A Theory for the Quasi-biennial Oscillation Based on Gravity Wave Breaking and Saturation

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Abstract

This paper describes a model for the quasi-biennial oscillation based on the theory of gravity wave breaking and saturation. In the model, gravity waves are forced at low altitude and propagate vertically upwards, their amplitude increasing with height. In regions where the waves begin to overturn, dissipation is added locally, so that the waves are prevented from breaking. This results in momentum flux divergence and acceleration of the zonal mean velocity.

1 Introduction

The quasi-biennial oscillation (QBO) is an alternating pattern of westerly and easterly zonal mean winds observed in the equatorial stratosphere. The oscillation has an average period of 26 months and a maximum amplitude of about 20 m s^{-1} and is symmetric about the equator occurring between 12° S and 12° N. In the past few decades, a number of theories have been developed to explain the QBO. It is well-known now that it is driven by wave-mean flow interactions of waves propagating upwards from the troposphere. There has been some debate as to the type of waves that are involved, i.e., large scale Rossby-gravity waves and Kelvin waves and/or mesoscale internal gravity waves. It is now generally recognized that to generate a large enough vertical momentum flux to drive the QBO, the gravity wave fluxes must be present. The review article of Dunkerton [1] discusses the role played by gravity waves in generating the QBO.

The QBO has been simulated quite successfully using one-dimensional models in which the waves interact with the mean flow but wave-wave interactions are ignored. Lindzen and Holton [2] initially postulated that the waves interact with the mean flow through absorption at a critical level where the mean velocity is equal to their phase speed. Oscillations in the mean velocity would result on specifying a suitable upper boundary condition to simulate an equatorial semi-annual oscillation (period $\sim$ 6 months). However, a few years later, they re-evaluated this theory including the effects of wave dissipation due to Newtonian cooling [3]. The mechanism by which dissipation of the waves could produce an oscillation of the mean flow was presented explicitly by Plumb [4] using a numerical model and by the laboratory experiments of Plumb and McEwan [5]. One of the key features emerging from [4] is that the period of the oscillation is inversely proportional to the square of the amplitude of the forcing.

In recent years, some GCMs (general circulation models) have been able to spontaneously produce QBOs (for example, [6], [7]). It is interesting to note that the majority of these oscillations have periods that quite closely approximate the period of the observed QBO. This is quite surprising given the fact that the GCMs all use different formulations, in particular, different gravity wave parameterizations, and hence they must certainly employ different forcing amplitudes (see, for example, [8], for a comparison of several GCMs, some of which are able to generate QBOs). According to the wave-mean flow interaction models [4], one would have expected their periods to differ greatly.

The motivation for this GFD Summer Project was to find an explanation for this. This report describes a simple model for the QBO that was developed in an attempt to better understand the relationship between the forcing amplitude and the period of the oscillation. The model is based on the idea of gravity wave breaking and saturation described by Lindzen [9]. The formulation of the model is similar to that of Plumb [4]. The main difference is that dissipation is introduced locally only when needed to prevent wave breaking. Because of the similarity with Plumb's model, a brief description of that model is given in section 3. In section 4, Lindzen's gravity wave saturation criterion is described. Results of the simulations are presented in section 5.

2 Generation of a QBO by dissipation

In this section, a brief review is given of the mechanism by which a QBO can be generated by dissipation of waves. The details are given in [4]. Note that the notation here differs slightly from that of [4]; in particular, the streamfunction here has been defined as the negative of the one used there:

$$\frac{\partial \psi}{\partial z} = -u, \qquad \frac{\partial \psi}{\partial x} = w,$$

so certain terms in the equations below are of opposite sign to the corresponding terms in [4].

The equations for the streamfunction ψ and the buoyancy σ (defined in terms of the density ρ as $-g\Delta\rho/\rho$) are

$$\frac{\partial \nabla^2 \psi}{\partial t} + \frac{\partial \sigma}{\partial x} - \nu \nabla^4 \psi + J(\psi, \nabla^2 \psi) = 0 \tag{1}$$

and

$$\frac{\partial \sigma}{\partial t} - N^2 \frac{\partial \psi}{\partial x} + \mu \sigma + J(\psi, \sigma) = 0. \tag{2}$$

For the moment, it is assumed that all the variables and parameters are dimensionless (see section 4). The equations are linearized about a mean state by writing $\psi(x, z, t) = \bar{\psi} + \epsilon \psi'(x, z, t)$ where $\bar{\psi}$ is the zonal mean and ψ' the disturbance streamfunction. The non-dimensional parameter ϵ then gives a measure of the magnitude of the perturbation quantities relative to the mean flow quantities. The computational domain is a rectangular region at the lower boundary of which a forcing of the form $\psi' = e^{ik(x-ct)}$ is applied. The solution then takes the form

$$\psi'(x, z, t) = \phi(z) e^{ik(x-ct)},$$

where the amplitude $\phi(z)$ of the disturbance satisfies the equation

$$\frac{\partial^2\phi}{\partial z^2} + \left\{\frac{N^2[1+i\mu/k(\bar{u}-c)}{(\bar{u}-c)^2} - \frac{\bar{u}_{zz}}{\bar{u}-c} - k^2\right\}\phi = 0. \tag{3}$$

Viscosity has been ignored in this last equation with the assumption that viscous dissipation of the waves is much less than thermal dissipation [4]. An equation for the evolution of the mean flow is obtained by averaging equation (1) over a wavelength $2\pi/k$:

$$\frac{\partial\bar{u}}{\partial t} - \nu\frac{\partial^2\bar{u}}{\partial z^2} = -\epsilon^2\frac{\partial}{\partial z}(\overline{u'w'}). \tag{4}$$

The numerical model comprises equations (3) and (4). To generate a QBO with this model, there must be two waves with phase speeds of opposite sign, i.e., $e^{ik_1(x-c_1t)}$, $e^{ik_2(x-c_2t)}$. We shall assume that $c_2 = -c_1$. It is assumed that $\bar{u}$ is slowly varying and $\bar{u}_{zz} \approx 0$ and that k^2 is sufficiently small that , equation (3) can be solved using the WKB method (see, for example, Bender and Orszag [10]) and the momentum flux for each wave calculated. These are shown to be:

$$F_n(z) = F_n(0)\exp\left\{-\int_0^z \frac{N\mu}{k(\bar{u}-c)^2}dz'\right\} \qquad n = 1, 2.$$

Note that, in the absence of dissipation, the momentum fluxes would be independent of height as required by the Eliassen-Palm non-acceleration theorem. The right hand side of equation (4) is the sum of the gradients of the momentum fluxes for the 2 waves. It was shown in [4] that the sign of $\overline{u'w'}$ is the same of the sign of the phase speed of the waves. Thus the wave with positive phase speed accelerates the mean velocity while the one with negative phase speed decelerates it.

3 Gravity wave breaking

Following Lindzen [9], but using the notation of section 2 (with $\mu = 0$), the criterion for wave breaking can be determined as follows. First, note that, with $\mu = 0$, the equation for the amplitude of the perturbation streamfunction can be written as

$$\frac{\partial^2\phi}{\partial z^2} + \lambda^2(z)\phi = 0,$$

where

$$\lambda^2(z) = \frac{\mathrm{N}^2}{(\bar{u}-c)^2} - \frac{\bar{u}_{zz}}{\bar{u}-c} - k^2.$$

If $\bar{u}$ is slowly varying and $\bar{u}_{zz} \approx 0$, then an approximate solution can be found for ϕ using the WKB method:

$$\phi \sim \lambda^{-1/2}e^{i\int\lambda dz}. \tag{5}$$

Now the density perturbation is given by

$$\rho' = -\frac{\bar{\rho}_z}{\bar{u}-c}\phi e^{ik(x-ct)}. \tag{6}$$

The condition for wave breaking is that the density gradient be positive, i.e.,

$$\epsilon \left| \frac{\partial \hat{\rho}}{\partial z} \right| \geq |\bar{\rho}_z| \tag{7}$$

To leading order,

$$\frac{\partial \hat{\rho}}{\partial z} \sim -\frac{\bar{\rho}_z}{\bar{u}-c} \lambda^{1/2} e^{i \int \lambda dz}; \tag{8}$$

so the waves break if

$$\frac{\epsilon \left| \lambda^{1/2} \right|}{|\bar{u}-c|} \geq 1. \tag{9}$$

This is approximately equivalent to saying

$$\frac{\epsilon \left| N^{1/2} \right|}{(\bar{u}-c)^{3/2}} \geq 1. \tag{10}$$

Thus, the waves are more likely to break if N is large or if $\bar{u}-c$ is small.

Consider now the case where μ, the diffusion coefficient in equation(3), is non-zero. Then

$$\lambda^2 \approx \frac{\mathrm{N}^2}{(\bar{u}-c)^2} \left(1 + \frac{i\mu}{k(\bar{u}-c)} \right)$$

and λ has an imaginary part. The detailed analysis of Lindzen [9] showed that the condition under which the dissipation would prevent the growth of $|\frac{\partial \rho}{\partial z}|$ is that the imaginary part of λ satisfies

$$|\lambda_I| \approx \frac{\partial}{\partial z} \left(\frac{\epsilon \left| N^{1/2} \right|}{(\bar{u}-c)^{3/2}} \right). \tag{11}$$

4 Numerical model and results

The model equations for our simulation are, as given in section 2,

$$\frac{\partial^2 \phi}{\partial z^2} + \left\{ \frac{N^2[1 + i\mu/k(\bar{u}-c)}{(\bar{u}-c)^2} - \frac{\bar{u}_{zz}}{\bar{u}-c} - \alpha k^2 \right\} \phi = 0 \tag{12}$$

for the amplitude of the perturbation streamfunction and

$$\frac{\partial \bar{u}}{\partial t} - \nu \frac{\partial^2 \bar{u}}{\partial z^2} = -\epsilon^2 \frac{\partial}{\partial z} (\overline{u'w'}) \tag{13}$$

for the time evolution of the zonal mean wind. The various quantities have been non-dimensionalized with respect to reference values as summarized in the table below. The range of values used in the simulations for each of the non-dimensional quantities is given as well as approximate values for some of the dimensional quantities. Asterisks (∗) denote dimensional quantities.

Non-dimensional	Dimensional	Reference quantities
$\bar{u}^*$	$\bar{u} \approx 0.1$	U
$c^* \sim 10 - 20\text{m s}^{-1}$	$c = \pm 0.2$	$U \sim 50 - 100\text{m s}^{-1}$
t^*	t	T
$z^* \leq 50\text{km}$	$z \leq 10$	$H \sim 5\text{km}$
x^*	$x \leq 2\pi$	L
ϕ^*	ϕ	φ

Table 1: Reference quantities used in the non-dimensionalization.

The non-dimensional parameter $\epsilon = \varphi/(LU)$ gives a measure of the amplitude of the perturbation at the forced boundary and $\alpha = H/L$ is the aspect ratio. The buoyancy frequency and hence the time scale T of the evolution of the disturbance and the mean flow are set by the choice of vertical scale H, the velocity scale U and the aspect ratio. The vertical scale is taken to be much smaller than the horizontal scale, so that $\alpha << 1$.

The equations were solved using standard second-order finite-difference approximations for the z-derivatives. The computational domain was the rectangular region $0 < x < 2\pi$, $0 < z < h = 10$. The initial mean velocity took the form of a jet centered at $z = 5$: $\bar{u}(z, 0) = b\text{sech}^2(z - 5)$, where b is a positive constant. Two waves of the form $e^{ik_n(x-c_nt)}$, with $k_1 = 1$, $c_1 = 0.2$ and $k_2 = 1$, $c_2 = -0.2$, were forced at $z = 0$ and propagated up to interact with the jet. The constant b was chosen to be less than 0.2, so that the Doppler-shifted phase speed $(\bar{u} - c)$ of both waves was non-zero everywhere. In other words, there was no critical level for either wave initially and, consequently, none could be generated at later times ([4]). At the upper boundary of the computational domain, a radiation condition was applied to prevent reflections at the boundary. The buoyancy frequency N was allowed to be dependent on z. It was chosen to satisfy

$$N^2 = N_0^2 e^{2z/h},$$

where N_0 is a constant. With this choice of N^2, the perturbation streamfunction varies like $e^{z/2h}$. The WKB-defined vertical wavenumber is approximately $N/(\bar{u} - c)$. With $c = \pm 0.2$, $N \sim O(1)$ and $h = 10$, there are then several vertical wavelengths within the computational domain.

The first few experiments were carried out with non-zero constant dissipation to reproduce the results of Plumb [4]. A QBO was generated with a period that depended on ϵ the amplitude of the forcing. Figure 1 shows time-height plots of the zonal mean velocity computed using this model. The set of parameters used in both graphs was as follows: $N_0 = 2.0$, $\mu = 10^{-3}$, $\nu = 5 \times 10^{-4}$, $\alpha = 2 \times 10^{-3}$. Initially the mean velocity profile took the form of a positive jet in the region $4 < z < 6$ and approxiamtely zero elsewhere. In Figure 1(a), this positive region has descended to $z = 0$ by $t \approx 2700$. This positive phase is followed by a negative phase (approximately $2700 < t < 5000$) and then another positive phase, and so on. The period of the oscillation appears to be about 5500 non-dimensional time units. This corresponds to a dimensional period of magnitude of about a month using the reference values given in Table 1. By decreasing ϵ from 0.1 to 0.05, the period of the oscillation was increased as predicted by the theory described in section 2. With a still smaller forcing amplitude, a more realistic period could be obtained.

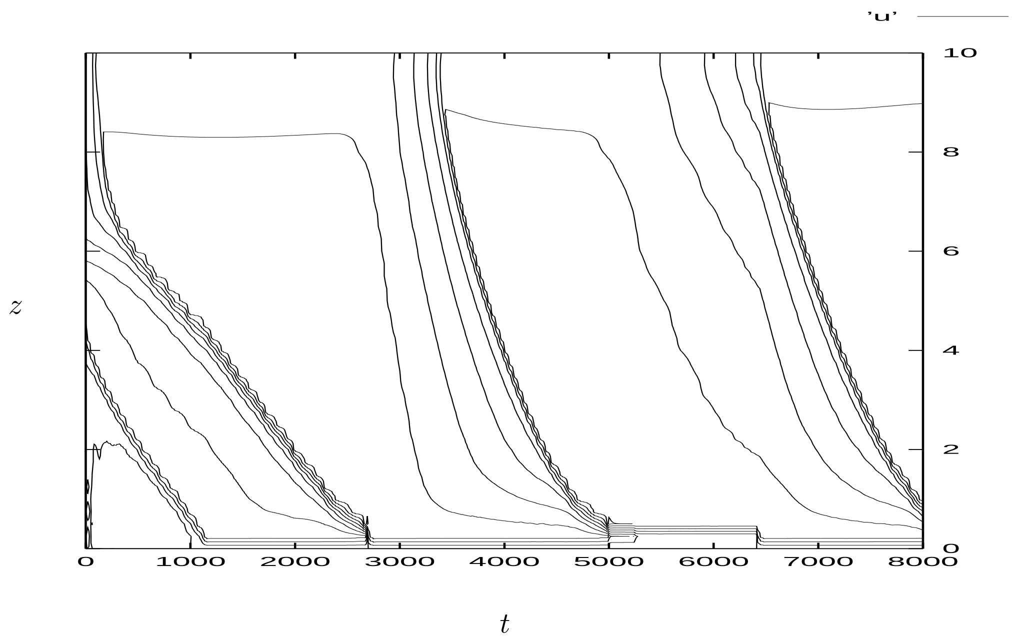
Having verified that the model could produce an oscillation when set up in this way with constant dissipation throughout the computational domain, we set about implementing the gravity wave saturation criterion described in section 3. With our choice of N^2, the criterion (11) becomes

$$|\lambda_I| \approx \frac{1}{2h} - \frac{3}{2}\left(\frac{1}{\bar{u}-c}\frac{\partial \bar{u}}{\partial z}\right). \tag{14}$$

The imaginary part of λ is known from the expression for λ and, from this, the dissipation coefficient μ required to prevent the waves from breaking may be evaluated. Clearly this μ depends on z; it is largest in regions where the waves are on the verge of breaking and small or zero elsewhere. The saturation technique can thus be summarized as follows. Initially μ is set to zero and the model run is started. Without dissipation, the momentum fluxes for the waves are independent of height and thus the only changes in the mean velocity result from the presence of the small but non-zero viscous term. The amplitude of the waves increases with height because of the exponentially increasing buoyancy frequency. Dissipation is introduced locally using (14) to prevent the waves from breaking. Thus, more dissipation is needed at higher altitudes and in particular near the center of the jet where $(\bar{u}-c)$ is smallest, but almost none near the ground. With such a set-up, the extent to which the mean velocity is accelerated should, to some extent, be insensitive to what occurs at low altitudes. It is conceivable then that the time scale of the evolution of the mean velocity would be independent of changes in the amplitude of the forcing.

The time-height plots resulting from this type of simulation are shown in Figure 2 for $\epsilon = 0.1$ and $\epsilon = 0.05$. Descending westerly and easterly phases are seen at higher altitudes; however the time scale of the descent is quite long and due to computational restrictions, the model has not yet been run for a long enough time to produce an oscillation. It is interesting to note that this time scale is almost the same for the two values of ϵ. It remains to be seen whether, if the simulations are continued to the point where an QBO-like oscillation results, the period of the oscillation would indeed be the same for the two values of ϵ.

(a)



(b)

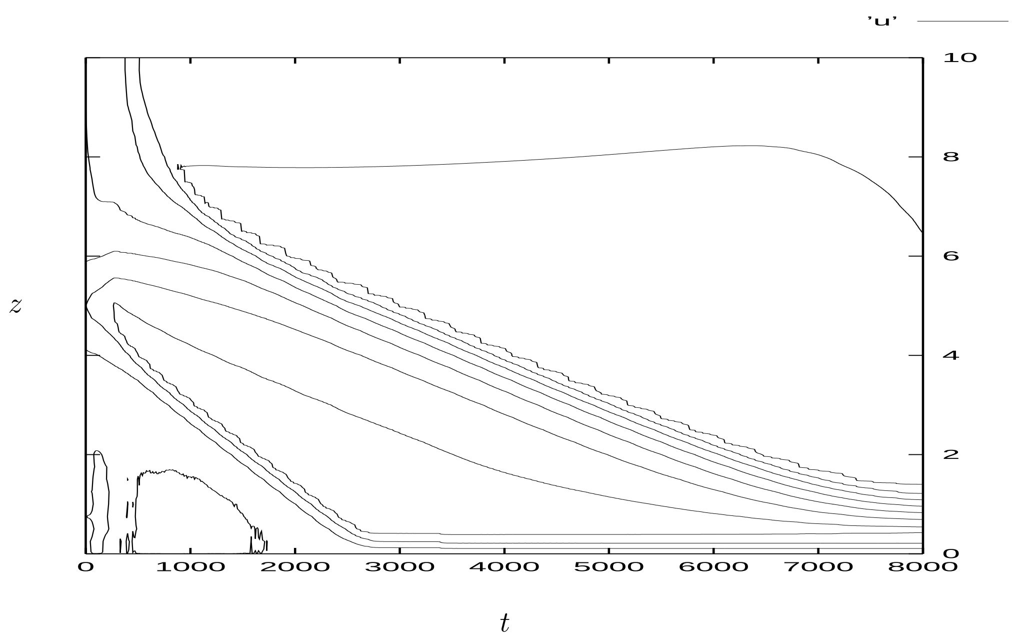


Figure 1: Time-height plots of the zonal mean velocity for the case of constant dissipation with (a) $\epsilon = 0.1$ and (b) $\epsilon = 0.05$

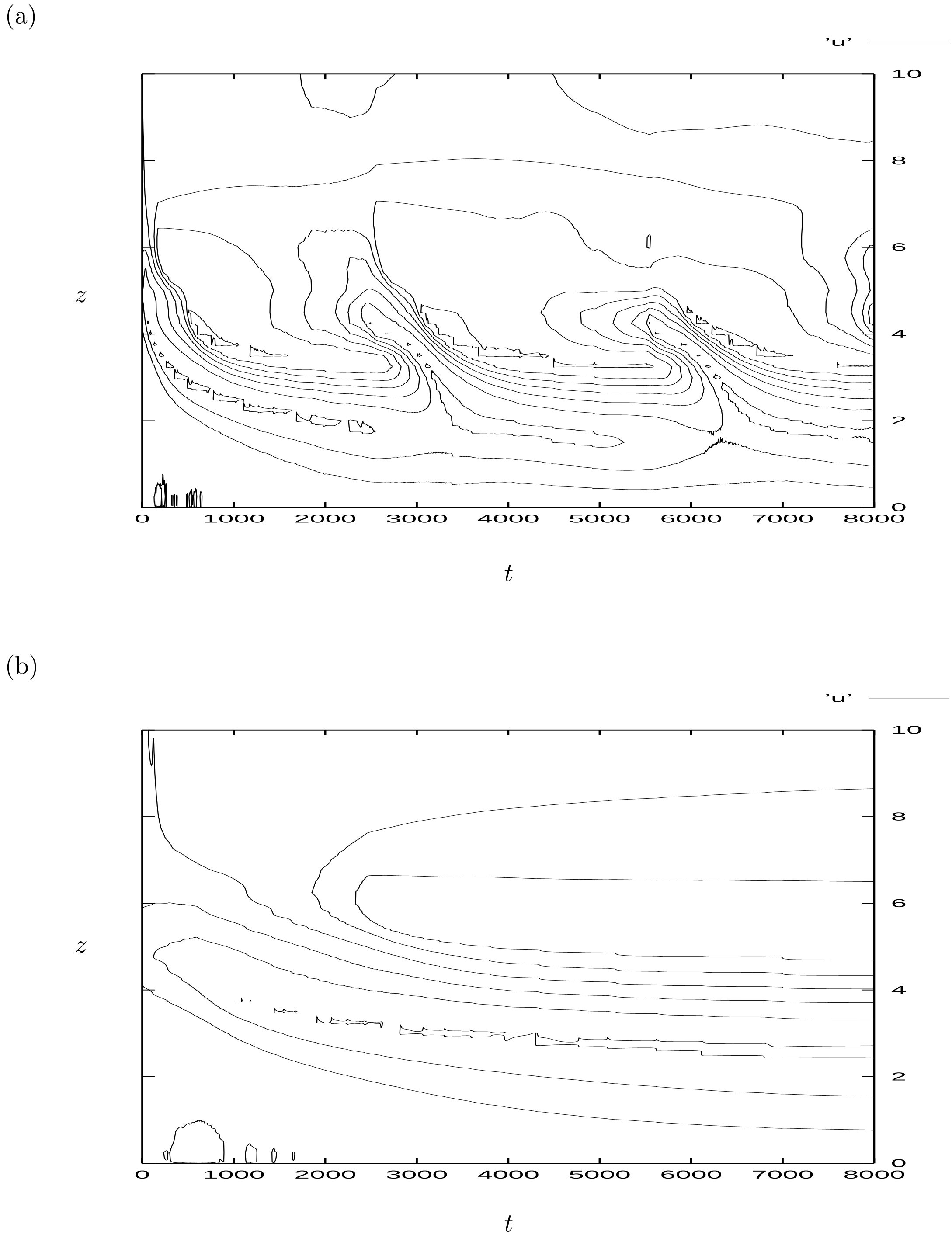


Figure 2: Time-height plot of zonal mean velocity with (a) $\epsilon = 0.05$ and (b) $\epsilon = 0.1$. Here, dissipation has been added to prevent the waves from breaking.

5 Conclusions

This report described a model for the QBO in which the waves are dissipated locally in regions where their amplitudes grow to the point of breaking. By adding dissipation to the model in this way, the mean flow is accelerated/decelerated locally and descending westerly and easterly phases of the zonal mean velocity result. The time-scale on which this occurs is apparently insensitive to changes in the forcing amplitude.

In section 1, it was mentioned that most GCMs that produce QBOs get the period approximately right although they use different wave parameterizations. This suggests that the period of the QBO must be independent of the amplitude of the waves at the forcing level. It is possible then that, in reality, the QBO is generated by a mechanism such as that described in this report. However, this report describes work in progress. At the time of writing, only a limited number of runs had been carried out. A more extensive series of simulations with a wider range of choices of the relevant parameters and a longer total computational time would certainly shed more light on the the relation between the period and the amplitude of the forced waves. Ultimately, a full non-Boussinesq model may be required for a complete analysis of the problem. Also it would probably be necessary to include the effects of the higher harmonics, i.e., wave-wave interactions, as well as wave-mean flow interactions.

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