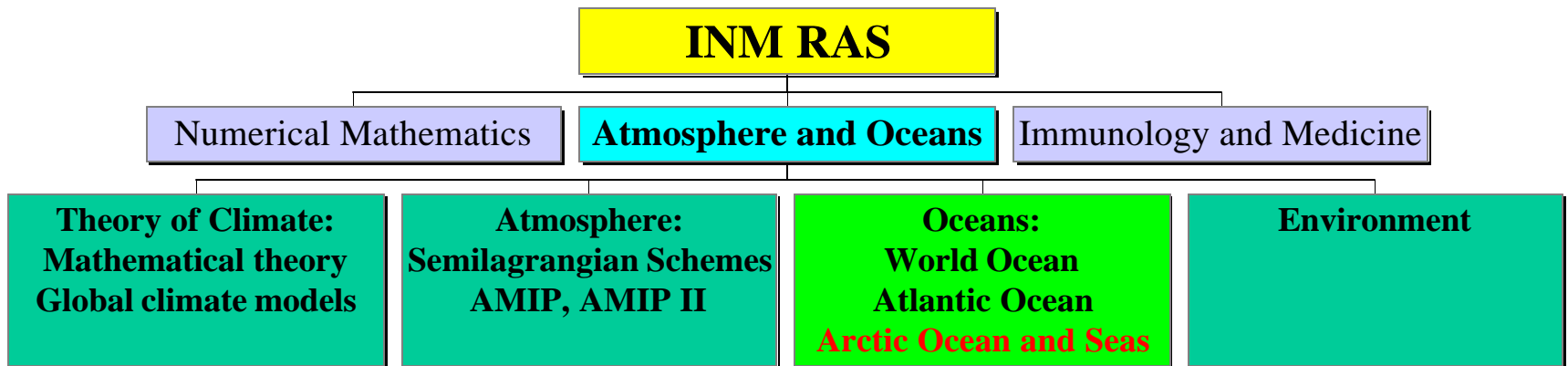


# 1. Introduction

# 1A. Author

- **Nikolay Yakovlev** (the other spelling is Nikolai Iakovlev).
- **Senior Scientist of the Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow.**
- **Gubkina str., 8, GSP-1, Moscow, 119991, Russia, INM RAS.**
- **Tel.: +007 (095) 938-3900**
- **Fax.: +007 (095) 938-1821**
- **iakovlev@inm.ras.ru**
- *Nikolay Yakovlev graduated Moscow Institute of Physics and Technology (MIPT) in 1984. Defended his Ph.D. Thesis in physics of atmosphere and hydrosphere in 1988. Specialist in numerical mathematics and large-scale oceanology. In Arctic Ocean modeling since 1996. A principal investigator and co-investigator of several projects on Arctic Ocean and its shelf seas, supported by Russian Academy of Sciences, Russian Foundation of Basic Researches and Ministry of Science and Technical Policy of Russian Federation. Now involved in the Arctic Ocean Model Intercomparison Project (AOMIP) activity.*

# 1B. Institute of Numerical Mathematics Russian Academy of Sciences. Main Structure



# 1C. Motivation. Physical Problems

- Institute of Numerical Mathematics has a good experience in global atmosphere and ocean modeling. Global atmosphere model, global ocean model, coupled atmosphere-ocean-permafrost model.
- Institute of Numerical Mathematics has a good experience in regional ocean and sea modeling - Black, Caspian, Barents, Okhotsk seas.
- **No Arctic and Antarctic sea ice in the global climate model. Pure sea ice in regional models.**
- To understand the large scale processes in ice-covered regions and their links with the regional and global climate we have to develop the coupled ocean-ice model and test it in a frame of an intercomparison project.
- **Arctic Ocean modeling - to understand mechanisms of the long-term natural variability of the Polar climate system.**

# 1D. Motivation. Numerical schemes and Coding

- **Finite -element spatial approximation is an alternative to the common finite-difference schemes used in MOM and POM OGCMs.**

Now almost all the OGCM are based on the MOM or POM codes. It means that all the models of the Arctic Ocean have the same dynamics. To evaluate the possible errors caused by pure numerical approximation it's necessary to take into account as many various models based on principally different schemes as possible.

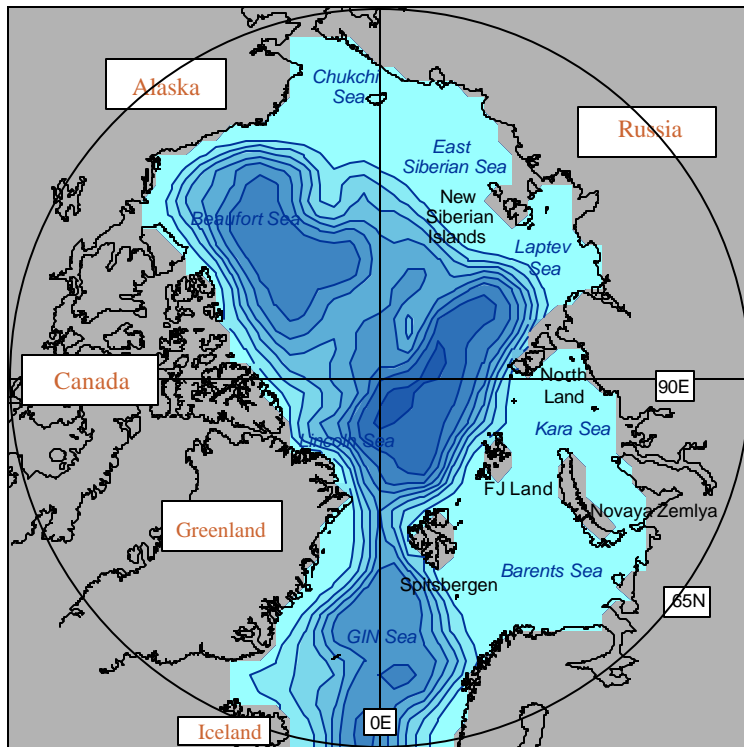
- **Time scheme should be stable for comparatively long time steps.**

Nonlinear processes of ice-water friction, ice dynamics caused by rheology and vertical turbulence processes require some special time schemes to achieve the high numerical performance.

- **The overall structure of the model should be transparent and flexible enough for any changes in physical parameterizations improvements.**

## **2. Numerical Model**

## 2A. General Layout



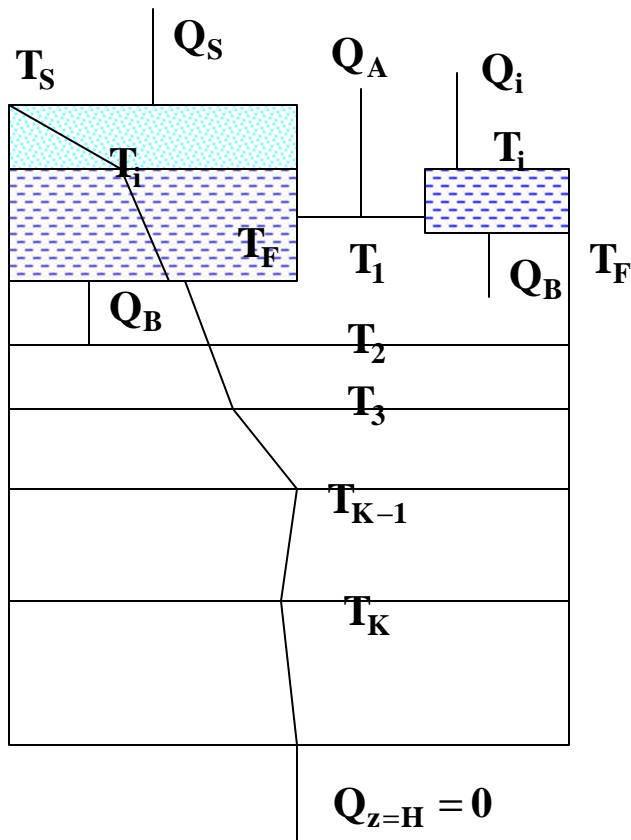
- Area of the Arctic Ocean north to 65N.
- North Pole shifted to the point 0N, 180E.
- 1 degree (~111.2 km) spatial resolution in the new coordinate frames.
- Z-coordinate vertical approximation, 16 levels.
- Three islands (Barents Sea).
- Four passages with the specified mass transports.
- Eight main rivers (both mass and salinity fluxes).

## 2B. Ocean Model

- **Primitive equations** with ordinary Boussinesque, hydrostatics and incompressibility approximations.
- **Linearized kinematics condition** at the ocean upper surface.
- **Sea level elevation** as an integral function of the model. This equation is derived on the **finite-dimensional level** thus providing mass conservation. **Implicit time scheme** for the sea level elevation problem - no time step restrictions due to surface gravity waves.
- Free slip boundary conditions at solid boundaries. Linear or quadratic friction at bottom.
- No heat and salinity fluxes at solid boundaries.
- **Specified barotropic mass transports** at open boundaries and at river estuaries  $V_n$ .
- Specified **salinity fluxes at river estuaries**:  $Q_{S,b} = -S \cdot V$ ,  $V$  - river flow velocity.
- Momentum fluxes with the quadratic ice drift drag at the upper surface.
- Heat and salinity fluxes at upper surface caused by snow\ice melting or freezing.
- **Heat and salinity fluxes**  $Q_{S,b} = S^* \cdot V_n$  at inflow side boundaries and  $Q_{S,b} = -S \cdot V_n$  at outflow ones.  $S^*$  is a specified salinity. The same is for T.
- **Vertical turbulence** parametrized by generalized **Prandtl theory**.



## 2C. Ice Thermodynamics



- Similar to *Parkinson and Washington (1979)* model. Linear profiles of temperature in snow and ice, thermodynamic equilibrium at the upper surface.
- Several gradations of ice thickness.
- Solution of the 1D thermodynamics problem for the whole snow-ice-water vertical column.
  1. Ocean water temperature profile from the surface to the bottom. Implicit time scheme.
  2. Snow-ice thermodynamic evolution.
  3. Total salinity flux at the ocean surface. Snow-ice melt and freezing, precipitation.
  4. Ocean water salinity profile from the surface to the bottom. Implicit time scheme.

## 2D. Ice Drift

- Ice mass and compactness transports for each thickness gradation.
- A simple mechanical redistributor: the thinnest ice — next thicker gradation (*I. Polyakov, personal communication*).
- **Elliptical visco-plastic rheology** (*W. Hibler 1979, M. Harder 1996*).
- **Vertical momentum exchange: implicit predictor-corrector time scheme for vertical vector** ( $u_{ice}, u_1, u_2, \dots, u_{KLast}$ ) (see sect. 2G).
- Zero velocity boundary conditions at solid boundaries, free drift flow at open boundaries.

## 2E. Transport Schemes

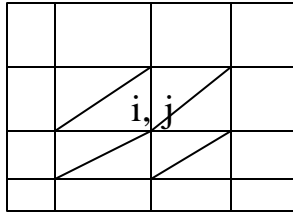
- Transport scheme for temperature and salinity (Hughes and Brooks 1979). Additional artificial diffusion:

$$\begin{aligned}\vec{\nabla} \cdot (A \vec{\nabla} q) \approx & \frac{1}{R^2 \sin^2 \mathbf{q}} \frac{\partial}{\partial \mathbf{l}} A_{11} \frac{\partial q}{\partial \mathbf{l}} + \frac{1}{R^2 \sin \mathbf{q}} \left( \frac{\partial}{\partial \mathbf{l}} A_{12} \frac{\partial q}{\partial \mathbf{q}} + \frac{\partial}{\partial \mathbf{q}} A_{12} \frac{\partial q}{\partial \mathbf{l}} \right) + \\ & + \frac{1}{R^2 \sin \mathbf{q}} \frac{\partial}{\partial \mathbf{q}} A_{22} \sin \mathbf{q} \frac{\partial q}{\partial \mathbf{q}}\end{aligned}$$

$$A_{11} = C \frac{u^2}{|\vec{u}|^2}, \quad A_{12} = C \frac{u \cdot v}{|\vec{u}|^2}, \quad A_{22} = C \frac{v^2}{|\vec{u}|^2} \quad |\vec{u}|^2 = u^2 + v^2 \quad C \approx \frac{1}{2} |\vec{u}| h$$

- Transport scheme for ice mass and compactness (N. Yakovlev).  
Scheme analogous to finite-difference upwind scheme and exact for uniform flow velocity field.

## 2F. Spatial Discretization



1. **Horizontal approximation** of model area - triangles derived from the rectangular discretization

2. **Vertical approximation** of model area - z-coordinate, bottom topography by stepwise function (like MOM)

3. Ocean **horizontal velocities, temperature, salinity, pressure** approximated by tensor products of 2D linear piecewise finite functions to 1D linear piecewise finite functions:

$$\Phi^h = \sum_{i,k} \Phi_{i,k} \mathbf{j}_i \mathcal{Y}_k$$

4. **Vertical velocity** is approximated by tensor products of 2D linear piecewise functions to 1D finite constant piecewise functions:

$$w^h = \sum_{i,k} w_{i,k} \mathbf{j}_i(x, y) \Pi_k(z) \quad \Pi_k(z) = \begin{cases} 1, & z \in [z_k, z_{k+1}] \\ 0, & z \notin [z_k, z_{k+1}] \end{cases}$$

5. Some special approximation for **ice deformation rate tensor components** in spherical coordinates.

# 2G. Time Scheme. General Structure

## Turbulence

Turbulent diffusion coefficients for momentum, heat and salt.

## Transport and Horizontal Diffusion

Explicit Matsuno time scheme. Modified transport scheme.

$$\mathbf{j}_t + L^h(\mathbf{j}) - D_{HJ}^h \mathbf{j} = 0$$

## Ice mass and compactness transport

Explicit Euler time scheme. Upwind transport scheme.

## Vertical diffusion of heat and salt, Ice thermodynamics.

Implicit time scheme.

## Vertical momentum exchange (ice and water).

Implicit predictor-corrector time scheme. Wind, ice-water drag, bottom friction.

$$\mathbf{j}_t = \nabla_z k \nabla_z \mathbf{j}$$

## Ice thickness distribution

The simplest distributor - thinner ice to the next thicker gradation.

## Rheology I

Explicit Euler scheme with small time steps ~ 1-2 min.

$$(\vec{u}_i)_t = R(\vec{u}_i, \vec{h}_i, \vec{A}_i), \quad t \in \left[ t_j, t_{j+1/2} \right]$$

## ... Time scheme

### Velocity-Pressure Adjustment

Implicit time scheme. GMRES for Level Elevation Solution.

$$u_t - fv = -\nabla_x P$$

$$v_t + fv = -\nabla_y P$$

$$\nabla_z P = g\mathbf{r}$$

### Ice velocity caused by sea level slopes

$$(u_i)_t - fv_i = -g\nabla_x \mathbf{z}$$

$$(v_i)_t + fu_i = -g\nabla_y \mathbf{z}$$

### Ice Pheology II

Explicit Euler scheme with small time steps 1-2 min.

$$(\vec{u}_i)_t = R(\vec{u}_i, \vec{h}_i, \vec{A}_i), t \in \left[ t_{j+1/2}, t_{j+1} \right]$$

### Vertical Velocity

$$\nabla_x^T u + \nabla_y^T v + \nabla_z^T w = 0$$

# **3. Preliminary Results**

## **Monthly Mean Climatology**

N.G. Yakovlev. A Coupled Model of Ocean General Circulation and Sea Ice Dynamics-Thermodynamics: Description and Arctic Ocean Climate Simulation Results. Submitted to *Izvestiya AN, Atmosphere and Ocean Physics*. (will be translated to English).

# Model Layout and Parameters

## Motivation: Test the ice model and the coupling method

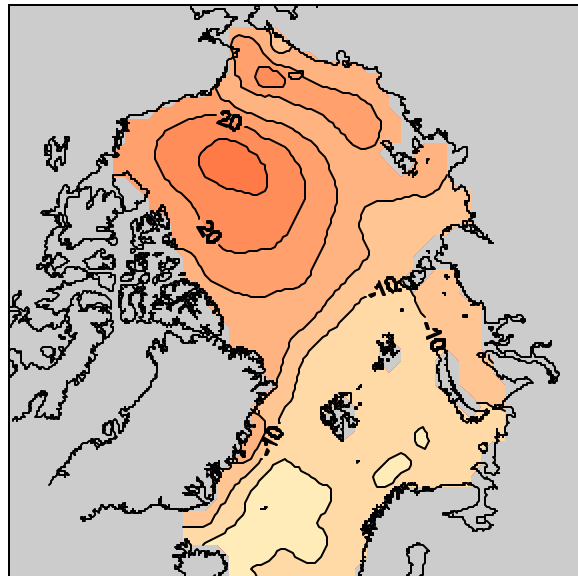
- **NCEP\NCAR monthly mean climatology** for shortwave and longwave radiation, air pressure and temperature, precipitation rate, latent heat.
- Seasonal ocean temperature and salinity (Polyakov and Timokhov, 1994). Empirical interpolation function by Nikoforov and Spaikher, 1980.
- Climate restoring with time scale **30 days**.
- 8 ice thickness gradations: 10, 30, 70, 120, 200, 400, 600 and > 600cm.
- Simplified sea ice rheology: **Cavitating fluid**.
- Ice ocean thermal interaction (Ebert and Curry, 1993):  $Q_{io} = r_0 c_w C_b(h_i)(T - T_F)$

$$?_b = \begin{cases} 1.26 \cdot 10^{-2} W_* / \sqrt{h_i}, & h_i < 3m; \\ 7.27 \cdot 10^{-3} W_*, & h_i \geq 3m, W_* \approx 1-3cm/sec \end{cases}$$

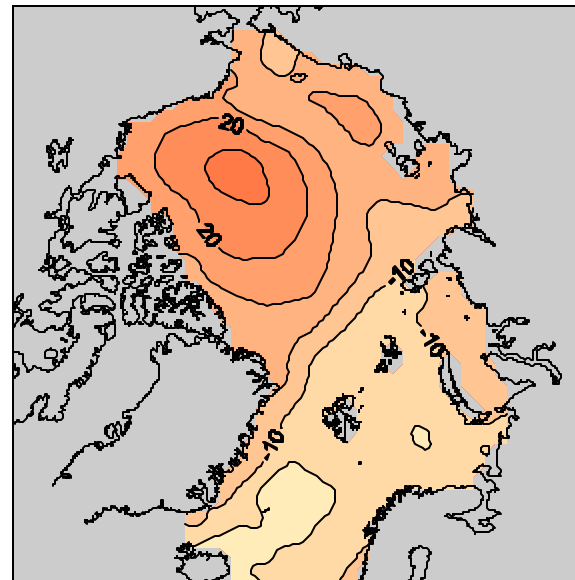


# Sea Level

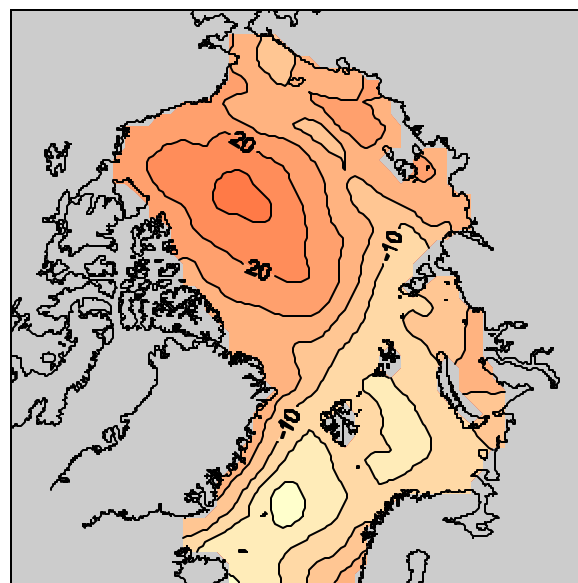
March



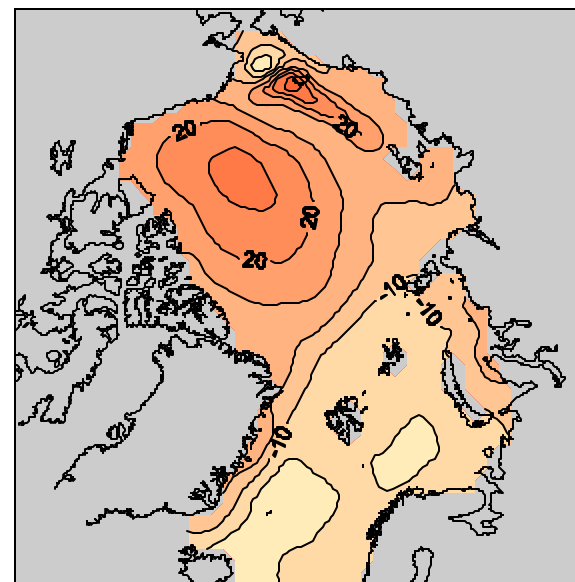
June



September

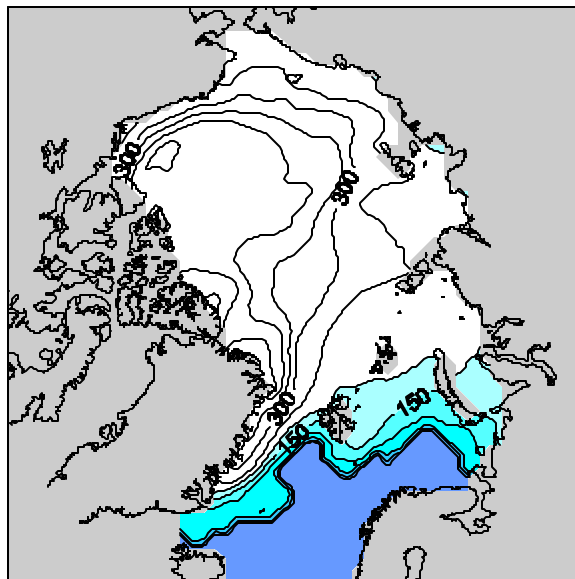


December

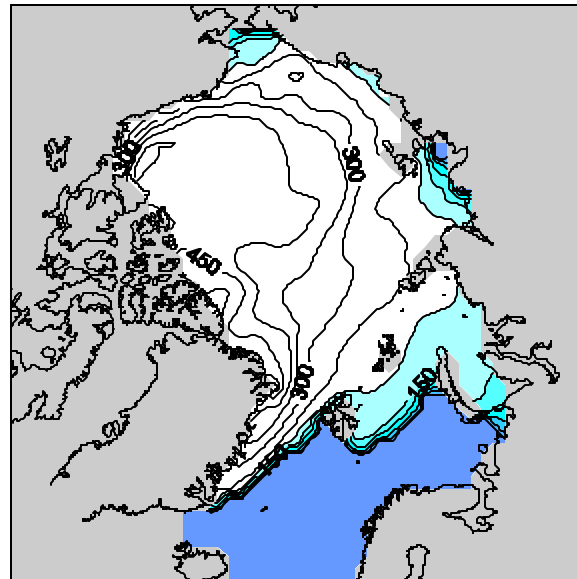


# Ice Thickness

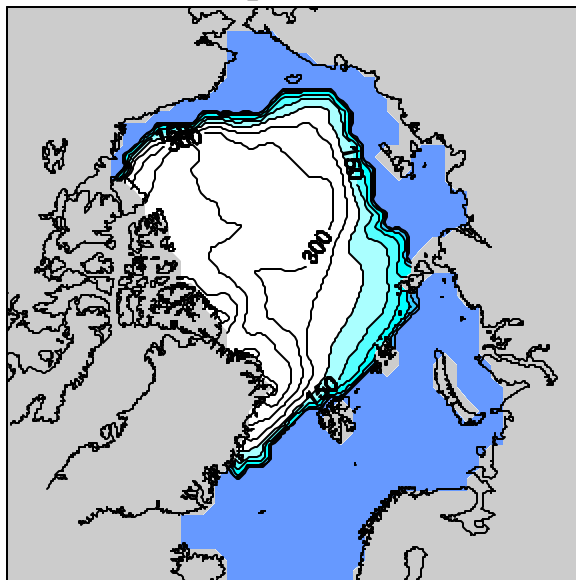
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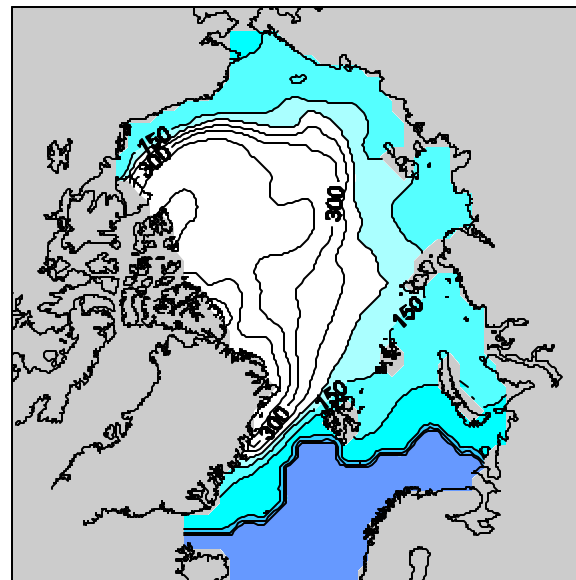
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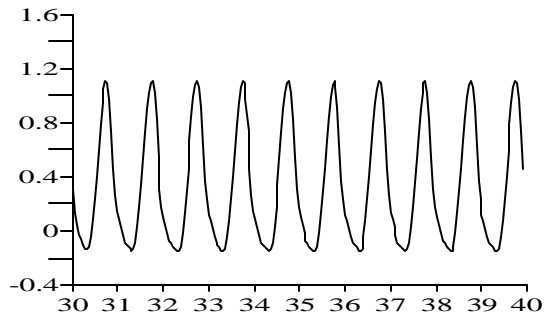


December

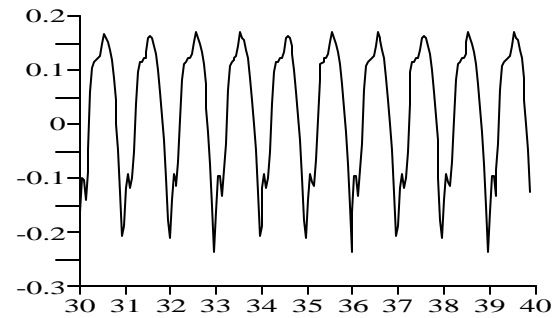


# Mass Transports through the Passages

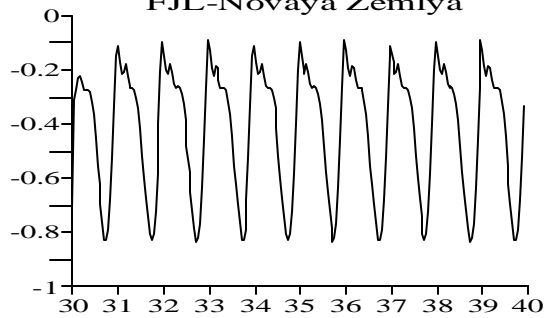
Fram Strait



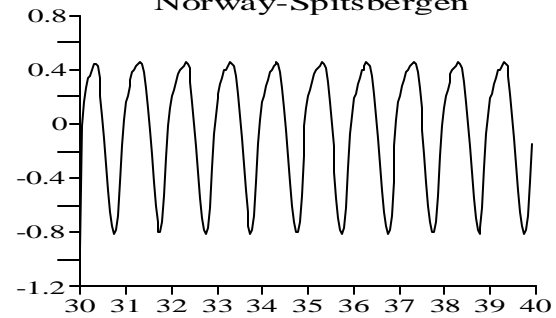
Karskiye Vorota



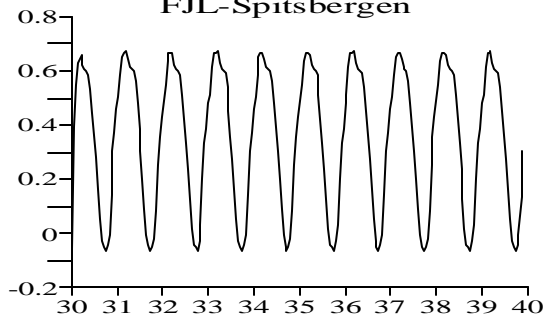
FJL-Novaya Zemlya



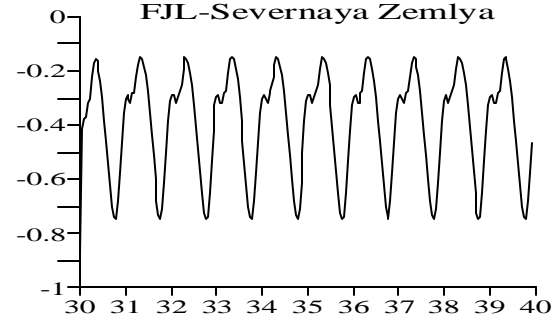
Norway-Spitsbergen



FJL-Spitsbergen

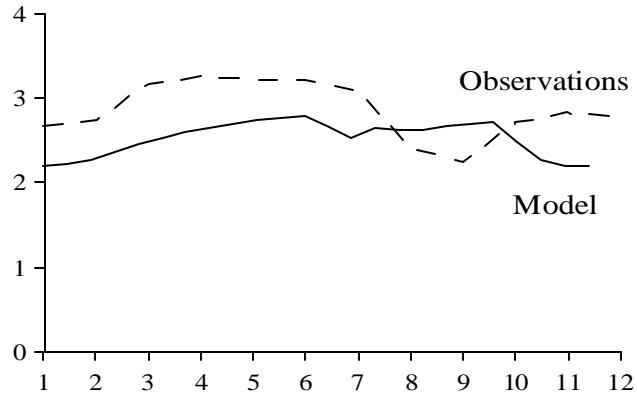


FJL-Severnaya Zemlya

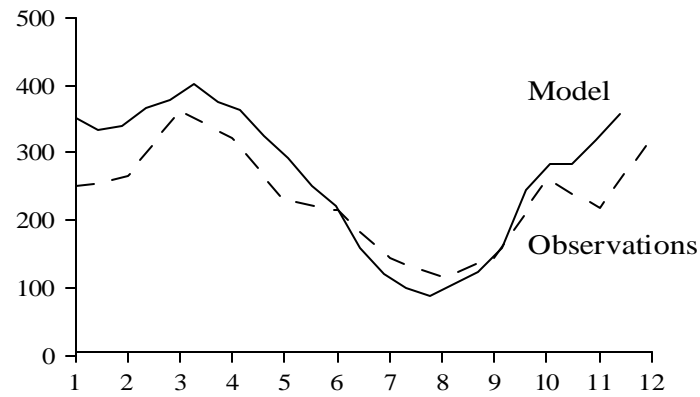


# Fram Strait

Ice Thickness in Fram Strait (m)



Ice Transport in Fram Strait ( $\text{km}^3/\text{month}$ )



# 4. AOMIP Activity

## NEW VERSION OF THE MODEL:

1. New atmosphere temperature and pressure (NCEP\NCAR Reanalysis): **daily data** were interpolated on the model grid.
2. New boundary conditions at **open boundaries** for temperature and salinity.
3. **Rivers** (both salinity and mass transports).
4. More realistic **ice rheology**.
5. New parameterizations for **latent heat and income longwave radiation** - time scheme for temperature.
6. Improved predictor-corrector time scheme for ice-ocean drag: time step up to **6 hours**.

## **Input parameters: AOMIP**

### **Model initialization:**

- Repeated some 10-20 times 1948 year of atmospheric forcing started with no ice, no ocean currents and ocean level slopes.
- Climate restoring to temperature and salinity with time scale 30 days.

### **Motivation for this preliminary stage:**

- Initial conditions for 1948-2002 AOMIP run.
- Test case to tune model parameters and parameterizations: Albedo model, ice-water heat fluxes, deep convection, vertical mixing, *etc.*