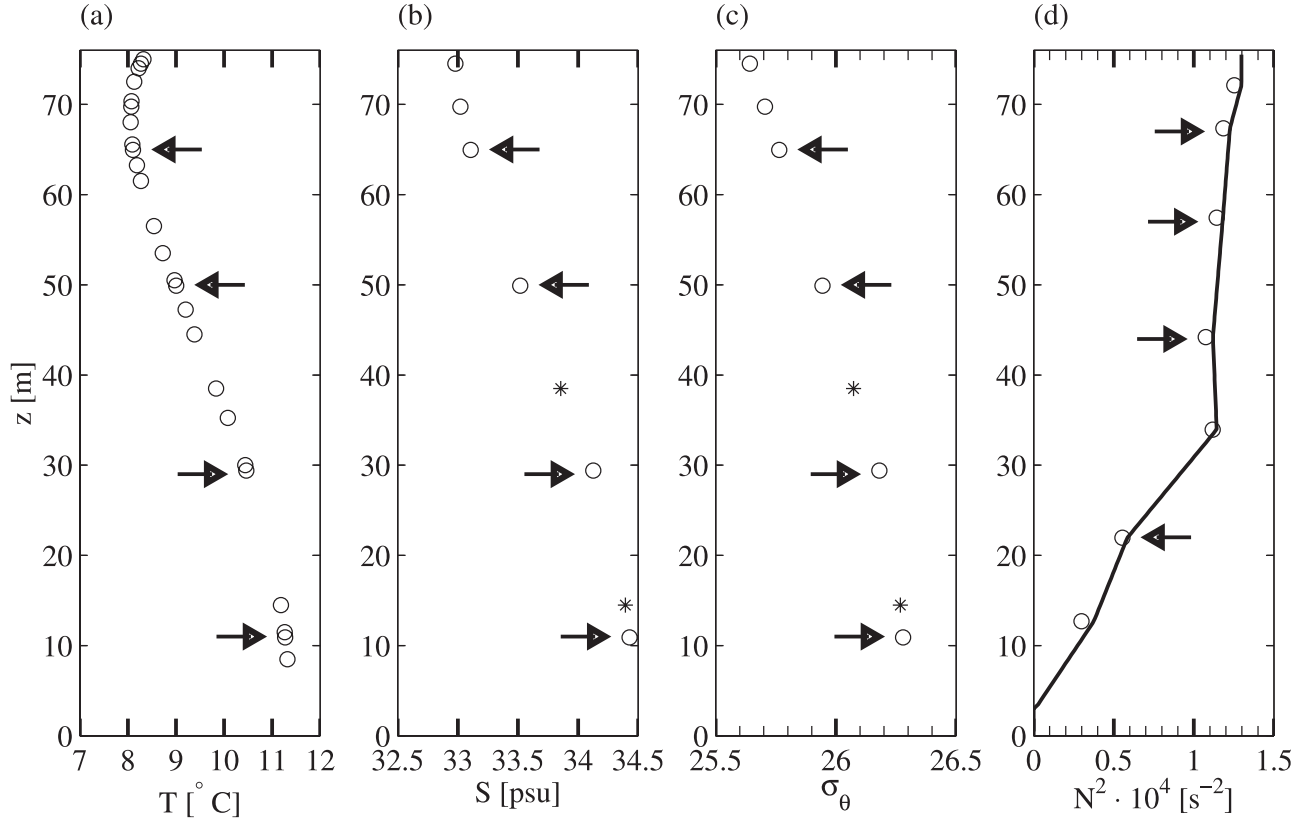


Figure 9. Vertical distribution of time-mean (a) temperature, (b) salinity, (c) potential density, and (d) buoyancy frequency squared for the SSF intrusion May 14–22. Arrows refer to elevations shown in Figure 11. The solid line in Figure 9d is the average N^2 distribution specified in MY2.5.

the investigation period throughout the water column (Table 2).

4. Strongly Stratified Conditions

4.1. Analysis Period

[32] We examine the tidal boundary layer for the stratified period 1000 GMT May 14 to 1000 GMT May 22, 1995, when typical buoyancy frequencies squared were $N^2 \sim 10^{-4} \text{ s}^{-2}$ (Figure 9d). The relatively short length of this analysis period (8 days) ensures that density and buoyancy frequency did not undergo major changes on timescales larger than tidal. This is important for our analysis, since effects of stratification on tidal flow dynamics are difficult to investigate if the analysis period is long enough to allow for significant subtidal variation of the density field.

[33] On the southern flank of Georges Bank, surface heat flux, surface winds, and migration of the SSF can all influence the density field. On-/off-bank migration of the SSF has particularly strong effects on density distribution, and happens on timescales of days to weeks. The analysis period chosen here is part of an SSF intrusion, i.e., an event when the base of the SSF moved from its average position near the 100-m isobath toward shallower water [Churchill *et al.*, 2003].

4.2. Temperature, Salinity, and Density Fields

4.2.1. Time-Mean Vertical Structure

[34] Measurements taken at ST1 show an intrusion of high-temperature and salinity water on May 6 which per-

sisted for about 16 days (Figure 10). Strong off-bank winds preceded the event between May 1 and 4, subsided temporarily on May 5, and picked up again on May 6–9 (Figure 7). The intrusion was bottom-trapped before May 14, and extended to the surface at later times. Near-bottom salinities increased from 32.6 psu before the event to 35.2 psu during the event, indicating that slope water (characteristic salinities 35–36 psu) displaced fresher Georges Bank water (characteristic salinities 32.2–33 psu).

[35] The second phase of the SSF intrusion (May 14–22) was marked by mostly calm winds (Figure 7) and strong stratification ($N^2 \sim 10^{-4} \text{ s}^{-2}$) in the upper half of the water column (Figure 9d). Average differences in temperature, salinity, and potential density between surface and bottom were about 3.5°C , 1.5 psu, and 0.7 kg m^{-3} , respectively (Figures 9a–9c). Salinity and density profiles in Figure 9 include estimates at $z = 14.5$ and 39 m. These estimates were derived from VMCM and TPOD temperature measurements assuming a linear T-S relation and utilizing the vertically interpolated coefficients from T-S fits at the SeaCAT depths. Fits were performed using 24 hours of hourly averaged data, with the centerpoint of the time window passing through the T-S data records.

4.2.2. Richardson Number and Vertical Mixing

[36] We computed estimates of Richardson number at the stratification depths shown in Figure 9d, following the procedure outlined above. Results indicate that $Ri < 0.25$ –1.0 about 40–90% of the analysis period in the lower ($z = 13$ and 22 m) water column, and 40–80% in the upper ($z = 67$ m) water column (Table 5). These regions are near

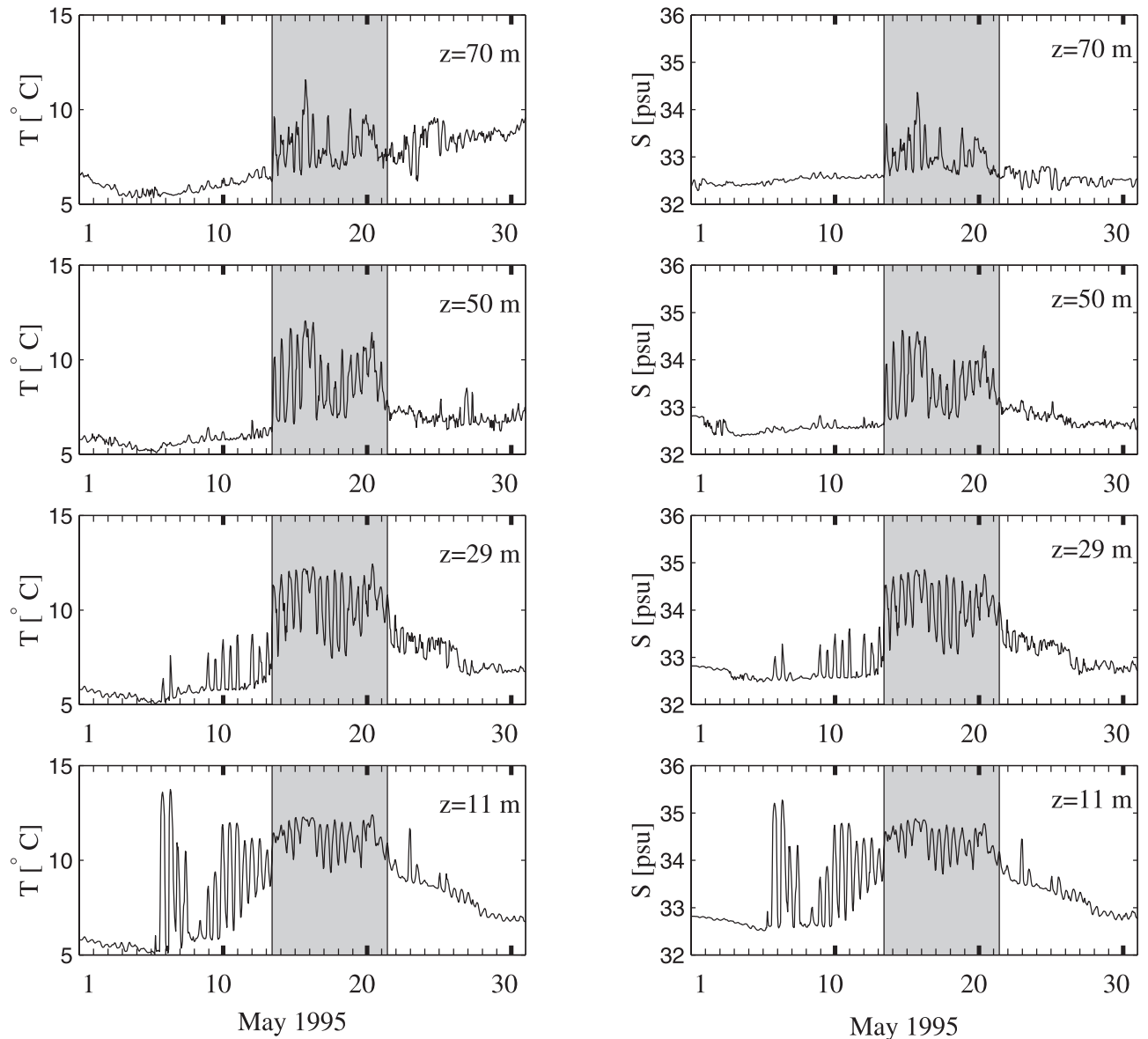


Figure 10. Time series of (left) temperature and (right) salinity during the May SSF intrusion from ST1 SeaCAT data at $z = 70, 50, 29$, and 11 m. Shaded areas mark the May 14–22 analysis period.

the bottom and surface boundaries where turbulence was produced by wind- and bottom-friction-induced momentum fluxes, respectively. At mid-depth, events with $Ri < Ri_c$ were less frequent. Table 5 suggests that less than about (10, 20, 40)% of all Ri estimates remain below (0.25, 0.5, 1.0) at $z = 34$ – 57 m. Estimates of overturning scale are 1–2 m and thus significantly smaller than the 9- to 15-m vertical resolution of the measurements in the pycnocline (Appendix A). Hence even the small percentage of time when $Ri < Ri_c$ observed here is a strong indication for $Ri < Ri_c$ on smaller, unresolved scales.

4.2.3. Tidal Variation

[37] Temperature, salinity and potential density in Figures 11a–11c display maxima at the reversal from flood to ebb ($\alpha = 90^\circ$) and minima at the reversal from ebb to flood ($\alpha = 270^\circ$). This behavior implies that the horizontal temperature, salinity, and density gradients were positive off-bank, in agreement with gradient estimates from ST1 and ST2

data. Compared to the earlier analysis period February 11 to March 11, salinity and density gradients have reversed.

[38] Buoyancy frequencies in the lower water column (< 30 m) were largest at the reversal from ebb to flood and

Table 5. Percent of Time That $Ri < Ri_c$ for the Stratified Period May 14–22 at N^2 Elevations ≤ 67 m Shown in Figure 9d^a

Meters Above Bottom	$Ri < Ri_c$, %		
	$Ri_c = 0.25$	$Ri_c = 0.5$	$Ri_c = 1.0$
67	43 ± 4	62 ± 4	78 ± 3
57	6 ± 3	18 ± 3	38 ± 5
44	10 ± 2	20 ± 3	40 ± 4
34	6 ± 1	14 ± 3	39 ± 4
22	43 ± 4	78 ± 3	94 ± 3
13	64 ± 4	80 ± 4	90 ± 3

^aResults are from Monte Carlo simulation: Time series of N^2 and U_z were perturbed by random instrument noise, and $Ri = N^2/U_z^2$ was computed 100 times. Listed are the average percentages when $Ri < R_c$ with 95% confidence levels.

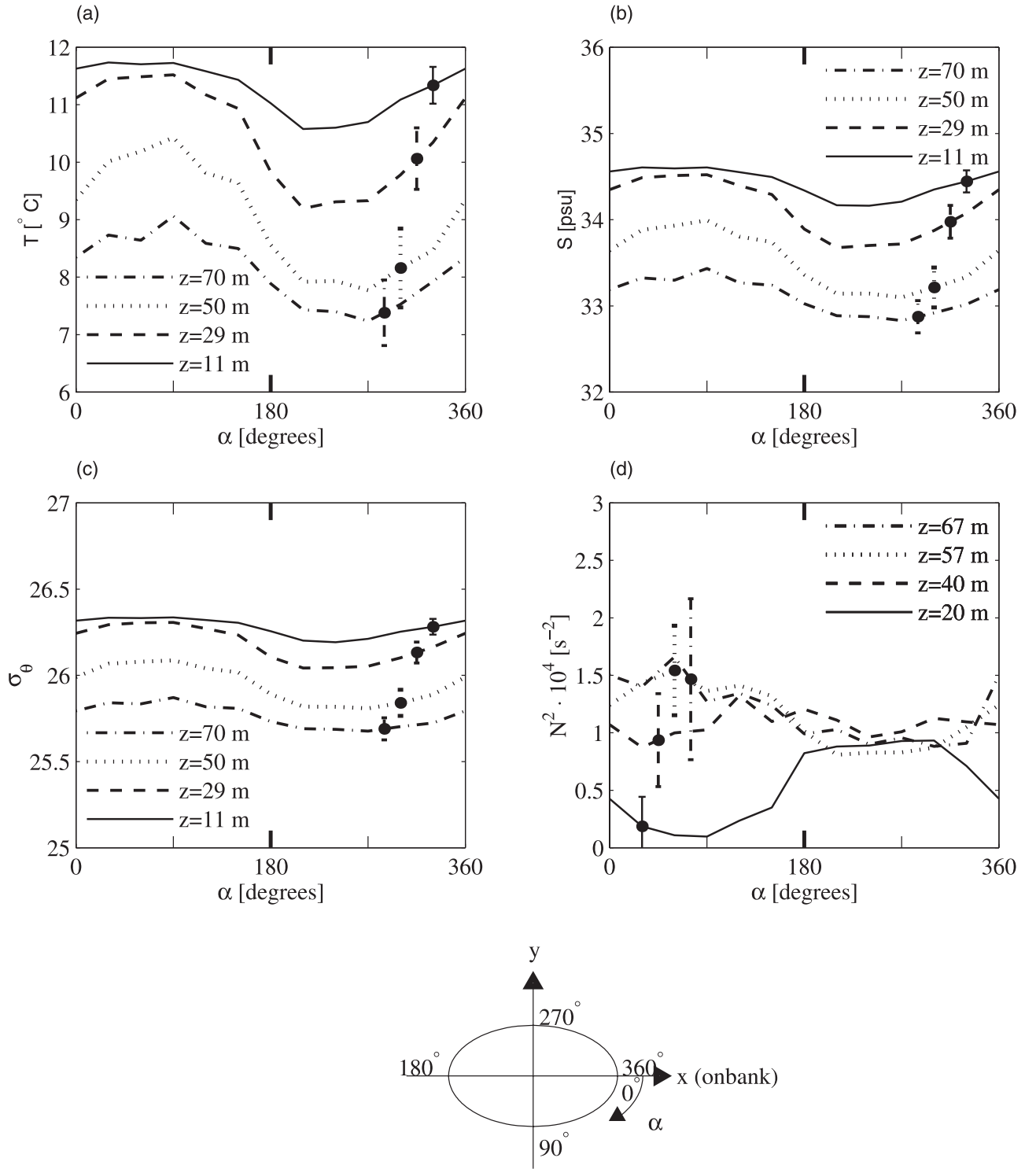


Figure 11. (a) Temperature, (b) salinity, (c) potential density, and (d) buoyancy frequency squared for May 14–22 as a function of depth-averaged tidal flow direction α (see schematic) divided in bins of 30° width. Results are shown at the four elevations marked by arrows in Figure 9, with lines connecting the mean values in each direction bin. Typical error bars are shown in each panel and denote the standard error at the 95% confidence level.

smallest at the reversal from flood to ebb (Figure 11d), indicating that the region north of ST1 was more stratified than the region to the south. The situation was reversed above 50 m, where tidal variation of N^2 implies advection of

more stratified water onto the bank during flood and less stratified water off the bank during ebb. At mid-depth (38–50 m), N^2 estimates remained nearly constant throughout the tidal cycle. The implication is that the closely spaced

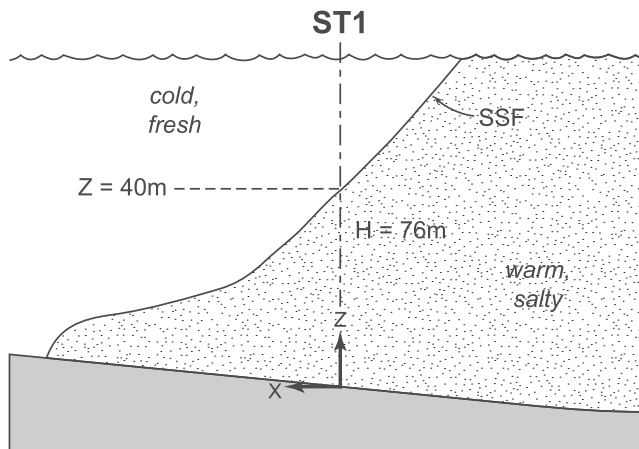


Figure 12. Schematic of the May SSF intrusion.

isopycnals defining the SSF intercepted the ST1 mooring site around 40 m above the bottom, causing large N^2 below the intercept to the north and above the intercept to the south (Figure 12).

4.3. Tidal Flow Dynamics

[39] We performed harmonic decomposition of hourly averaged VMCM data for May 14–22 (BASS data were not available for this period). Results for U_{Maj} and U_{Min} were larger by about 5 cm s^{-1} than for the February 11 to March 11 analysis period (Figure 13d). The explanation is that May 14–22 coincided with spring tide conditions, and that the short investigation period (8 days) does not allow for resolution of the spring-neap cycle. Our results show that velocity amplitudes were strongly sheared in the lower water column, reached a local maximum near $z = 30 \text{ m}$, and were nearly homogeneous for $z > 50 \text{ m}$ (Figures 13a and 13d). Since no mid-depth velocity maximum was observed during the nearly homogeneous analysis period, we conclude that its appearance in Figures 13a and 13d is a direct result of strong stratification. To confirm this conclusion, we produced an artificial velocity time series using the resolved rotary components for the nearly homogeneous period February 11 to March 11 to perform a tidal prediction. Next, we found the tidal constituents of the predicted time series for May 14–22. Results show no velocity maximum at mid-depth, indicating that for similar tidal forcing such a maximum occurs only if the water column is stratified (Figures 13a and 13d). Our results are in agreement with earlier observations from the North Sea, which displayed mid-depth current peaks similar to Figure 13a during stratified conditions [Maas and van Haren, 1987]. In both cases, buoyancy forcing restricted the turbulent momentum transfer across the pycnocline.

[40] On the basis of the ratio of boundary-layer scale heights $\frac{\delta^-}{\delta^+} = \frac{\omega+f}{\omega-f} = 5.2$, the boundary layer of the clockwise-rotating component R^- extends farther into the stratified interior than that of the counterclockwise component R^+ . Hence stratification mostly affects R^- , as explained conceptually by Souza and Simpson [1996]. In agreement with theory, the mid-depth current maximum impacts only R^- to a visible degree (Figure 13a). Also, as a result of vertical stratification, Φ^- leads stronger in the lower water column than during nearly homogeneous conditions

(Figure 13c). Veering of Φ^+ toward the bottom is slightly positive (corresponding to phase-lead), but barely distinguishable from zero within error limits (Figure 13b). The observed phase-advance of the near-bottom currents is about 20 (40 min) for May 14–22 (Figure 13f) as opposed to about 10 (20 min) for February 11 to March 11 (Figure 5f). Within error limits, changes of ellipse inclination with depth are $< 3^\circ$ (Figure 13e).

4.4. Model-Data Comparison

[41] Results from 2LK fail to reproduce the observed velocity maximum at mid-depth and the pronounced phase-advance of the current vector near the bottom (Figure 13). The model's behavior is explained by the fact that buoyancy forcing does not enter the model, and thus solutions cannot account for the effects of stratification. Also, due to the absence of buoyancy effects, 2LK overestimates velocity shear in the lowest 15 m of the water column and predicts bottom stress values that exceed observations by 28–52% (Table 4). Since bottom tripod data were not available between April and July, we estimated bottom stress from VMCM measurements at $z = 6 \text{ m}$ using $c_D = 2 \times 10^{-3}$ based on the 1995 winter and summer BASS deployments [Werner et al., 2003].

[42] In better agreement with observations, MY2.5 predicts a local maximum of R^- (and hence U_{maj} and U_{min}) near $z = 25 \text{ m}$ as well as pronounced phase veering of Φ^- and Φ in the lower water column (Figure 13). Bottom stress estimates from MY2.5 are within 2–7% of observations (Table 4). Despite the model's ability to incorporate the effects of stratification, model results underpredict the observed current amplitudes at mid-depth. One reason is that the model allows for almost no mixing in an approximately 25-m-thick region between the turbulent bottom and surface boundary layers. This is apparent from the predicted eddy viscosity distribution, which suggests K_m nearly reduces to molecular background diffusion between $z = 30\text{--}60 \text{ m}$ (Figure 8b). Hence turbulent momentum transfer is almost nonexistent in the pycnocline according to these model results. The consequence is an abrupt transition from the predicted velocity maximum near the top of the turbulent bottom-boundary layer to a nearly frictionless interior region at mid-depth. Our Ri analysis does not support such behavior, but suggests that mixing takes place at all depths (section 4.2.1).

5. Observations and Model Predictions of Turbulent Dissipation

[43] For better evaluation of MY2.5's performance in the pycnocline, we compare model predictions of turbulent dissipation rate ϵ to estimates from microstructure measurements taken at ST1. Measurements were made on April 29 and May 1, 1995, preceding the May SSF intrusion [Burgett, 1997; Burgett et al., 2001], and produced 6- and 10-hour's worth of good data, respectively. Surface wind conditions were calm to medium strong, with time-mean wind stress values around 0.4 dyne cm^{-2} (April 29) and 1.4 dyne cm^{-2} (May 1). Measurements on May 1 were made immediately before the onset of strong off-bank winds.

[44] For both microstructure measurement periods, the time-mean density profiles were nearly homogeneous in the

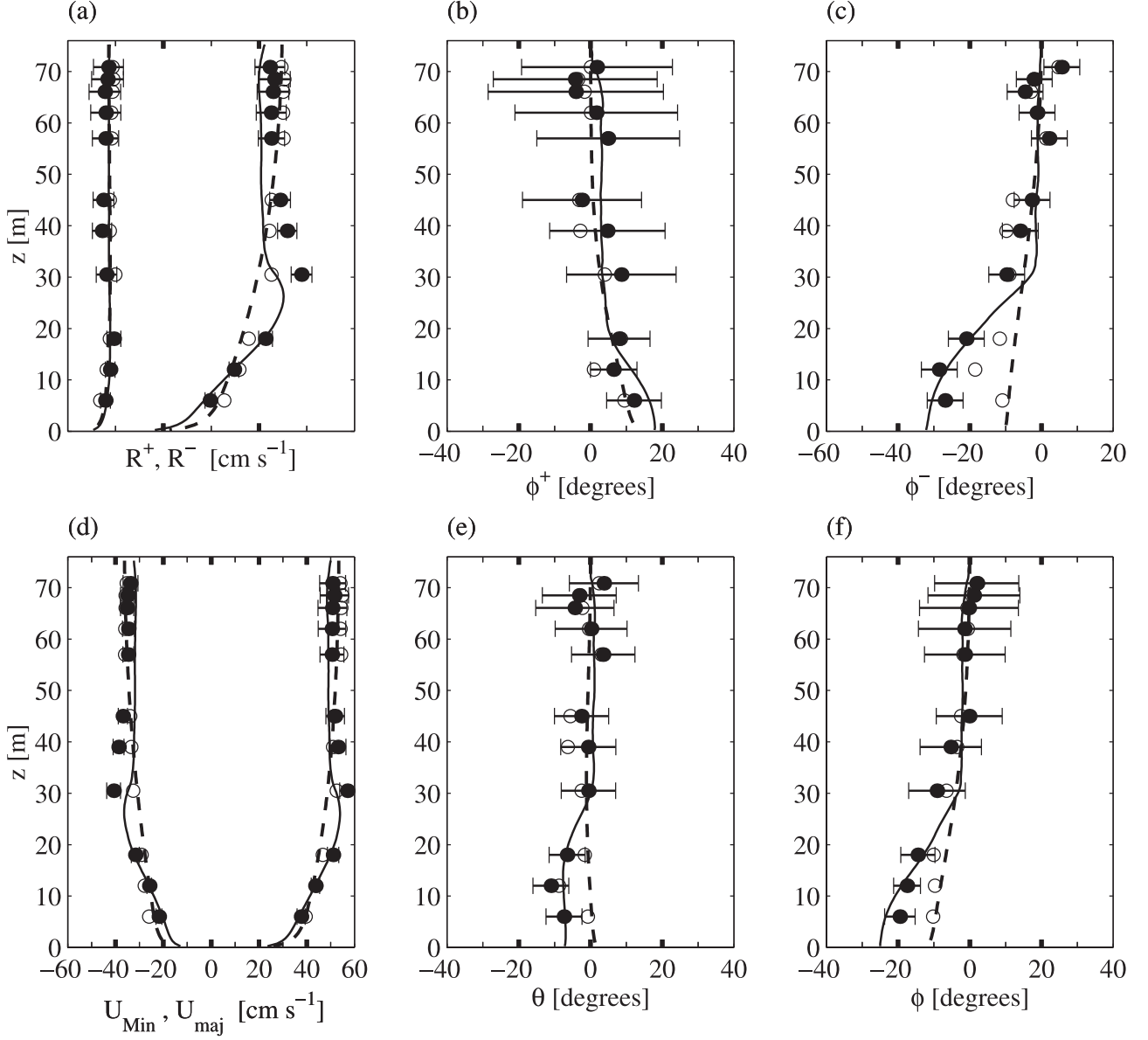


Figure 13. Profiles of M_2 current ellipse parameters for May 14–22: (a) magnitudes of (smaller values) R^+ and (larger values) R^- , phase angles (b) Φ^+ and (c) Φ^- (d) amplitudes U_{Maj} and U_{Min} , (e) inclination θ , and (f) phase of the tidal velocity vector ϕ . Phases and inclination are displayed as veering with respect to the surface (vertically averaged velocity vectors at top five VMCMs). Error bars denote the standard error of the tidal fit at the 95% confidence level, or, for $\Phi^\pm$ and θ , the compass uncertainty (depending on which is larger). Open circles are from tidal prediction based on results for February 11 to March 11 as described in the text. Also shown are model results from 2LK (dashed line) and MY2.5 (solid line).

lower 55–60 m of the water column (Figure 14a). Buoyancy frequency squared reached peaks near $N^2 \sim 10^{-4} - 10^{-3} \text{ s}^{-2}$ at $z = 60\text{--}65 \text{ m}$ and approached zero near the surface (Figure 14b). Dissipation rates were smallest in the lower pycnocline and increased toward the bottom and surface as a result of bottom friction and wind-induced turbulence (Figure 14c).

[45] As described in section 3.4.1, we ran MY2.5 for combined wind and pressure forcing for the period February 1 to May 30, with the model density and N^2 prescribed using stratification estimates interpolated from hourly aver-

aged SeaCAT data for the entire period augmented with hourly averaged microstructure data during the two periods when microstructure profiles were taken at ST1. Model predictions of turbulent dissipation rate averaged over the 6- and 10-hour measurement periods are in overall good agreement with microstructure estimates below the main pycnocline and predict the observed increase toward the surface (Figure 14c). In the pycnocline, the model underestimates observations by several orders of magnitude.

[46] Since the dissipation rate estimates represent an average of individual profiles taken over 1 hour, one could

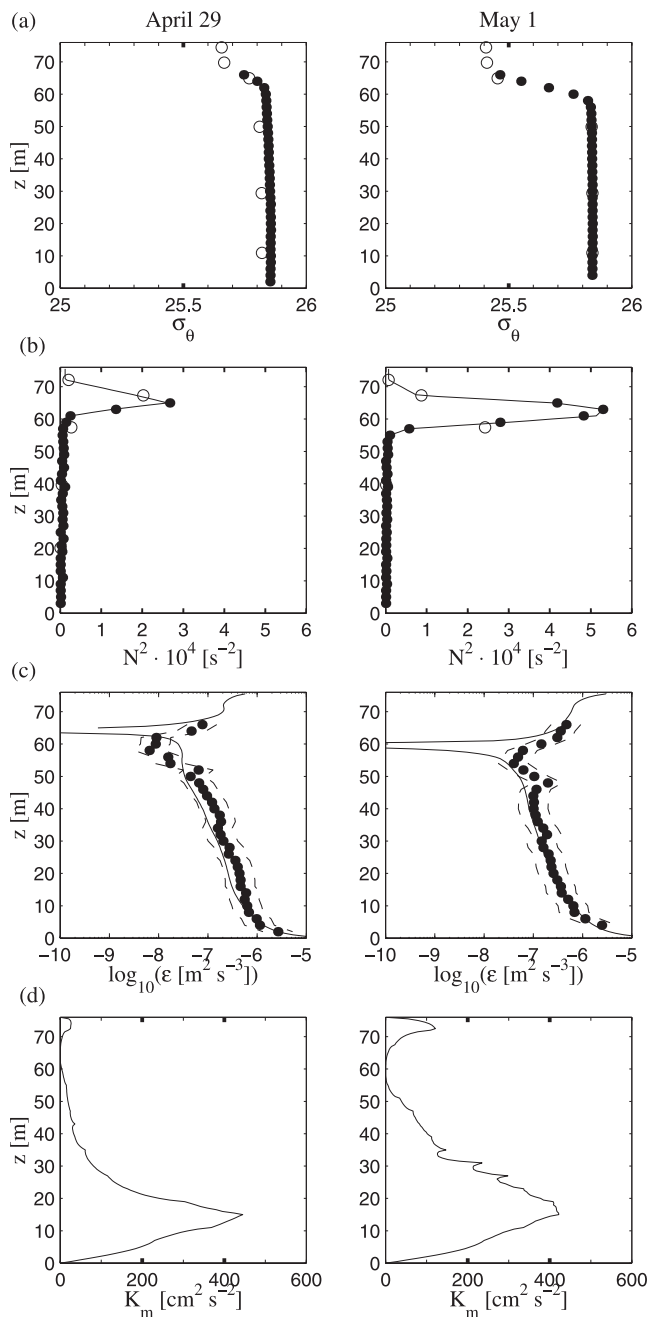


Figure 14. (a) Potential density, (b) buoyancy frequency squared, and (c) turbulent dissipation rate on a semilogarithmic scale from ST1 microstructure measurements for (left) April 29 and (right) May 1. All data are time averaged. Open circles are from SeaCAT measurements. Solid lines in Figure 14b are the average N^2 distributions specified in MY2.5. Dashed lines in Figure 14c are measurement uncertainties, and solid lines are results from MY2.5. Also shown are (d) eddy viscosity profiles from MY2.5.

imagine that a thin layer of near-zero mixing (as predicted by MY2.5) could be carried up and down by vertical motion of the pycnocline with timescales less than 1 hour. The resulting hour-averaging could result in a vertical smearing

of the dissipation rate estimates, effectively reducing or eliminating any sign of the thin near-zero mixing layer. To test this conjecture, individual dissipation rate profiles were closely examined, and no evidence found to support the existence of such a layer in the pycnocline.

[47] In summary, the predicted vertical distribution of ϵ emphasizes the behavior of MY2.5 to allow for little or no mixing across the strongly stratified pycnocline. This is also made clear by the predicted eddy viscosity distribution, which is equal or close to molecular diffusion at those elevations where N^2 is largest (Figure 14d).

6. Model Discussion

[48] For the weakly stratified conditions during February 11 to March 11, both the advanced MY2.5 and simplistic 2LK models compare well with observations. However, a meaningful test of any turbulent closure model has to take into account the respective model's performance during strongly stratified conditions. Simple models such as 2LK with eddy viscosity parameterizations that do not depend on N^2 are destined to fail under such conditions. On the other hand, the advanced MY2.5 incorporates the effects of stratification by solving for turbulent kinetic energy, mixing length, and Richardson-number-dependent stability functions that combine to give the eddy viscosity K_m , and if the model solves for density, the diffusion coefficient K_h . The stability functions are designed such that K_m and K_h decrease with increasing Ri and reduces to molecular background level once $Ri \geq 0.19$ [Mellor and Yamada, 1974], a criterion roughly consistent with $Ri_c = 0.25$ from linear instability theory.

[49] Our model-data comparison of current profiles and turbulent dissipation rates suggests that the Mellor-Yamada level 2.5 turbulent closure scheme implemented in MY2.5 does not allow for sufficient, if any, mixing across the main pycnocline. This is in agreement with Stacey *et al.* [1999], who found that MY2.5 underpredicted observations of turbulent kinetic energy in a partially stratified estuary. Simpson *et al.* [1996] also reported underestimation of turbulent dissipation in the pycnocline of the Irish Sea by lower-order, boundary-layer schemes developed by Mellor and Yamada (i.e., MY2.2, MY2.0) using diagnostic, instead of prognostic, turbulent length-scale parameterizations.

[50] At least two factors can explain MY2.5's underestimation of turbulence production in the pycnocline: First, the model's behavior may be related to the assumption that K_m and K_h reduce to molecular background level if $Ri \geq 0.19$, as opposed to using a larger Ri_c , such as $Ri_c = 1.0$ from stability analysis of nonlinear flows [Abarbanel *et al.*, 1984, Canuto *et al.*, 2001]. Second, physical processes not incorporated in the model may enhance mixing. Among such processes are nonlinear internal waves with frequencies near N , which are known to produce shear instabilities at the density interface and lead to enhanced mixing in the pycnocline [MacKinnon and Gregg, 2003]. Hydrostatic models, including MY2.5, are severely limited in their ability to resolve such high-frequency waves explicitly, and the intermittent enhanced current shear and mixing caused by such wave features are not included implicitly in the MY2.5 closure parameterization. For this reason,

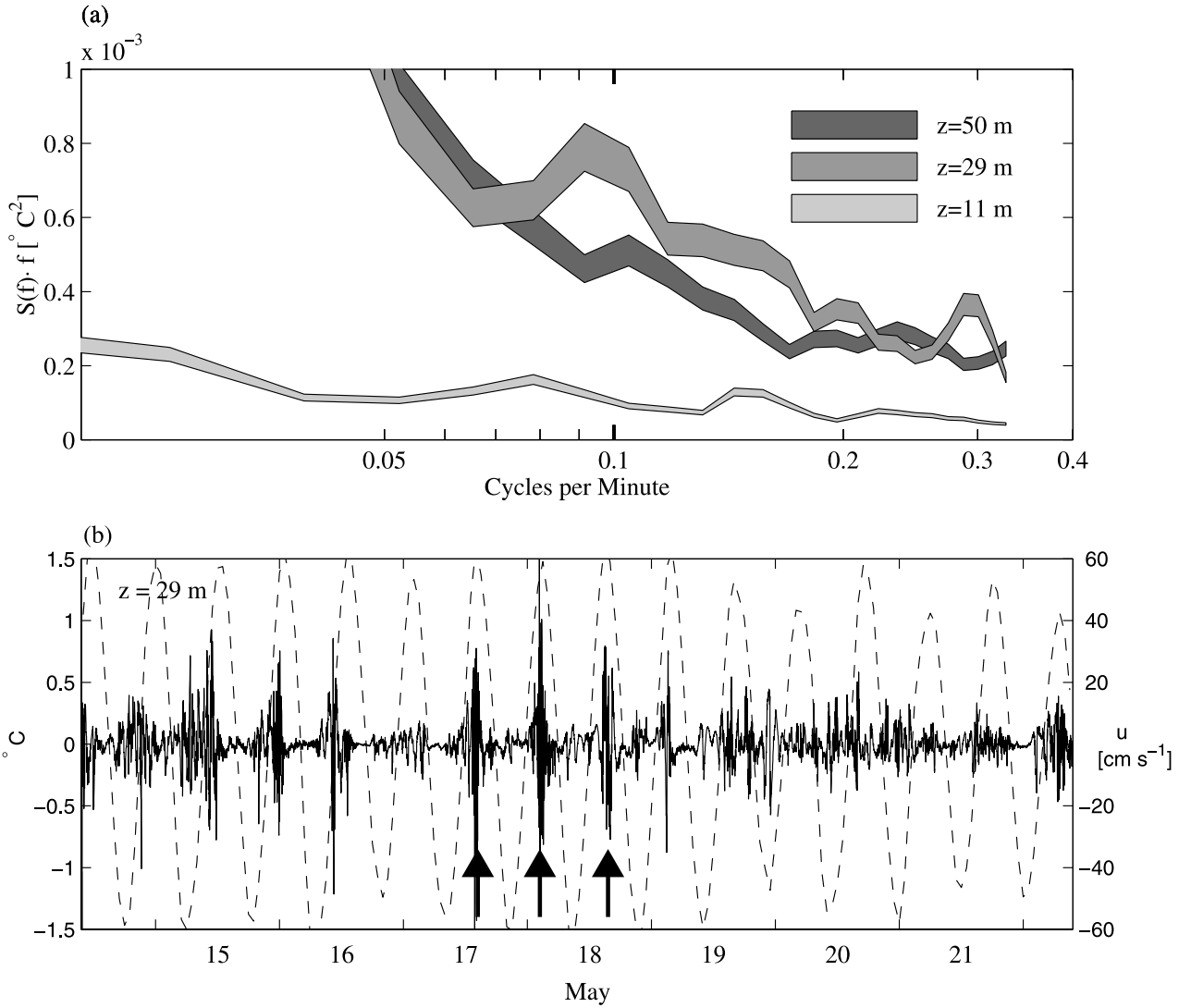


Figure 15. (a) Variance-preserving frequency spectra of high-pass-filtered temperature measurements within 95% confidence limits (shaded areas) at $z = 50$, 29, and 11 m, and (b) time series of high-pass-filtered temperature at the base of the pycnocline ($z = 29$ m) for the May 14–22 analysis period. High-pass-filtered data are the difference between unaveraged (1.5-min) SeaCAT measurements and the low-pass-filtered time series using a filter operator with a 120-min half-power period. Dashed lines in Figure 15b give the cross-bank velocity component where subtidal variation >33 hours is removed.

meaningful evaluation of MY2.5 requires that the presence of internal waves be investigated.

7. Discussion of Internal Waves

[51] Internal waves can occur over a wide range of temporal and spatial scales, for example, as near-inertial oscillations, internal tides, to high-frequency nonlinear solitary waves. Our velocity data do not display significant variance in the inertial frequency band, so that we exclude the possibility of strong inertial motion. Internal tides in their first baroclinic mode should result in large M_2 current shear throughout the pycnocline, i.e., vertically sheared current amplitudes accompanied by strong phase veering. Amplitude and phase of the semidiurnal tidal currents for May 14–22 were nearly uniform in the upper 20–30 m

despite large N^2 , suggesting that internal tides were very weak or not present during this period (Figure 13). Tidal analysis indicates that similar conclusions apply to those times in April and May when microstructure data were taken (not shown).

[52] At higher frequencies, unaveraged (1.5-min), high-pass-filtered temperature data for May 14–22 reveal a spectral peak near 10 min at $z = 29$ m (Figure 15a), corresponding to the approximate location of the base of the main pycnocline (Figure 9d). This peak is indicative of high-frequency internal waves, which passed ST1 during flood and were most pronounced near $z = 29$ m (Figure 15b). In agreement with previous observations over the northeast U.S. continental shelf, we conclude that these features represent large-amplitude internal solitary-like waves, which are generated near the shelfbreak by the barotropic tidal flow

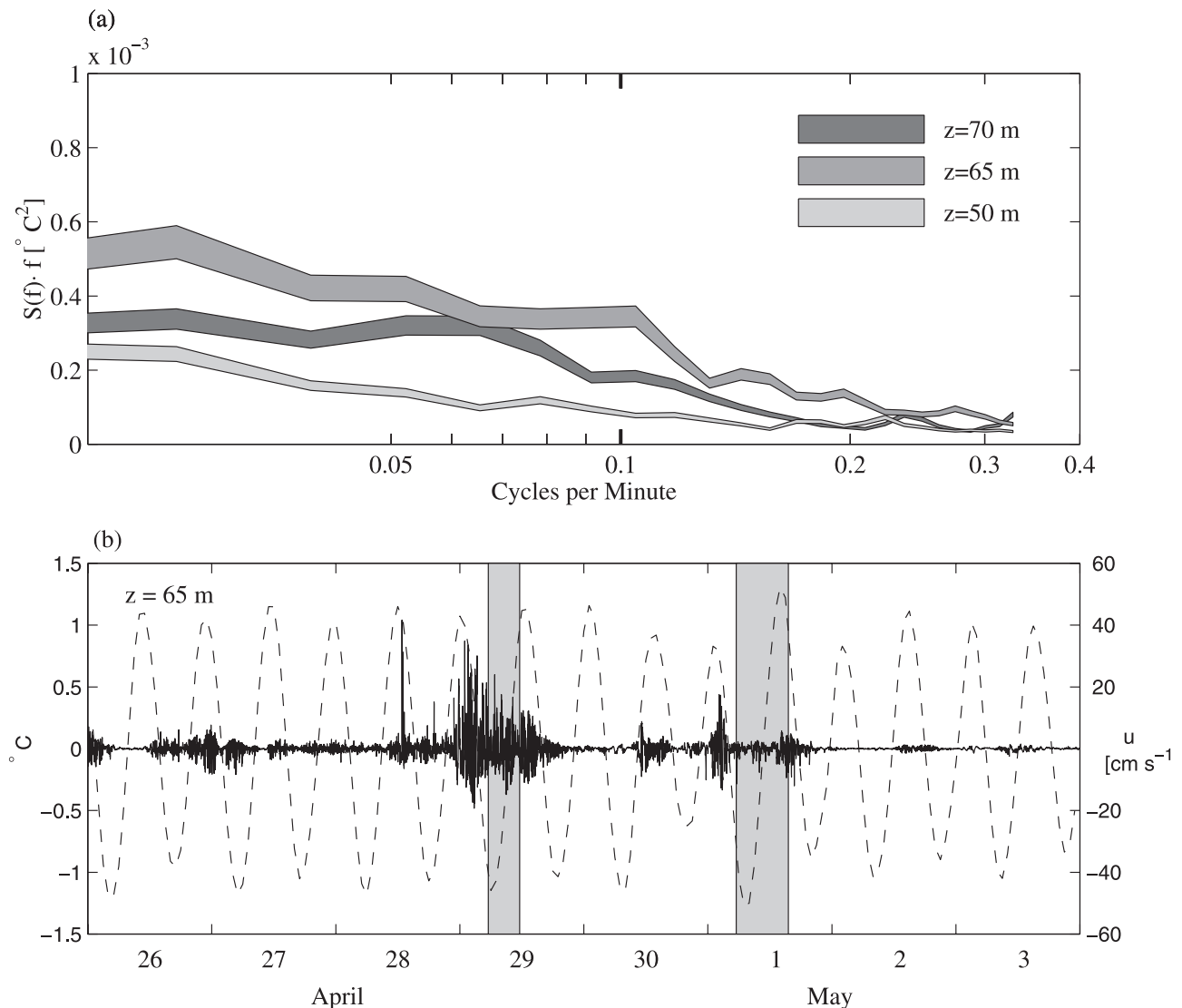


Figure 16. (a) Variance-preserving frequency spectra of high-pass-filtered temperature measurements within 95% confidence limits (shaded areas) at $z = 70$, 65 , and 50 m, and (b) time series of high-pass-filtered temperature in the pycnocline ($z = 65$ m) for April 26 to May 3. High-pass-filtered data are the difference between unaveraged (1.5-min) SeaCAT measurements and the low-pass-filtered time series using a filter operator with a 120-min half-power period. Dashed lines in Figure 16b give the cross-bank velocity component where subtidal variation >33 hours is removed, and shaded areas indicate when microstructure data were taken.

and propagate on-bank [Wiebe *et al.*, 1999; Colosi *et al.*, 2001]. The occurrence, timing, and amplitudes of these waves at ST1 were quite variable during this period, due in part to significant variability in the local stratification, i.e., N^2 . Hence MY2.5's underestimation of turbulence production during the May SSF intrusion may have several explanations: missing implementation of temporarily enhanced current shear due to high-frequency internal waves, inadequacies of the turbulence closure scheme related to the implicit assumption that $Ri_c = 0.19$, or both.

[53] Temperature data also show high-frequency variability between the end of April and beginning of May. During times coinciding with microstructure measurements, such variability occurred primarily in the main pycnocline at $z = 65$ m (Figure 16b), with rough estimates of vertical excursions of the pycnocline ranging from 5 to 15 m with

periods near the local buoyancy period (6–12 min), consistent with the occurrence of high-frequency internal waves. In contrast to the mid-May period (Figure 15), the occurrence of high-frequency temperature variability during April 26 to May 3 is much less regular (Figure 16b). During this April–May period, N^2 based on the density difference between $z = 11$ and $z = 70$ m varied over the range $1-7 \times 10^{-5} \text{ sec}^{-2}$, indicative of large variations in the internal wave guide that helps explain the weak frequency dependence of high-pass-filtered temperature (Figure 16a).

8. Summary and Conclusions

[54] We presented above observations and model predictions of the M_2 tidal bottom boundary layer at a 76-m-deep study site on the southern flank of Georges Bank (bottom

slope = 8×10^{-4}), where currents are dominated by the semidiurnal tidal components. The largest semidiurnal component is the rotary M_2 , which propagates on-bank as a Sverdrup wave with a depth-averaged major axis of $40\text{--}50 \text{ cm s}^{-1}$. This component generates most of the bottom stress and controls the overall structure of the bottom boundary layer on tidal and longer time scales except during extreme surface forcing. Our observational and numerical analysis covers nearly homogeneous and stratified conditions representative of winter and spring, respectively, and yields the following conclusions:

[55] • Ri estimates for the 27.6-day-long winter analysis period (February 11 to March 11, 1995) are less than (0.25, 0.5, 1.0) during more than about (65, 75, 85)% of the time in the weak winter pycnocline ($N^2 \sim 10^{-5} \text{ s}^{-2}$), and more than 90% of the time near the surface and bottom. This result applies despite the limited vertical resolution (15–20 m) of density estimates at mid-depth, indicating that strong turbulent mixing occurred throughout the water column much of the time. Observations of tidal boundary layer height give $\frac{\delta}{\delta_T} \sim 7$ for M_2 based on rotary component analysis, in reasonable agreement with $\frac{\delta}{\delta_T} = \frac{\omega+f}{\omega-f} = 5.2$ from simple theory based on a logarithmic velocity layer close to the bottom.

[56] • For highly turbulent winter conditions such as investigated here, the observed M_2 velocity distribution and bottom stress can be reproduced closely with one-dimensional models using either a simplistic linear-constant eddy-viscosity parameterization or the advanced Mellor-Yamada level 2.5 turbulence closure scheme. This result supports earlier conclusions by *Davies* [1991] and *Davies and Xing* [1995], who found that simple eddy-viscosity parameterizations can compete with advanced higher-order schemes when the fluid is homogeneous. In agreement with theory, the linear-constant eddy viscosity model reproduces the observations most closely if the sublayer height is approximately equal to the observed thickness of the logarithmic layer.

[57] • M_2 velocity profiles during a shelf-slope front intrusion (May 14–22, 1995) display several features known to result when vertical stratification (here $N^2 \sim 10^{-4} \text{ s}^{-2}$) causes a reduction in mixing. Among such features are the reduced height of the boundary layer, pronounced phase-advance of the current vector with depth, and occurrence of a velocity maximum at the base of the pycnocline, in agreement with previous observations and conceptual models [e.g., *Maas and van Haren*, 1987; *Souza and Simpson*, 1996; *van Haren*, 2000].

[58] • The stratified M_2 tidal boundary layer cannot be modeled with the simplistic linear-constant eddy viscosity model, which does not incorporate buoyancy effects in the parameterization of eddy viscosity. As a result, this model not only shows poor agreement with the observed velocity distribution, but also overestimates bottom stress by up to 52%. On the other hand, the advanced MY2.5 model succeeds in predicting strong phase veering with depth as well as a mid-depth velocity maximum, and gives bottom stress magnitudes that are within 7% of observations. Nevertheless, the performance of MY2.5 is limited by an abrupt adjustment from the strongly sheared turbulent bottom-boundary layer to a nearly nonturbulent ($K_m \sim \nu$) region above, which separates the mid-water column from

the turbulent surface layer. Existence of such a frictionless interior is not supported by our measurements, which suggest that some mixing occurs throughout the water column. This follows from Ri estimates that are smaller than (0.25, 0.5, 1.0) less than about (10, 20, 40)% of the time at mid-depth ($N^2 \sim 10^{-4} \text{ s}^{-2}$), more than about (40, 75, 90)% of the time below the pycnocline, and more than about (40, 60, 75)% of the time near the surface. The coarse vertical resolution of density data (9–15 m in the pycnocline) exceeds the expected scale of density overturns by a factor of about 5–10, so that even the limited number of events with $Ri < 0.25\text{--}1.0$ observed here is a strong indicator of $Ri < 0.25\text{--}1.0$ on smaller, unresolved scales.

[59] • MY2.5 model results for 2 days (April 29 and May 1, 1995) with well-defined density interfaces ($N^2 \sim 10^{-4} \text{--} 10^{-3} \text{ s}^{-2}$) also suggest that $K_m \sim \nu$ between the turbulent surface and bottom boundary layers. For these times, predicted dissipation rates in the pycnocline are several orders of magnitude smaller than estimates from microstructure measurements. This result, together with model results for May 14–22, suggest that MY2.5 underestimates turbulent mixing in the presence of strong stratification. Similar conclusions were drawn by *Stacey et al.* [1999] based on comparison of MY2.5 to turbulent kinetic energy measurements in a partially stratified estuary, and by *Simpson et al.* [1996] for lower versions of MY2.5 (i.e., MY2.2, MY2.0).

[60] • We observed high-frequency internal waves (with periods of minutes) during flood for the spring analysis period May 14–22, when they were most pronounced at the base of the pycnocline. This corresponds approximately to the elevation where K_m from MY2.5 first approaches ν . High-frequency internal waves were also observed during April 29 and May 1. Hence missing parameterization of temporarily enhanced shear by high-frequency internal waves is one possible reason explaining MY2.5's underprediction of mid-depth turbulence during the May SSF intrusion and the earlier comparison periods.

[61] • The model-data comparison presented here shows that the basic features of the stratified tidal boundary layer are predicted reasonably well by our one-dimensional MY2.5 model in the less-stratified region below the main pycnocline. This is encouraging, since recent numerical studies of the circulation on Georges Bank all feature the MY2.5 closure scheme. Near the top of the tidal boundary layer, the MY2.5 model appears to underestimate turbulent mixing. We have mentioned two possible causes, the omission of high-frequency internal-wave mixing in the model parameterization and the use of a Ri_c too low to describe the transition from molecular to turbulent mixing in oceanic flows. Other possibilities include three-dimensional processes, for example, horizontal advection of turbulence from topographic features [*Yoshida and Oakey*, 1996]. Additional work, both observational and numerical, is needed to examine this question of mid-level mixing in the coastal ocean.

Appendix A: Vertical Scale of Density Overturns

[62] A meaningful scale for the largest density overturns is $L_B = 2\pi(\epsilon/N^3)^{1/2}$ [*Gregg et al.*, 1993], where ϵ is the rate

of turbulent dissipation and $(\epsilon/N^3)^{1/2}$ is the Ozmidov scale [Ozmidov, 1965]. We estimate the Ozmidov scale from microstructure measurements made at ST1 on April 29, May 1, and June 11–15, 1995, by *Burgett et al.* [2001]. Their data show that vertical stratification comparable to the mid-depth buoyancy frequencies describing our nearly homogeneous ($N^2 \sim 10^{-5} \text{ s}^{-2}$) and stratified ($N^2 \sim 10^{-4} \text{ s}^{-2}$) investigation periods coincided with dissipation estimates $\epsilon \sim 0.5\text{--}1 \times 10^{-7} \text{ m}^2 \text{ s}^{-3}$. These estimates yield first-order approximations $L_B \sim 8\text{--}11 \text{ m}$ and $L_B \sim 1\text{--}2 \text{ m}$, respectively. The vertical distance between density estimates used for estimation of N^2 is 15–20 m (9–15 m) at mid-depth during winter (spring), respectively (Figures 3c and 9c). Given the vertical resolution of our data, we conclude that density overturns were resolved most of the time during the nearly homogeneous period February 11 to March 11, but very rarely during the stratified period May 14–22.

Appendix B: Review of Rotary Components

[63] The counterclockwise (+) and clockwise (−) rotary components of a tidal current are given by [Soulisby, 1983]

$$\mathbf{R}_j^\pm = R_j^\pm e^{i\Phi_j^\pm}, \quad (\text{B1})$$

where $R^\pm$ is velocity magnitude and $\Phi^\pm$ is phase angle with respect to the on-bank (+x) reference axis at the center of the investigation period. Similar decompositions can be performed on the horizontal pressure gradient and bottom stress vectors yielding the rotary components (divided by density) $\mathbf{P}^\pm$ and $\boldsymbol{\tau}_b^\pm$, respectively. In the following, the principle of rotary components is explained using the example of a tidal current.

[64] Summation over all resolved constituents yields the velocity vector

$$\mathbf{u} + i \times \mathbf{v} = \sum_{j=1}^M (\mathbf{R}_j^+ e^{i\omega_j t} + \mathbf{R}_j^- e^{-i\omega_j t}). \quad (\text{B2})$$

In equation (B2), u and v are across- and along-bank velocity components, respectively, M is number of resolved constituents, and ω_j is the radian frequency of the j th component. Current speeds along the major (Maj) and minor (Min) axes of the tidal ellipse can be evaluated from

$$\begin{aligned} U_{Maj} &= R^+ + R^-, \\ U_{Min} &= R^+ - R^-, \end{aligned} \quad (\text{B3})$$

where $U_{min} > 0$ ($U_{min} < 0$) denotes counterclockwise (clockwise) rotation of the velocity vector, and $e = \frac{|U_{Min}|}{U_{Maj}}$ is eccentricity. The inclination of the tidal ellipse with respect to the onbank (+x) reference axis is

$$\theta = 0.5 \times (\Phi^+ + \Phi^-), \quad (\text{B4})$$

with $\theta > 0$ counterclockwise from +x. The phase of the tidal velocity vector

$$\phi = 0.5 \times (\Phi^- - \Phi^+) \quad (\text{B5})$$

is evaluated at the center of the investigation period and gives the time $t = \frac{\Phi}{\omega}$ of maximum current.

[65] Error limits for the rotary components of tidal velocity and bottom stress are derived from the residual spectrum, i.e., the spectrum of measured minus predicted currents/bottom stress. For the M_2 tide, the residual variance is summed over a frequency band centered at the M_2 frequency, divided by the bandwidth $\pm 0.17 \times 10^{-4} \text{ s}^{-1}$, and multiplied by 2 to give the standard error at the 95% confidence level. The bandwidth $\pm 0.17 \times 10^{-4} \text{ s}^{-1}$ approximately corresponds to twice the width of the semidiurnal-frequency band defined by the M_2 (12.42 hours), N_2 (12.66 hours), and S_2 (12 hours) constituents. Hence our choice of bandwidth accounts for smearing of the residual spectrum across the three major semidiurnal constituents independent of the length of the data record that is analyzed. Confidence limits for Φ^+ , Φ^- , and θ reflect the larger of the tidal fit or compass uncertainties. Compass uncertainties of the VMCMs are 5° , and the combined compass/sensor alignment error of BASS is estimated to be 8° . Error limits of Φ are from tidal fits, since uncertainties of ellipse orientation have no impact on phase prediction.

Appendix C: Two-Layer-K Model (2LK)

[66] The 2LK uses the eddy viscosity parameterization,

$$K_m = \begin{cases} \kappa \bar{u}_* z & \text{for } z \leq l \\ \kappa \bar{u}_* l & \text{for } z > l \end{cases}, \quad (\text{C1})$$

where the overbar denotes the time-average over 12.42 hours and l is a sublayer height. Evaluation of equation (C1) takes place after completion of each tidal cycle, and the predicted eddy viscosity profile is used to determine the stress vector during the following flood and ebb. According to equation (C1), K_m does not vary on tidal timescales but remains constant over the course of 12.42 hours. This approach was taken based on previous analytical and model results indicating that temporal variation of eddy viscosity has little effect on the first harmonic of the predicted flow [Trowbridge and Madsen, 1984; Davies, 1990].

[67] Meaningful parameterizations of sublayer height assume $l = a \times \delta$, where δ is a bottom boundary layer thickness, and a is an empirical constant [Trowbridge and Madsen, 1984]. For steady planetary and rectilinear oscillating flows, representative scale expressions are $\delta \simeq \frac{\kappa H_*}{f}$ and $\delta \simeq \frac{\kappa H_*}{\omega}$, respectively [Grant and Madsen, 1986]. In the present case of rotating tidal currents with $\frac{\delta^-}{\delta^+} \sim 7$, the clockwise (δ^-) boundary layer dominates the current distribution, so that $\delta = \frac{\kappa H_*}{\omega_{M_2} - f}$ is a characteristic boundary-layer scale. The empirical constant a is not well known, and its specification is somewhat arbitrary. Trowbridge and Madsen [1984] suggest $a = \frac{1}{6}$ for nonrotational rectilinear flow, because model results using this value are in good agreement with laboratory experiments by Jonsson and Carlsen [1976]. Similar scaling was applied by Beardsley et al. [1995] to investigate rectilinear tidal currents on the Amazon shelf. For the rotating tidal flow on Georges Bank, $a = \frac{1}{6}$ overpredicts the observed bottom stress and current magnitudes in the bottom boundary layer. Numerical experi-

ments show model results agree well with observations if $a = \frac{1}{20}$, so that we use

$$l = \frac{1}{20} \times \frac{\kappa \bar{u}_*}{\omega_{M_2} - f} \quad (C2)$$

to determine the sublayer height. With $\bar{u}_* \sim 1.0 \text{ cm s}^{-1}$ from model results and observations, equation (C2) yields $l \sim 4 \text{ m}$, in close agreement with the suggested 3-m thickness of the logarithmic layer at ST1 [Werner et al., 2003].

Appendix D: Mellor-Yamada Level 2.5 Model (MY2.5)

[68] The MY2.5 turbulence closure model used here is a one-dimensional version of the Blumberg-Mellor hydrodynamic circulation model [Blumberg and Mellor, 1987], modified to include mixing lengthscale limitation by stable stratification [Galperin et al., 1988]. As part of the turbulence closure scheme, the model solves the turbulent kinetic energy and turbulent lengthscale equations to predict the mixing coefficients of momentum (K_m) and density (K_h).

[69] Physical processes sustaining vertical stratification on Georges Bank are at least two-dimensional, so that time evolution of density cannot be predicted by a one-dimensional model. Hence the version of MY2.5 used here does not solve for density, but utilizes a prescribed time series of ρ , instead. Specification of density in the vertical and in time guarantees that the model uses a realistic buoyancy distribution to estimate buoyant destruction of turbulence, which in turn enters the model's parameterization of K_m . The need for a priori specification of ρ follows from scaling: Assuming along-bank density gradients are small, the density equation reduces to

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} = \frac{\partial}{\partial z} \left(K_h \times \frac{\partial \rho}{\partial z} \right). \quad (D1)$$

Taking the vertical derivative of equation (D1) gives

$$\frac{\partial N^2}{\partial t} + u \frac{\partial N^2}{\partial x} - \frac{g}{\rho} \frac{\partial u}{\partial z} \frac{\partial \rho}{\partial x} = \frac{\partial^2}{\partial z^2} (K_h N^2). \quad (D2)$$

The second term in equation (D2) describes across-bank advection of buoyancy, and the third term represents tidal straining. Accurate prediction of N^2 with a one-dimensional model is possible only if temporal evolution of N^2 (first term in equation (D2)) and vertical diffusion (last term in equation (D2)) balance to lowest order. In the present case, characteristic variations of N^2 over a M_2 tidal cycle are $(\Delta N^2) \sim 0.04 \times 10^{-4} \text{ s}^{-2}$ in the winter pycnocline (Figure 4d), so that the time-derivative scales

$$\frac{\partial N^2}{\partial t} \sim \omega_{M_2} \times (\Delta N^2) \sim 5.6 \times 10^{-10} \text{ s}^{-3}. \quad (D3)$$

Taking the tidal excursion $\ell_{M_2} = \frac{2U}{\omega_{M_2}} \sim 6 \text{ km}$ as a representative horizontal scale, where $U \sim 40 \text{ cm s}^{-1}$

is the crossbank velocity amplitude (Figure 5d), gives for the advection term

$$u \frac{\partial N^2}{\partial x} \sim U \times \frac{(\Delta N^2)}{\ell_{M_2}} \sim 2.7 \times 10^{-10} \text{ s}^{-3}. \quad (D4)$$

Tidal straining has no importance in the pycnocline where M_2 current shear is small, but may be significant in the lower part of the water column. With typical density variations $\Delta(\sigma_\theta) \sim 0.02$ between ebb and flood (Figure 4c) and $\ell_{M_2} \sim 6 \text{ km}$, the straining term amounts to

$$\frac{g}{\rho} \frac{\partial u}{\partial z} \frac{\partial \rho}{\partial x} \sim \frac{g}{\rho} \frac{U}{\Delta z} \frac{\Delta(\sigma_\theta)}{\ell_{M_2}} \sim 4.4 \times 10^{-10} \text{ s}^{-3}. \quad (D5)$$

In equation (D5), $\Delta z \sim 30 \text{ m}$ corresponds to the approximate extent of the sheared bottom boundary layer. Scaling of the diffusion term on the right-hand side of equation (D2) requires estimation of K_h . MY2.5 gives $K_h \sim K_m$, so that a representative value according to model results is $K_h \sim 400 \text{ cm}^2 \text{ s}^{-1}$ (Figure 8a). With $N^2 \sim 0.5 \times 10^{-5} \text{ s}^{-2}$ in the pycnocline and $\Delta z \sim 20 \text{ m}$ as an approximate pycnocline thickness (Figure 3d), the diffusion term scales like

$$\frac{\partial^2}{\partial z^2} (K_h \times N^2) \sim \frac{K_h \times N^2}{(\Delta z)^2} \sim 5.0 \times 10^{-10} \text{ s}^{-3}. \quad (D6)$$

According to equations (D3)–(D6), all terms in equation (D2) are of similar magnitude. Hence a one-dimensional balance between the time-derivative and diffusion terms does not hold even to lowest order.

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