# **12.521** Computational Geodynamics Modeling

# **Overview of the boundary-element method**

## 1 Finite element versus boundary element methods

Crouch & Starfield (1983) provided an excellent overview of the essence of the boundary element method. Numerical methods for solving boundary value problems can be divided into two classes: (a) those that require approximations to be made throughout the region R, such as finite element and finite difference methods; and (b) those that require approximations to be made only on the boundary C, such as the boundary element methods.



In boundary element methods, only the boundary C is divided into elements. The numerical solution builds on the analytical solutions that have already been obtained for simple singular problems. This solution satisfies the specified boundary conditions at each element on C. Because each of the singular solutions satisfies the governing partial differential equations in R, there is <u>no need to divide R into a network of elements</u>.

If we divide C into N elements, then we seek N singularity solutions which, when superimposed, will give the required conditions at the midpoint of each element. The system of equations to be solved (total number N) is much smaller than the system needed to solve the same problem using finite element method, although the equations are no longer sparse. Once these equations are solved, the solution at any point in R can be constructed by Green's functions. Because it exploits an analytical solution that holds true throughout R, a boundary element method is potentially <u>more accurate</u> than the finite element method.

## 2 Boundary element method source codes

The following two software packages are based on the boundary element method (BEM) and can be used to investigate 3-D stress, strain, and displacement fields of geological problems such as earthquakes, faulting, dikes, underground caves, etc. Both codes are available for academic use without fees.

#### 2.1 Program 1: "Coulomb 3.2"

This code was developed by Shinji Toda, Ross Stein, and Jian Lin, as well as earlier by Geoffrey King, based on the program structure of Crouch & Starfield (1982) and 3D Green's functions of Okada (1992).

Jian Lin will provide the students of this class Coulomb 3.2 version, which can be run on Matlab using Mac, PC, and other computers. You can download a complete package of software, manual, and detailed examples from the following USGS web page:

http://earthquake.usgs.gov/research/modeling/coulomb/overview.php

You will need this software to complete your homework assignment #1.

#### Merits:

1) This is a very user friendly program and will require only minimal introduction on how to use it. It is probably one of the most commonly used boundary element program by earthquake research communities in the US, Europe, and Japan.

2) All results can be seen graphically on the computer screen as well as being saved as text output files.

#### Shortcomings:

You can only specify slip (not stress) boundary conditions.

#### References:

- King, G.C.P., R.S. Stein and J. Lin, Static stress changes and the triggering of earthquakes, *Bull. Seismo. Soc. Amer.*, *84*, 935-953, 1994.
- Toda, S. R. S. Stein, P. A. Reasenberg and J. H. Dieterich, Stress transferred by the Mw=6.9 Kobe, Japan, shock: Effect on aftershocks and future earthquake probabilities, *J. Geophys. Res.*, 103, 24,543-24,565, 1998.
- Lin, J. and R.S. Stein, Stress triggering in thrust and subduction earthquakes, and stress interaction between the southern San Andreas and nearby thrust and strike-slip faults, J. Geophys. Res., 109, B02303, doi:10.1029/2003JB002607, 2004.

#### 2.2 Program 2: "3d-def"

The software can be downloaded from the following web page. This page also provides a detailed online user's manual.

http://www.ceri.memphis.edu/3ddef/guide.html

### Merits:

This is a fully 3-D boundary element program that allows you to specify displacement or stress boundary conditions. Thus it can be applied to a wider range of geological problems than Coulomb.

## Shortcomings:

1) You need to invest much more time to learn how to use this program. If a geological problem can be solved by Coulomb, you will find that it is much easier to use Coulomb unless you are quite familiar with "3d-def" already.

2) This code package does not have nearly as nice graphic output options as Coulomb.

### References:

- Bodin, P. and J. Gomberg, Triggered seismicity and deformation between the Landers, California, and Little Skull Mountain, Nevada, earthquake, *Bull. Seismo. Soc. Amer.*, 84, 835-843, 1994.
- Gomberg, J. and M. Ellis, Topography and tectonics of the central New Madrid seismic zone: Results of numerical experiments using a three-dimensional boundary-element program, J. Geophys. Res., 99, 20,299-20,310, 1994.
- Kilb, D., Ellis, M., Gomberg, J., and Davis, S., On the origin of aftershocks following the 1989 Loma Prieta, California, earthquake, *Geophys. J. Intl.*, 128, 557-570, 1997.
- Behn, M.D., J. Lin, and M.T. Zuber, Evidence for weak oceanic transform faults, *Geophys. Res. Lett.*, 29(24), 2207, doi:10.1029/2002GL015612, 2002.