Geophys. Astrophys. Fluid Dynamics, 1983, Vol. 24, pp. 63–68
0309-1929/83/2401-0063 $18.50/0

A Note on Nonlinear Flow over Obstacles

L. J. PRATT†

Woods Hole Oceanographic Institution, Woods Hole, Massachusetts 02543, U.S.A.

(*Received November 15, 1982*)

A simple "laboratory" flow is described which exhibits the bistable behavior observed in more complicated geophysical flows. The behavior is present in a blocking experiment involving a shallow, homogeneous flow over an obstacle. It is demonstrated that within a certain parameter space, the blocking tendency depends not only on the height b_0 of the obstacle but on the history of the flow as well. The implied hysteresis confirms behavior predicted by Baines and Davies (1980, p. 239). It is further shown that the hysteresis is associated with a nonuniqueness in steady solutions which is, in turn, attributed to the existence of stationary blocking bores upstream of the obstacle.

In the study of nonlinear fluid flow over topography, a common experimental procedure is to place an obstacle in an initially uniform flow and note the adjustment that takes place and the asymptotic state reached after long time. This is essentially the approach taken by Long (1954, 1970) and Houghton and Kasahara (1968), among others, hereafter referred to as L1, L2, and HK respectively. Although the sudden appearance of the obstacle in the fluid is not a realistic forcing mechanism for geophysical flows, the experiments are of value in developing intuition into the nonlinear adjustment to other types of forcing and, in particular, the mechanism of blocking. The intuitive value of the results is greatly enhanced if robustness

†Present address: Graduate School of Oceanography, Narragansett Bay Campus, University of Rhode Island, Kingston, R.I. 02881, U.S.A.

can be shown with respect to the initial conditions. For example, the asymptotic state should depend upon the height of the obstacle but not, say, on the details of the way in which the obstacle is placed in the fluid. This consideration becomes particularly relevant in view of the nonuniqueness of steady nonlinear solutions whereby several steady states may exist for the same upstream conditions and obstacle height. This note deals with a case in which such lack of robustness occurs within a certain parameter space as a consequence of nonuniqueness.

Consider a shallow, inviscid, homogeneous stream of fluid with uniform velocity u_0 and depth h_0 above a flat bottom. At time $t=0$ an obstacle of height b_0 appears on the bottom and the fluid is forced to adjust. The nature of the adjustment and the asymptotic state depend upon b_0/h_0 and the initial Froude number $F_0=u_0(gh_0)^{-1/2}$ in a way indicated by the well-known diagram reproduced in Figure 1. If $(b_0/h_0, F_0)$ lies to the left of the curve BAF then the asymptotic state (as $t\rightarrow\infty$) retains the essential features of the initial flow. To the right of the curve BAE the initial state becomes partially blocked and the asymptotic state consists of a hydraulically controlled flow which is critical ($F=u(gh)^{-1/2}=1$) over the obstacle's sill. The curve BAE represents the threshold beyond which the initial flow has insufficient energy to surmount the obstacle without a decrease in flow rate and is given by HK as

$$b_0/h_0=1-\tfrac{3}{2}F_0^{2/3}+\tfrac{1}{2}F_0^2.$$

To the left of BAE continuous, steady solutions can be found which have *upstream* Froude number F_0 and which are not hydraulically controlled.

The experimental results of L1, L2 and HK indicate that the blocking process described above is accomplished by excitation of a bore which moves upstream from the obstacle leaving behind a partially-blocked flow. However the speed of an upstream-propagating bore can be zero if the initial flow is supercritical ($F_0>1$), a fact which lead L2 to define a separate threshold AF given by

$$\frac{b_0}{h_0}=\frac{(8F_0^2+1)^{3/2}}{16F_0^2}-\tfrac{1}{4}-\tfrac{3}{2}F_0^{3/2}.$$

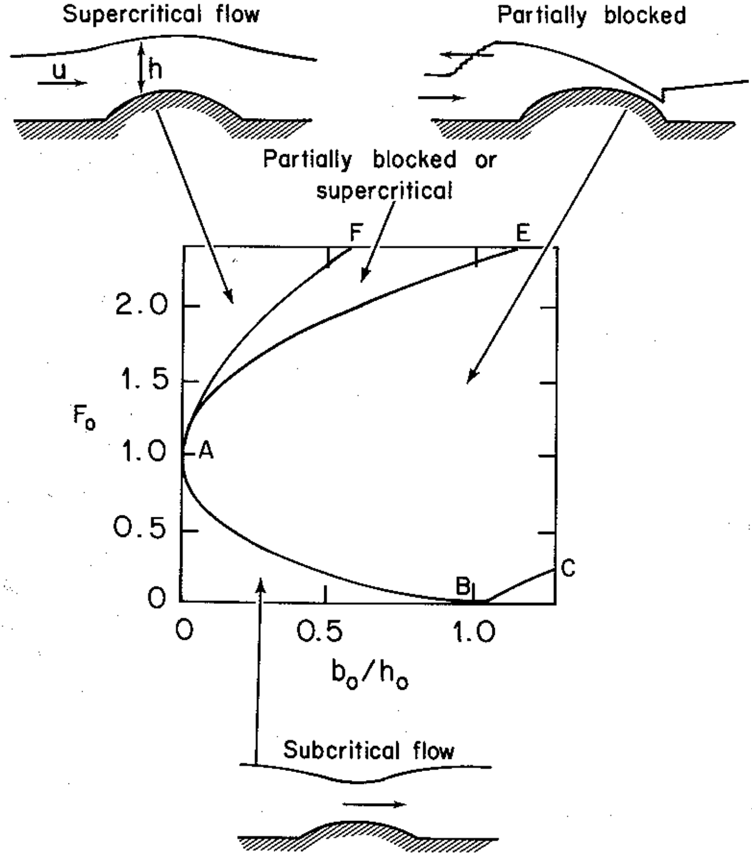

FIGURE 1 Simplified version of Baines and Davies (1980) Figure 2 showing asymptotic states for initial Froude number $F_0 = u_0/(gh_0)^{1/2}$ and relative obstacle height b_0/h_0.

Along AF a bore which connects a supercritical upstream flow to a hydraulically controlled flow over the obstacle will have zero propagation speed. To the right of AF such bores propagate upstream. Thus, two asymptotic states are possible along AF, the first being a continuous supercritical flow and the second a flow which is supercritical upstream of the obstacle but connected discontinuously to a hydraulically controlled flow. Similarly, two asymptotic states—an (unblocked) supercritical flow and a (partially blocked) controlled flow—are possible *within the region* EAF. The question of interest here concerns which asymptotic state will arise in a given experiment. According to L1 and L2, the flow in EAF should be partially blocked. According to HK, it should not.

To investigate this dilemma the following initial-value calculation was performed. Starting with a uniform, supercritical flow an obstacle was slowly grown from zero height to a height b_0 and the fluid response computed numerically using a shock-resolving Lax–Wendroff method identical to the one used by HK. For b_0/h_0 lying to the left of AF, the initial flow is unobstructed, as expected. More interestingly the flow remains unobstructed when b_0/h_0 lies in region EAF—a result that runs contrary to what might be inferred from L2. Indeed, partial blockage occurred only after b_0/h_0 is increased so as to lie to the right of AE. The bore formation and resulting blockage is shown in Figure 2(a). If, however, after raising b_0/h_0 to lie to the right of AE, the obstacle is slowly lowered to the point where b_0/h_0 lies again in EAF, the bore continues to move upstream and the partially blocked asymptotic state is established (Figure 2(b)), this time in agreement with L2. To reestablish the supercritical state, it is necessary to lower the obstacle so that b_0/h lies to the left of AF as shown in Figure 2(c, d). A similar chain of events can be produced for a *fixed* obstacle height by slowly varying the upstream Froude number.

We note that the hysteresis apparent in the above chain of events was intuitively predicted by Baines and Davies (1980) but, to the knowledge of the author, has never been verified. The results point out the danger in using the region AEF of Figure 1 to predict shallow flow response to more realistic forcing. Similar behavior will undoubtedly occur in experiments with more complicated rotating and/or stratified fluids in which the presence of blocking waves can lead to nonuniqueness of steady solutions. The presence of "bistable" states in currents (such as the Kuroshio south of Japan) have been well documented. It should also be noted that the dependence of the solution on the history of the flow may not follow the above scenario if the obstacle is introduced into the fluid impulsively. The nonhydrostatic effects of such an introduction may, for example, lead to partially blocked flow in region EAF, whereas slow introduction of the obstacle would lead to an unobstructed flow there.

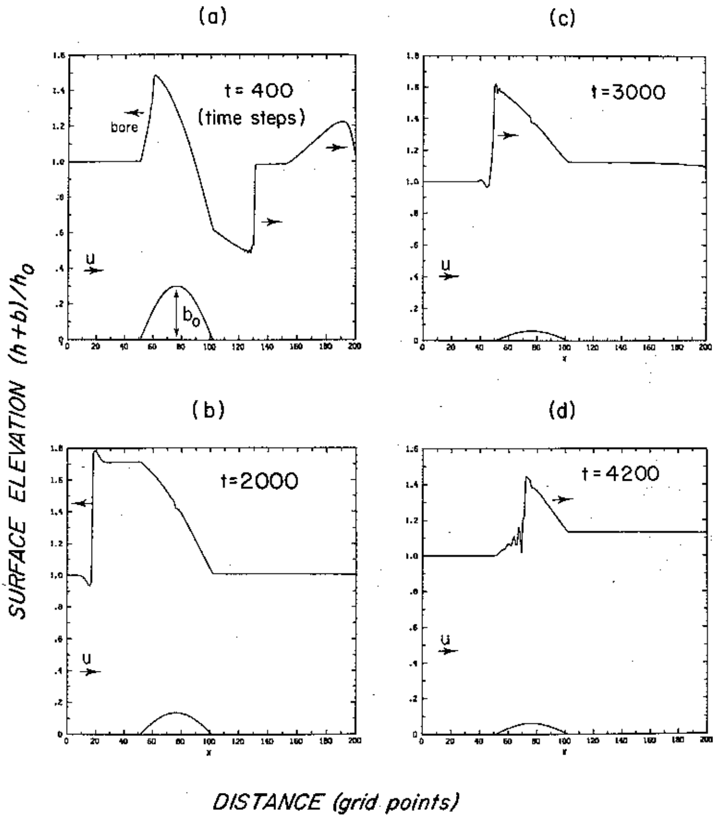

FIGURE 2 Hysteresis of surface of initial flow with Froude number $F_0 = 1.5$. The critical values of the obstacle height (from Figure 1) are 0.160 (from curve AE) and 0.112 (from AF). (a) The obstacle is grown so that $b_0 = 0.3$ causing a blocking bore to be generated. (b) b_0 is lowered to 0.135. However, the bore continues upstream, leaving behind a hydraulically controlled flow. (c) b_0 is lowered to 0.75, causing the bore to reverse. (d) The bore passes back over the obstacle.

Acknowledgements

This work was supported by the National Science Foundation under Grant OCE-25405. Computations were performed at the National Center for Atmospheric Research. The author wishes to thank N. Hogg, J. Pedlosky, J. Whitehead, G. Flierl, and E. Mollo-Christensen, M. Hall, and M. Lucas. W.H.O.I. Contribution No. 5226.

References

Baines, P. O. and Davies, P. A., "Laboratory studies in topographic effects in rotating and/or stratified fluids," In *Orographic Effects in Planetary Flows*, GARP Pub. No. 23, World Met. Org. (1980).

Houghton, D. D. and Kasahara, A., "Nonlinear shallow fluid flow over an isolated ridge," *Comm. Pure Appl. Math.* **21**, 1–23 (1968).

Long, R. R., "Some aspects of the flow of stratified fluids II, Experiments with a two-fluid system," *Tellus* **5**, 42–58 (1954).

Long, R. R., "Blocking effects in flow over obstacles," *Tellus* **22**, 471–480 (1970).