

Estimates of North Atlantic Ventilation and Mode Water Formation for Winters 2002–06

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ABSTRACT

Lagrangian estimates for ventilation rates in the Gulf Stream Extension using Argo and World Ocean Circulation Experiment/Atlantic Climate and Circulation Experiment (WOCE/ACCE) float data, scatterometer (QuikSCAT) wind stress satellite observations, and altimetric [Archiving, Validation, and Interpretation of Satellite Oceanographic data (AVISO)] sea surface height (SSH) satellite observations from 2002 to 2006 are presented. Satellite winds and estimates of surface geostrophic currents allow the inclusion of the effects of currents on wind stress as well as their impact on the Ekman pumping. The presence of large surface geostrophic currents decreases the total Ekman pumping, contributing up to 20% where the Gulf Stream makes its two sharpest turns, and increases the total Ekman pumping by 10% or less everywhere else. The ageostrophic currents may be as large as 15% of the geostrophic currents, but only in proximity of the Gulf Stream.

Using currents and mixed layer depths (MLDs) that are either climatological or vary from year to year, obducted water tends to originate along the Gulf Stream, while subducted water tends to originate to its south. However, using time-varying MLDs for each year, subduction varies significantly, sometimes oppositely from obduction. The 18° Water (EDW) subducts in different locations and is distributed differently each year but tends to be located in the Sargasso Sea. Vertical pumping is the only dominant factor in ventilation closer to the coast where MLDs are shallower and lighter parcels are subducted. Vertical pumping contributes up to 20% of the several hundreds of ventilated meters per year around the Gulf Stream and less elsewhere. Using a temperature- or density-based criterion for estimating the MLDs, especially along the coasts and north of 45°N, obduction estimates differ by up to 25%. The horizontal and temporal structure of the MLDs is the primary factor that controls the tens of sverdrups of ventilation (and a few sverdrups of EDW subduction).

1. Introduction

The upper ocean that separates the atmosphere and the permanent thermocline consists of an Ekman layer, a mixed layer, and a seasonal thermocline. Water parcels gain their identities in the mixed layer and keep their identities to a great extent in the thermocline. To understand climate variability and water mass forma-

tion, it is important to know about water mass exchange rates between the surface and the permanent thermocline. The ocean can exchange properties such as heat with the atmosphere and may store these properties for decades or longer through transfer to the thermocline (Williams 2001).

Subduction is the mass flux from the seasonal thermocline into the permanent thermocline. Obduction is the (irreversible) mass flux into the seasonal thermocline from the permanent thermocline. These processes may also be referred to as ventilation. The top of the seasonal thermocline is defined as the shallowest depth at which the mixed layer's base is mixed within a given

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year. The top of the permanent thermocline is defined as the deepest mixed layer depth within a given year. Seasonally, the entire column of water down to the base of the seasonal thermocline has been mixed, but may not be mixing.

Because temperature and salinity are not conserved when water parcels are mixed, the top of the permanent thermocline is the relevant boundary when considering conservation of physical properties of water parcels. It was believed by Iselin (1939) that Ekman pumping in the late winter (March) forced water to cross the base of the wind-generated Ekman layer and flow along isopycnals thereafter. Stommel (1979) hypothesized that taking all mixed layer parameters from late winter can bypass the complication of the seasonal cycle and thus the seasonal thermocline can be ignored. Climatological estimates using Stommel's hypothesis assume water parcels subducted from the summer mixed layer are generally reentrained as the mixed layer deepens the following winter. This is usually referred to as Stommel's mixed layer demon.

In addition to the sign difference, obduction and subduction are different in three ways, according to Qiu and Huang (1995). 1) Different physical processes govern them. Temperature and salinity properties, as determined by the mixed layer processes, will be carried by the subducted water to the permanent thermocline. On the other hand, water loses its identity via convective mixing when water is obducted. 2) Subduction and obduction have different annual cycles. Subduction occurs after late winter when the mixed layer restratifies, and obduction occurs between late fall and early winter when the mixed layer is deepening. 3) A mean annual subduction rate is an estimate of how much water enters the permanent thermocline, while a mean annual obduction rate is an estimate of how much water comes from the permanent thermocline.

Local maxima in the subduction of water occur where there are "mode" waters for which the water column is nearly vertically homogeneous in density. The mode waters of interest here are also characterized by temperatures between 17° and 19°C (Worthington 1959). Because of this property, its alternate name is "Eighteen Degree Water" (EDW). This well-defined mode water is important for the storage of heat and chemical constituents in the North Atlantic. While surface heat fluxes largely determine the sea surface temperature (SST) variability in most of the World Ocean, heat transport by geostrophic flow makes a contribution comparable to surface heat fluxes in the northwestern North Atlantic (Dong and Kelly 2004). Dong and Kelly have also shown that the interannual heat transport is primarily driven by geostrophic currents, not by Ekman currents.

EDW is a link between the ocean and atmosphere. Production of EDW is believed to be connected to the North Atlantic Oscillation (NAO), sea surface density anomalies near the Gulf Stream, and deep-water formation in the North Atlantic subpolar regions (Curry and McCartney 2001). The EDW typically forms just south of the Gulf Stream where it cools (to the lower end of the 17°–19°C range) and recirculates. Years with greater EDW production tend to have EDW closer to 17°C, while years with lower production have warmer EDW as the EDW mixes with warmer recirculating water (Kwon 2003, see chapter 5). EDW is also an important component of the near-surface overturning circulation and for transferring biogeochemical properties between the near-surface and the ocean interior since EDW is depleted of nutrients and rich in carbon content (Palter et al. 2005).

Joyce et al. (2000) found that the Gulf Stream (northern latitude) position, NAO index, and EDW potential vorticity (or lack of EDW production) are highly correlated with one another with a zero lag time. The EDW potential vorticity is also highly correlated with large-scale atmospheric forcing. Hence, three factors have been thought to influence EDW variability: 1) Gulf Stream position, 2) ocean–atmosphere heat fluxes, and 3) temperature and salinity anomaly advection. However, Dong et al. (2007) has found that both oceanic advective convergence—the product of heat fluxes—and the area of the outcropping 17°–19°C isotherms, not variability of surface heat flux, play the most important roles in EDW variability. The oceanic advective convergence indicates the importance of 1) and 3) while the area of the outcropping 17°–19°C isotherms indicates the importance of preconditioning. Since the NAO affects the SSTs, a link between the NAO and EDW may be drawn if it can be shown that the SSTs affect EDW volume.

Kwon and Riser (2004) have shown that the EDW volume reaches its annual peak in the winter. Using profile estimates, they show that the formation rate of EDW is on the order of 4–5 Sv ($\text{Sv} \equiv 10^6 \text{ m}^3 \text{ s}^{-1}$) from profile estimates, as opposed to the 15–20 Sv Speer and Tziperman (1992) estimated from air–sea fluxes using the Walin (1982) formulation. The Walin (1982) formulation diagnoses the EDW formation rates by using diabatic forcings as a function of outcropping isopycnals. No circulation or mixing is included, so mixing of density within the interior is ignored. Nurser et al. (1999) found both deepening of the winter mixed layer and lateral mixing within the mixed layer (including entrained denser waters into lighter ones) to be potentially significant at midlatitudes, albeit less significant than mixing in sustained upwelling regions in the tropics. The discrepancy between the estimates of Kwon and Riser (2004) and Speer and Tziperman (1992) may be caused by an error

TABLE 1. The data we used in our calculations.

Quantity	Winds	SSH	MLD, σ_θ , EDW, shear	Shear
Source	QuikSCAT	AVISO	Argo, WOCE/ACCE	WOA
Dates	Jan 2002–Aug 2006	Jan 2002–Aug 2006	January 2002–June 2007	Pre-1994
Frequency	Daily	Weekly	Every 10 days	Monthly

in the Walin (1982) formulation or inaccurate air–sea fluxes, but most probably can be accounted for by the lateral mixing, or eddy processes, that result in lateral transport in the mixed layer near the Gulf Stream. These processes have patchy spatial distributions to which bulk parameters may not do justice.

Here we build on the work of Qiu and Huang (1995) who made subduction rate estimates for the North Atlantic (and Pacific) with climatological estimates for mixed layer depths (MLDs), climatological estimates for currents, and the Hellerman and Rosenstein (1983) wind stress data (which has been noted to overestimate the wind stress by 30%; Harrison 1989). We recalculate Ekman pumping, as Xie (2004) did with Quick Scatterometer (QuikSCAT) wind stress data from 1999 to 2003, but include the impact of upper-ocean geostrophic currents on Ekman velocities. We include winds, currents, observationally derived MLDs, and vertical shear to calculate (EDW) ventilation rates and compare with previous estimates. Section 2 describes the data used in our Ekman pumping and (EDW) ventilation rate estimates. We end with results in section 3 before summarizing.

2. Data and methods

Owing to recent realizations of satellite and profiler systems, we are able to investigate not only interannual variability in a few selected locations but also intra-annual variability across the Gulf Stream and Sargasso Sea. QuikSCAT winds are available daily from July 1999 and mapped as in Kelly et al. (1999; more information available online at <http://airsea.jpl.nasa.gov/DATA/QUIKSCAT/wind/>); Archiving, Validation, and Interpretation of Satellite Oceanographic data (AVISO) sea surface heights (SSHs) are available weekly from October 1992 (<http://www.aviso.oceanobs.com/en/data/products/sea-surface-height-products/global/msla/index.html>); Argo temperatures and salinities are available every 10 days from the year 2000 (http://www.usgodae.org/ftp/outgoing/argo/geo/atlantic_ocean/); and World Ocean Circulation Experiment/Atlantic Climate and Circulation Experiment (WOCE/ACCE) mixed layer depths and temperatures are from 1997 through 2003. *World Ocean Atlas 1994* (WOA) temperature and salinity climatologies are available for each month of the year (<http://www.cdc.noaa.gov/data/gridded/data.nodc.woa94.html>). Our data are summarized in Table 1.

a. Surface currents

The AVISO SSH product, on a $\frac{1}{3}^\circ \times \frac{1}{3}^\circ$ grid, also includes error maps, which indicate significant errors located between the tracks of Ocean Topography Experiment (TOPEX)/Poseidon and *Jason-1*. Because of these errors, surface geostrophic flow calculated from the SSH anomalies shows signals that reflect the satellite track patterns. The anomalies have high-frequency noise and coverage errors in them while the mean does not, as the mean was derived from historical and hydrographic float (surface) data following Singh and Kelly (1997; more information available online at <http://kkelly.apl.washington.edu/mat1/meanssh.html>). Because more structural detail would be lost if the sum of the anomalies and mean SSHs is filtered, we first filter the anomalies and then add on the mean SSHs (not filtered by us). We use spatial low-pass filtering on the SSH anomaly data after temporally averaging over three-week blocks of time. We use a half power of six longitudinally (between 80° and 30°W) and a half power of five latitudinally (between 20° and 55°N), low-pass filtering once in each direction with a Butterworth filter. In regions where the errors are greater than their corresponding SSH anomalies, we assume that these SSH anomalies are zero, which is typically when errors exceed 20 cm. The only places with greater errors are along the coast (with errors sometimes as high as 50 cm). We set our final SSH to be the sum of these low-passed anomalies and the mean AVISO SSH.

The resulting currents are derived from the geostrophic relationships

$$u_g = -\frac{g}{f} \frac{\partial \eta}{\partial y}, \quad (1)$$

$$v_g = \frac{g}{f} \frac{\partial \eta}{\partial x}, \quad (2)$$

where $g = 9.81 \text{ m s}^{-2}$; f is the Coriolis parameter, which we allow to vary with latitude; and η is the SSH. These currents do not show a track pattern. Additional filtering was applied to mask over the jump between land and nonzero currents. If the difference between the currents between two grid points was 0.25 m s^{-1} or larger, the currents from that longitude to the westernmost grid point and within a box of three latitudinal grid points are set to zero. The mean annual geostrophic surface current structure maps out the Gulf Stream (lhs of Fig. 1).

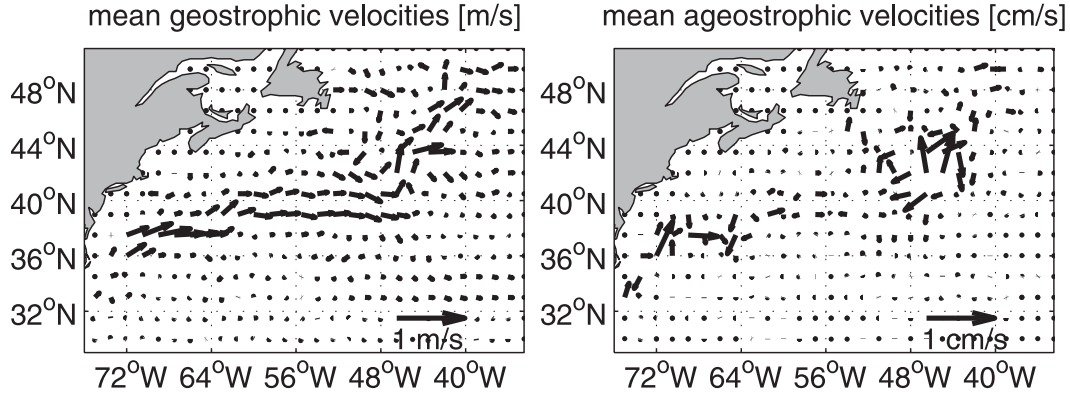


FIG. 1. Average of the geostrophic and ageostrophic (as estimated from the first-order quasigeostrophic equations described in the appendix) currents from AVISO in the Gulf Stream region over 1999–2006.

This objective map gives smaller SSH anomalies in regions of larger errors, which in turn gives slower current speeds.

b. Shear

To obtain the vertical structure of the currents, we low-pass filter the potential densities averaged monthly from January 2002 to December 2006 using the same procedure as in low-pass filtering the SSH: once with a half power of six longitudinally (between 80° and 30°W) and a half power of five latitudinally (between 20° and 55°N). We find these potential densities using WOA (T, S) data when the errors associated with objectively mapped Argo (T, S) data exceed 50 m as calculated according to the objective mapping method of Le Traon (1990). Then we use the thermal wind equations,

$$\frac{\partial u_g}{\partial z} = \frac{g}{f\rho_o} \frac{\partial \rho_\theta}{\partial y}, \quad (3)$$

$$\frac{\partial v_g}{\partial z} = -\frac{g}{f\rho_o} \frac{\partial \rho_\theta}{\partial x}, \quad (4)$$

where ρ_θ is the potential density and ρ_o is the mean density over our domain.

We grid these current shears with 50-m vertical resolution, vertically integrate, and then use the AVISO surface currents from the previous section as a reference. The ratio of the currents at some depth to currents at the surface is capped at one and filled with the spatially averaged ratios for this depth if greater than one (which occurs at less than 10% of grid points). These currents are low-pass filtered once in both horizontal directions with a half power of 4. Last, we fit logarithmic curves to the profiles of these ratios and low-pass filter with a half power of 4 again for a small shear profile.

c. Ekman pumping

One way that water parcels may obduct or subduct is by vertical advection or vertical pumping through Ekman currents (appendix). Ekman pumping is defined by $-w_e \equiv -\nabla \cdot \mathbf{U}_e$, as a convergence of Ekman transports \mathbf{U}_e , which leads to a downward motion. An upward motion results from positive Ekman suction, w_e . Usually, the Ekman transports are approximated with wind momentum input uncoupled from currents:

$$U_e = \frac{1}{\rho f} \tau^y, \quad (5)$$

$$V_e = -\frac{1}{\rho f} \tau^x, \quad (6)$$

here called the simple Ekman balance (appendix). This analytical approximation, which Qiu and Huang (1995) and Xie (2004) have used for w_e , is valid in most regions, but the assumptions fail in the Gulf Stream. If surface geostrophic currents are strong, the vertically averaged flow in the Ekman layer satisfies (appendix)

$$\begin{aligned} \frac{\partial U_e}{\partial t} + U_e \frac{\partial u_g}{\partial x} + u_g \frac{\partial U_e}{\partial x} + V_e \frac{\partial u_g}{\partial y} + v_g \frac{\partial U_e}{\partial y} - f V_e \\ = \frac{\tau^x}{\rho} - \tau_{\text{damp}} U_e, \end{aligned} \quad (7)$$

$$\begin{aligned} \frac{\partial V_e}{\partial t} + U_e \frac{\partial v_g}{\partial x} + u_g \frac{\partial V_e}{\partial x} + V_e \frac{\partial v_g}{\partial y} + v_g \frac{\partial V_e}{\partial y} + f U_e \\ = \frac{\tau^y}{\rho} - \tau_{\text{damp}} V_e. \end{aligned} \quad (8)$$

We have used the approximation of Qiu and Huang (1995) and Xie (2004) for a first guess to the Ekman pumping and impose the simple Ekman balance at the

coasts. Such regions serve as boundaries to the regions in which the Qiu and Huang approximation is not valid and help define a well-posed problem. To numerically solve for a converged solution, we have applied a time difference method with time steps $\Delta t = \frac{1}{2}$ h. The geostrophic currents are held fixed for each week of AVISO SSH data and QuikSCAT wind data. To find the Ekman solutions, we time step, including accelerations as well as damping terms, $\tau_{\text{damp}} U_e$ and $\tau_{\text{damp}} V_e$. The damping factor τ_{damp} should be short relative to the temporal resolution of our data, a week, and long relative to the time steps Δt . Running for $2/\tau_{\text{damp}}$ days, the solution reaches steady state. This yields the Ekman currents, and from this we calculate w_e . The root-mean-squared differences between using $\tau_{\text{damp}} = \frac{1}{1} \text{ day}^{-1}$ and $\tau_{\text{damp}} = 1/1.675 \text{ day}^{-1}$ for u_e , v_e , and w_e , respectively, are $0.0020 \pm 0.0014 \text{ m s}^{-1}$, $0.0016 \pm 0.0009 \text{ m s}^{-1}$, and $2.55 \times 10^{-7} \pm 1.19 \times 10^{-7} \text{ m s}^{-1}$. The solution does depend on τ_{damp} , but we choose $\tau_{\text{damp}} = 1/1.675 \text{ day}^{-1}$ to ensure enough time for the Ekman layer to set up.

d. Subduction rate approximation

Subduction is defined as positive when fluid descends through the thermocline, so subduction is positive when w_e is negative. This can be seen in the expression for approximate mean annual subduction rates (Marshall et al. 1993; Qiu and Huang 1995):

$$S_{\text{ann}} = -\left(w_e - \frac{\beta}{f} \int_{-\delta_e}^0 v_g dz\right) + \frac{1}{\tau} (h_{m,0} - h_{m,1}). \quad (9)$$

Here τ is one year and $h_{m,0}$ and $h_{m,1}$ are the March MLDs for the year in which we want to compute the mean subduction rate and the year after that, respectively. The first term represents vertical pumping and the second term represents lateral induction. Average rates of obduction may be computed similarly but with MLD information from one year and the year previous. Over a period, τ , an average subduction rate may be computed via

$$S_{\text{avg}} = -\frac{1}{\tau} \int_{t_0}^{t_1} w_{tr} dt - \frac{1}{\tau} \int_{t_0}^{t_1} \left(\frac{\partial h_m}{\partial t} + u_{tr} \frac{\partial h_m}{\partial x} + v_{tr} \frac{\partial h_m}{\partial y} \right) dt, \quad (10)$$

where t_0 and t_1 are the initial and final times (functions of space and year in our parcel tracking model to be discussed later), $\tau = t_1 - t_0$, w_{tr} and (u_{tr}, v_{tr}) are the vertical and horizontal velocities at the base of the mixed layer, h_m is the maximum depth of the mixed layer in the given year, and the $\partial h_m / \partial t$ term averages to zero over a sufficiently long τ (Qiu and Huang 1995).

e. MLDs

Water parcels may also obduct or subduct by movement of the MLDs relative to a water parcel that allows the MLD to entrain (obduct) or detrain (subduct) this parcel (9). The MLDs are calculated using Argo and WOCE/ACCE data. When available, we use adjusted temperatures and salinities and only measurements with quality “1” ratings (http://www.coriolis.eu.org/cdc/argo_rfc.htm). The floats with World Meteorological Organization identification number 852 from 2004 onward have been excluded owing to a nonconstant cold bias discovered in February of 2007 (<http://argo.jcommops.org>). These floats were fairly uniformly distributed in the North Atlantic, leaving coverage as shown in Fig. 2. Thus, while we expect the MLDs to be biased low including these data and to be higher excluding these data, the MLDs should be better with their exclusion.

We use the shallowest pressures at which the criterion, $\Delta T > 0.3^\circ\text{C}$, is satisfied unless stated otherwise. Here, ΔT is the difference between the surface temperature and the in situ temperature at some depth. Alternatively, $\Delta \rho_\theta > 0.125 \text{ kg m}^{-3}$ may be used as a criterion where $\Delta \rho_\theta$ is the difference between the surface density and the potential density at some depth. Using the $\Delta T > 0.3$ criterion to estimate the MLDs yields results that look qualitatively different from those obtained using $\Delta \rho_\theta > 0.125$ as a criterion along the coasts north of 45°N . Despite how we should be using the $\Delta \rho_\theta > 0.125$ criterion, our WOCE/ACCE data lacks salinity, so we settle with the $\Delta T > 0.3$ criterion.

For each grid point, we calculate a simple two-sample t statistic with

$$t = (\overline{\text{MLD}}_T - \overline{\text{MLD}}_{\rho_\theta}) \sqrt{\left(\frac{\sigma_T^2}{n_T} + \frac{\sigma_{\rho_\theta}^2}{n_{\rho_\theta}} \right)^{-1} - 1}, \quad (11)$$

with degrees of freedom

$$df = \left(\frac{\sigma_T^2}{n_T} + \frac{\sigma_{\rho_\theta}^2}{n_{\rho_\theta}} \right)^2 \times \sqrt{\left[\frac{1}{n_T - 1} \left(\frac{\sigma_T^2}{n_T} \right)^2 + \frac{1}{n_{\rho_\theta} - 1} \left(\frac{\sigma_{\rho_\theta}^2}{n_{\rho_\theta}} \right)^2 \right]^{-2}}, \quad (12)$$

σ^2 representing the sample temporal variance where the MLDs are not zero, $n = 48$ representing the number of months in our sample, T subscripts corresponding to estimates derived from the $\Delta T > 0.3$ criterion, and ρ_θ subscripts corresponding to estimates derived from the

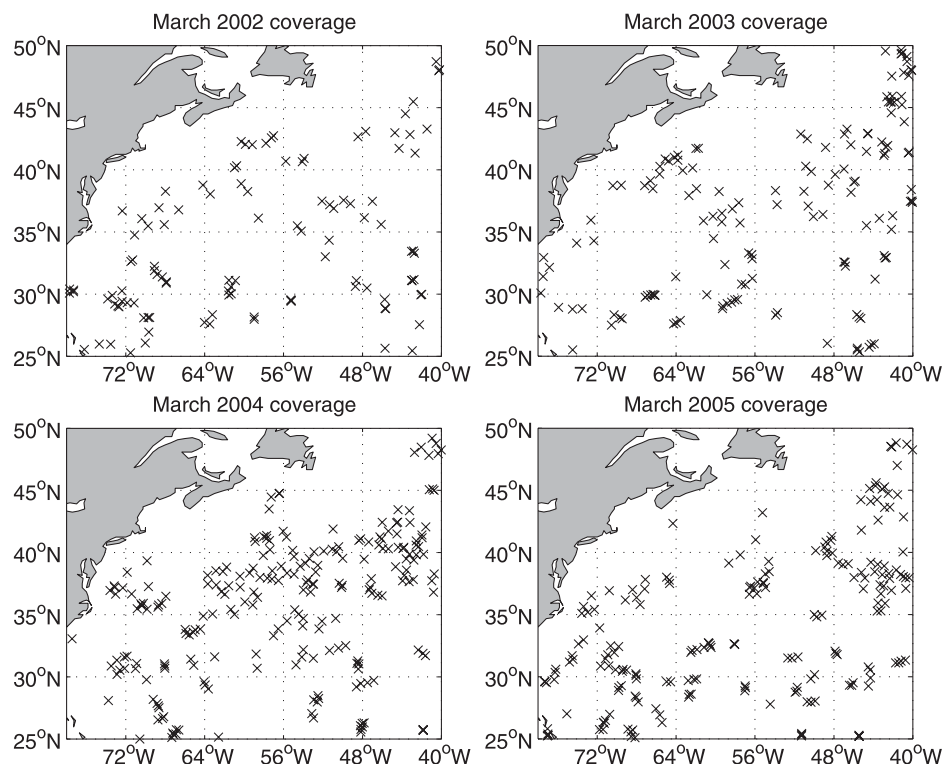


FIG. 2. Locations of non-852 Argo floats that reported data in the month of March from 2002 to 2005.

$\Delta\rho_\theta > 0.125$ criterion (Snedecor and Cochran 1989). We reject the null hypothesis that the (averages of the) MLDs are the same along the coasts and in the northeast region of our domain using the two criteria to the 95% confidence limit. In these regions, using the $\Delta\rho_\theta > 0.125$ criterion yields deeper MLDs. This is consistent with where salinity profiles are relatively homogeneous or compensate for the temperature change in the vertical.

The MLD estimate based on each of the two criteria are objectively mapped according to the method of Le Traon (1990), building on the work of Bretherton et al. (1976). The number of grid points that had valid estimates (nonzero MLD anomalies) was largest in 2004 because that year had fewer number 852 floats than subsequent years. The number of points our objective mapping routine uses on average in a given month to describe the data are 1572 in 2002, 1605 in 2003, 1614 in 2004, 1597 in 2005, 1604 in 2006, and 1625 in 2007. We set our large-scale basis function to be $1 + x + y + x^2 + y^2$ and use a Poisson covariance function and a decorrelation length scale of 150 km.

The boundary between the seasonal and permanent thermoclines is defined to be the maximum MLD, h_m , over one year time. Here, one year is the time from when the MLD is deepest in one year to when the MLD is deepest in the next year at each horizontal grid point

(e.g., Fig. 3). This can be shorter or longer than one year and this variability of time between maximum MLDs greatly influences the result. A concern is whether our results will be an artifact of better data coverage in later years. The means and standard deviations of the time-averaged MLDs and their time-averaged associated objective mapping errors calculated according to the method of Le Traon (1990) are shown in the top and bottom half of Fig. 4. Because the spatial variance from objective mapping is constant, $O(10\,000\text{ m}^2)$, the errors are relatively small.

f. Parcel tracking

Following the method of Qiu and Huang (1995), fluid parcels are tracked from a given initial position and time until they are obducted or run into the coast, with time steps of 7 days. We track the parcel within the upper 1000 m and within the domain $27^\circ\text{--}53^\circ\text{N}$, $78^\circ\text{--}32^\circ\text{W}$. The parcels are placed on a 2° latitude by 2° longitude by 100-m depth grid system and tracked for each horizontal and vertical grid point. The geostrophic velocities of these water parcels are calculated subject to the vertical shear at each of these grid points. The purposes of this $2^\circ \times 2^\circ \times 100\text{ m}$ grid are to 1) resolve the vertical shear in the geostrophic flow and 2) start following water parcels from each of these grid points.

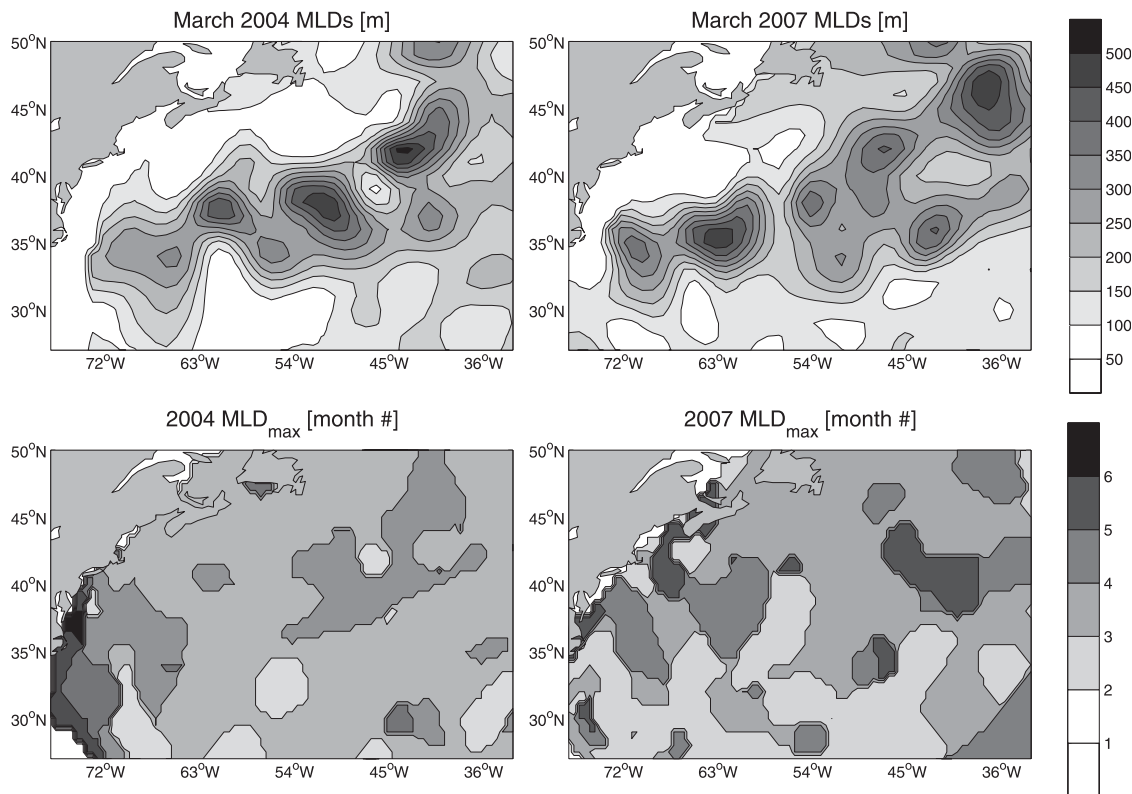


FIG. 3. The MLDs in (top) March and (bottom) the month numbers (e.g., March = 3) when the MLDs reach a maximum in late winter/early spring of (left) 2004 and (right) 2007. These are the years with the most and least spatial coverage by non-WMO 852 Argo floats from when WMO 852 Argo floats errors were introduced.

To track the parcels, a fourth-order Runge–Kutta integration was used. The numerical time integrations have been performed with Matlab’s ode45 (fourth-order Runge–Kutta) routine at a relative tolerance of 1%. The velocities were integrated along the trajectory of each water parcel, $u_{tr} = (u_g + u_e)/[111\,200 \times \cos(\pi\lambda/180)]$, $v_{tr} = (v_g + v_e)/111\,200$, and

$$w_{tr} = \begin{cases} w_e z/\delta_e + \frac{\beta}{f} \int_{-z}^0 v_g dz, & \text{if } 0 > z > \delta_e \\ w_e + \frac{\beta}{f} \int_{-z}^0 v_g dz, & \text{if } \delta_e > z, \end{cases}$$

where λ is degrees latitude, $\delta_e = 40$ m is the Ekman layer depth (below which u_e and v_e are 0), 111 200 is the conversion factor from m s^{-1} to deg s^{-1} , f is the Coriolis parameter allowed to vary with latitude as $\sin(\pi\lambda/180)$, and β is the latitudinal derivative of f . Finally, we assume that flows just below the mixed layer are geostrophic.

We find the depths at which water columns cross h_m to find the thickness of each water parcel that gets transferred to or from the thermocline over τ , the amount of time it takes for the mixed layer to be deepest from one year to the next. Here τ is the interval of time between

releases of water parcels. A subducted (Fig. 5, top) or obducted (Fig. 5, bottom) water parcel is defined by its top and bottom. For subducted parcels, the bottom is the maximum MLD in its first year of being tracked and the top is determined by tracking the parcel. Likewise, for obducted parcels, the top is the maximum MLD in its first year of being tracked and the bottom is determined by tracking the parcel. We interpolate from a $2^\circ \times 2^\circ \times 100$ m grid onto a $1/2^\circ \times 1/2^\circ \times 20$ m grid to find the top of subducted parcels and the bottom for obducted parcels. The 20-m vertical resolution comes from our approximate error estimates in regions where ventilation is predicted to be nonzero. We have performed the same calculation with a 10-m resolution and obtained results that are different by at most 10 m yr^{-1} , less than our error. We assume that water parcels that go outside of our model domain neither subduct if they are above the MLD nor obduct if they are below the MLD when this occurs.

g. Parcel identities

Water parcel properties (temperature, salinity, etc.) are constant below the mixed layer and only change

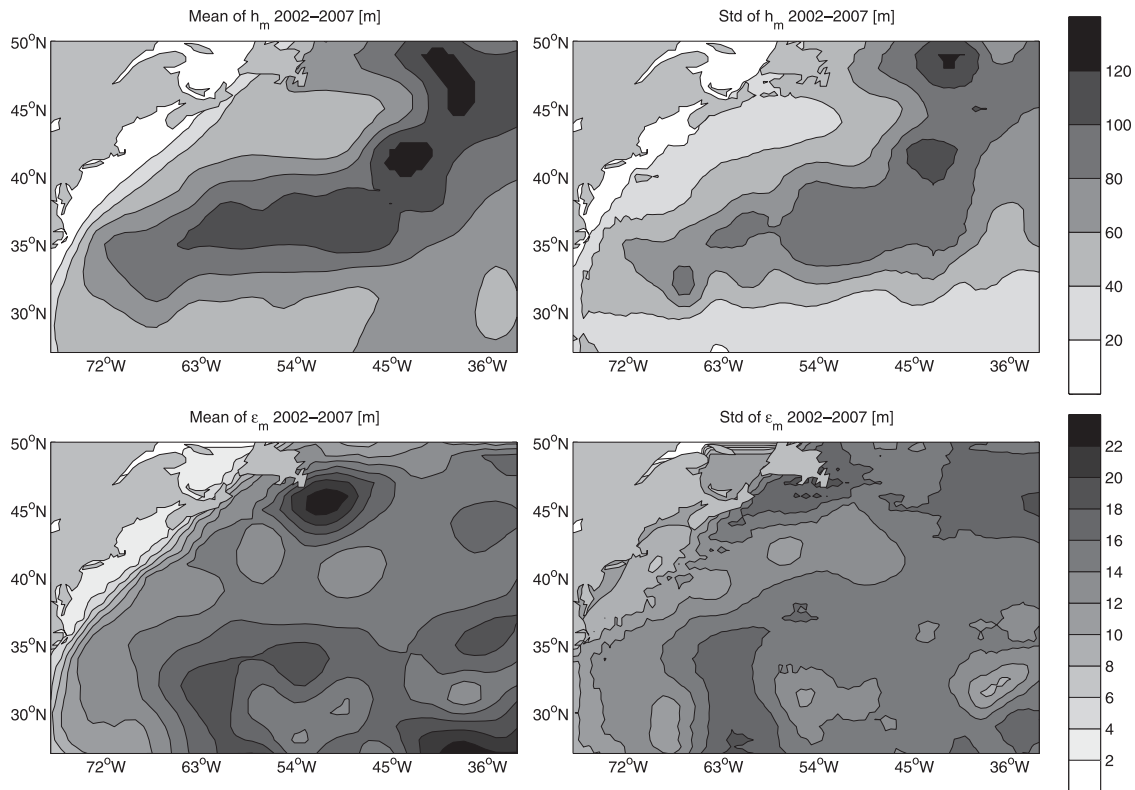


FIG. 4. (top left) Mean of the MLDs over 2002 to 2007 in meters. (top right) Standard deviation of the MLDs over 2002–07 in meters. (bottom left) Mean of the associated objective mapping errors over 2002–07 in meters. (bottom right) Standard deviation of the associated objective mapping errors over 2002–07 in meters.

in the mixed layer. A parcel's properties, once in the permanent thermocline, are identical to those when and where they subduct in our Lagrangian trajectory calculation, so a parcel's properties are set once out of the mixed layer. Due to mixing at the surface, if a parcel comes back into the mixed layer, its properties are no longer assumed to be constant.

The waters that are EDW 1) have temperatures between 17° and 19°C , 2) are west of 40°W and north of 25°N , and 3) have a vertical temperature gradient $dT/dz < 0.006^{\circ}\text{C m}^{-1}$. Kwon and Riser (2004) impose criterion 2 onto their region to calculate EDW volume. This makes at most a negligible 1% difference in volume given the two other properties of EDW. We estimate that the equivalent criterion of 3 in terms of potential vorticity, fN^2 , for Brunt–Väisälä frequency $N^2 = -g/\rho(\partial\rho/\partial z)$, Coriolis parameter f , and $g = 9.81 \text{ m s}^{-2}$ is $|fN^2| < 1 \times 10^{-7} \text{ s}^{-3}$.

We calculate the vertical thickness of EDW and multiply by the area of a grid cell at each grid point to find the volume of EDW in a given month from objectively mapped thicknesses of Argo and WOCE/ACCE temperature data. This objective mapping procedure is the same as the one for the MLDs. We calculate EDW

ventilation rates by using the objectively mapped temperatures from Argo and WOCE/ACCE data to identify the EDW as we track it. We do not use WOA data substitutions here because this reduces the interannual variability in the record. By tracking how much EDW enters the permanent thermocline each year, we can infer the mean annual EDW subduction rates. We do the same with EDW exiting the permanent thermocline to get the mean annual EDW obduction rates. The EDW ventilation rates are dependent upon the aforementioned mixing assumption of our model, but the EDW volume is not since it is calculated directly from data.

h. Error

Using an estimated variance σ_h^2 on the order of $10\,000 \text{ m}^2$ for the MLDs, we may find an upper bound on the error in ventilation rate between two consecutive years. To do this, we set $(\Delta h_{\text{err}})/\tau = e_m$. Here $(\Delta h_{\text{err}})^2 = \epsilon_{m,0}^2 + \epsilon_{m,1}^2$ is the sum of squared MLD errors for two consecutive years. Both $\epsilon_{m,0}^2$ and $\epsilon_{m,1}^2$ are computed by multiplying the associated errors from objective mapping in percent of variance by σ_h^2 . We may also estimate the ventilation rates using the lateral induction term in the Qiu and Huang (1995) formulation (9) with $(\Delta h_m)/\tau$.

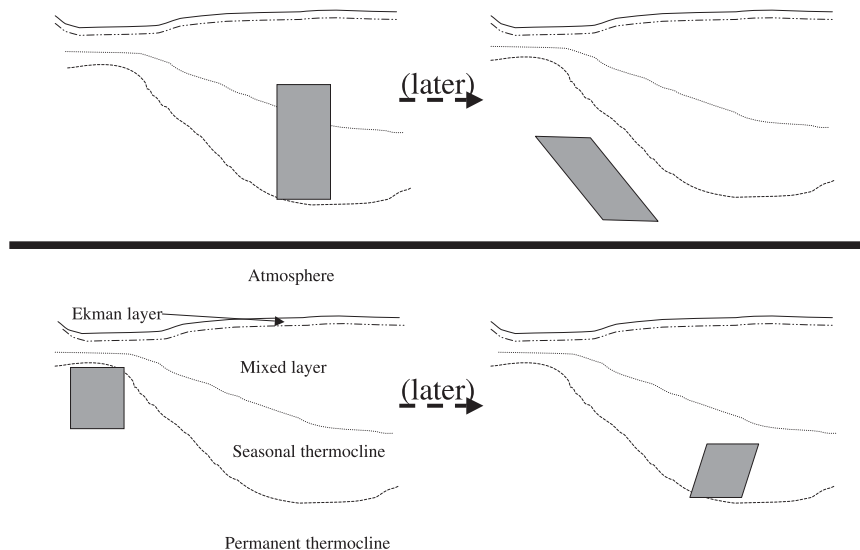


FIG. 5. The atmosphere, surface Ekman layer, mixed layer, seasonal thermocline, and permanent thermocline are shown to represent an idealized cross section across a strong density gradient. The gray parallelograms are approximate representations of the water columns that either subduct or obduct. Here h_m is the boundary between the seasonal and permanent thermocline. (top) The water column subducts and stays in the permanent thermocline. (bottom) The water column obducts. The dashed arrow indicates time evolution, the interval it takes for the MLD to be deepest from one year to the next. For simplicity, h_m is the same in each year.

The spatial distribution of $(\Delta h_{\text{err}})/\tau$ is similar to that of ε_m in Fig. 4 but patchier. The errors are largest around 60° and 37.5°W in the south and off the coast of Newfoundland. By multiplying each $(\Delta h_{\text{err}})/\tau$ and $(\Delta h_m)/\tau$ (“analytical” estimates tabulated in Table 3) by the area of a grid cell at each gridcell point and summing all of the grid points where ventilation is nonzero, we may get volume transport estimates for error and ventilation. We find that the volume transport error estimates can be as large as 50% of our volume transport ventilation estimates.

3. Results

a. Ventilation analysis

By taking advantage of new datasets, we are able to make new estimates for the quantities needed to estimate the ventilation of the thermocline in the North Atlantic. The ageostrophic currents are small in most places relative to the geostrophic ones (right-hand panel of Fig. 1). The first-order corrected ageostrophic currents only get up to 15.8% of the geostrophic currents at any given place or time and almost never get above 10%. The greatest geostrophic current contributions to the Ekman transports come from terms with derivatives in the geostrophic flow (e.g., $U_e \partial u_g / \partial x$). The contributions from the derivatives in the Ekman transports are neg-

ligible, both by scaling arguments and direct calculation, especially outside of the Gulf Stream.

Accounting fully for the geostrophic currents numerically, w_e is noticeably different when calculating w_e directly from the wind (Fig. 6). The advection of Ekman velocities by geostrophic currents decreases w_e by 20% where the Gulf Stream makes its sharpest turns (consistent with our ageostrophic velocities being largest there). The advection of Ekman velocities by geostrophic currents increases w_e by about a 10% elsewhere. Comparing our w_e with Xie (2004), who also used QuikSCAT winds, we see similar structure and magnitude, whereas w_e of Qiu and Huang (1995) is only half as big in the Gulf Stream. There is less pumping south of the Gulf Stream in the Sargasso Sea and less suction north of the Sargasso Sea than in Xie (2004).

Parcels travel at most 5° latitudinally but may travel as many as 35° longitudinally in a year. The means and standard deviations of longitudinal and latitudinal distances water parcels travel each year in our model are given in Table 2. Lateral induction, $(\Delta h_m)/\tau$, dominates both subduction and obduction away from the coast, contributing a few hundreds of meters per year while vertical transport contributes at most 75 m yr^{-1} in the Qiu and Huang (1995) formulation (9). However, vertical transport via w_e becomes quite significant close to the coasts.

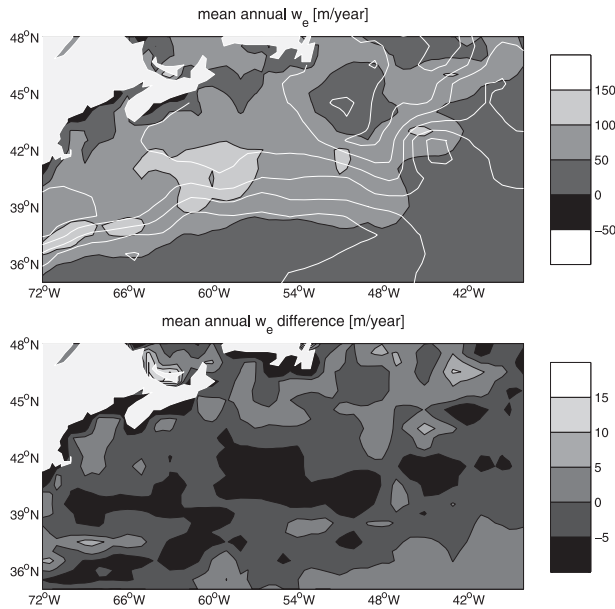


FIG. 6. The average of Ekman pumping (m yr^{-1}) in the Gulf Stream and Sargasso Sea regions, including (top) currents (solved numerically) over 1999 to 2006 with mean SSH contours in white and (bottom) the difference between the average of w_e (m yr^{-1}) including currents and that without currents in the same region over the same time period.

With shear in weekly currents and monthly MLDs for individual years from 2002 to 2005, our ventilation rate estimates are shown in Fig. 7. This may be compared to our use of the Qiu and Huang (1995) formulation with associated errors (Table 3). Using the Qiu and Huang (1995) formulation, the errors are much less than ventilation rate estimates everywhere except the coast of Newfoundland and two southern regions of our domain. Because of these errors, the estimated magnitudes of obducted water parcels from along the coast of Newfoundland are questionable.

By taking the monthly average of the Argo-derived MLDs over 2002–06, annual average of the currents with shear over 1999–2006, and annual average of the vertical velocities over 1999–2006, we may compute climatological ventilation rates (Fig. 8). Vertical shear in the geostrophic flow has a second-order effect on the ventilation rates. This shear may cause up to a 10% decrease in subduction but is not clearly out of our range of error. To see this, an average monthly MLD climatology has been constructed over 2002–06 and used to calculate ventilation both with and without shear (Fig. 8, top and second from top).

Comparing the use of our $\Delta T > 0.3$ criterion (Fig. 8, top) with the $\Delta\rho_\theta > 0.125$ criterion (Fig. 8, second from bottom), we find up to a 30% decrease in obduction using the former because h_m is smaller in the northeast

TABLE 2. Means and std dev of the longitudinal and latitudinal distances ($^\circ$) water parcels travel in a given year through our model.

Distances	Year			
	2002	2003	2004	2005
Mean of longitudinal distance	6.24	6.34	6.40	6.19
Std dev of longitudinal distance	6.87	7.07	7.55	7.48
Mean of latitudinal distance	1.11	0.989	0.963	1.07
Std dev of latitudinal distance	1.48	1.24	1.37	1.32

region of our domain where we estimate more obduction than subduction. The difference between total subduction and obduction is closer to zero using the $\Delta\rho_\theta > 0.125$ criterion. Comparing our estimates with and without Ekman currents (Fig. 8, top and bottom), we find up to a 15% decrease in ventilation when $w_e = 0$.

We may also compare the climatology using the $\Delta T > 0.3$ criterion with shear to the ventilation rates that we get by using the monthly Argo-derived MLDs (not averaged over 2002–06), average of currents over 1999–2006 with shear, and average of vertical velocities over 1999–2006, and finally taking the average of these ventilation rates over 2002–06 (Fig. 9). We find that the ventilation rates calculated using mean currents are about 5% smaller and using mean MLDs are about 45% smaller. The lateral induction term of (9) is mostly responsible for this (nonlinear) difference in ventilation using means instead of weekly currents and monthly MLDs.

More specifically, the MLDs are not deepest in March every year (Fig. 3, bottom). For example, one year may have a deepest MLD in April and the next may have a deepest MLD in February at a given horizontal grid point, as governed by convective cooling and stratification strength. Then the ratio of Δh_m to τ may not stay constant from year to year. Here, τ is the time from when h_m is deepest one year to when h_m is deepest in the following year. The other nonlinear component to the ventilation rate estimation pertains to spatial distributions of currents and MLDs. The MLDs within the Gulf Stream are not as deep as they are to its south and east; as stratification is strong within the Gulf Stream, the negative buoyancy forcings are sufficiently counteracted by advected heat. The subduction rates are nonzero south and east of the Gulf Stream (Fig. 7) where the MLDs deepen most (Fig. 3, top). Their magnitudes are consistent with Qiu and Huang (1995).

On the other hand, the imbalance of subduction and obduction over our time series warrants further attention: as Table 4 shows, over our four years there is a mean net subduction (subduction minus obduction) of 0.6 Sv of water with more subduction than obduction only in 2004 and 2005. A mean net subduction greater than zero would imply a shoaling of MLDs over the time

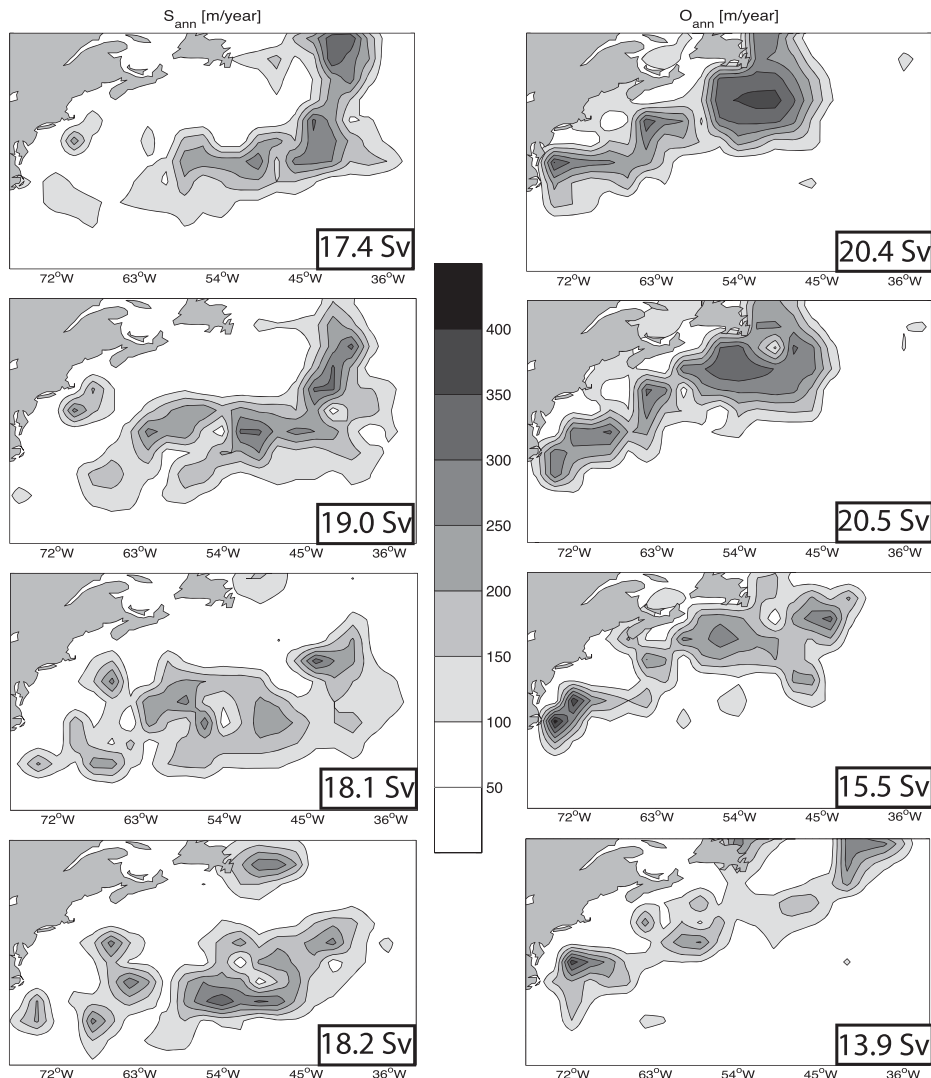


FIG. 7. The mean annual (left) subduction (m yr^{-1}) and (right) obduction (m yr^{-1}) rates for (top) 2002–03, (second from top) 2003–04, (second from bottom) 2004–05, and (bottom) 2005–06 using monthly varying MLDs and weekly varying currents with shear. The corresponding volume transport (Sv) is in the lower right-hand corner box of each plot.

period, but this is sensitive to the domain over which this is estimated. It should be noted that, if the $\Delta\rho_\theta > 0.125$ criterion were used, the MLDs in the northeast region would be deeper. There would be less subduction and more obduction. We have not tracked parcels north of 50°N where MLDs get deep enough to also yield more obduction that may account for the remaining discrepancy.

Last, we cannot estimate how long water parcels reside in the thermocline after subduction because we only have four years of data that can be tracked. However, we see that some of the parcels that reside in the thermocline for more than a year can be identified as EDW. Table 4 shows that there is a net subduction of EDW greater than zero for each of the four years, while the

EDW volume stays relatively stable. The EDW is not likely to travel north of 50°N and obduct. Any EDW that travels south of 25°N is unlikely to obduct. We may hypothesize that, by properly taking mixing into account, this EDW ventilation imbalance will diminish, but we cannot test this with our model.

b. Density class analysis

The volume transport rates in our domain for σ_θ up to 28.2 kg m^{-3} are in Table 4. The subducted waters come from regions south and southeast of the Gulf Stream, while obducted waters come from regions along the Gulf Stream and closer to shore. Most of the subducted waters are slightly less dense than the obducted waters

TABLE 3. Spatial mean and std dev of subduction and obduction and rms error: “analytical” [$(\Delta h)/\tau$ from Qiu and Huang (1995)] vs modeled (—indicates insufficient data).

Estimation	Year				
	2002	2003	2004	2005	2006
Analytical rms error (m yr ⁻¹)	74.7	57.6	41.2	48.3	44.1
Analytical subduction mean (m yr ⁻¹)	45.1	58.7	67.3	46.5	46.3
Analytical subduction std dev (m yr ⁻¹)	50.5	68.5	69.8	40.2	51.6
Modeled subduction mean (m yr ⁻¹)	51.8	57.5	52.8	54.9	—
Modeled subduction std dev (m yr ⁻¹)	59.3	57.5	49.9	56.3	—
Modeled obduction mean (m yr ⁻¹)	64.6	63.9	49.6	47.1	—
Modeled obduction std dev (m yr ⁻¹)	79.9	73.2	57.9	53.9	—

(Fig. 10), consistent with Qiu and Huang (1995). There is generally a right-skewed Gaussian-shaped distribution of ventilation as a function of density and a secondary peak in subduction for some years. The reason for this double peak in subduction is the local subduction minimum in the 27.2 σ_θ density class (seen barely in 2002, in 2003, and markedly in 2005), consistent with observations that there is a local minimum in oxygen concentration there (Richards and Redfield 1955). It is unclear why this local minimum disappears in 2004.

Waters denser than EDW, of 27.0 kg m⁻³ and above, are subducted where the ageostrophic currents are relatively large on an annual average as well as on a winter average, accelerating the subduction process into the Gulf Stream. Waters less dense than 22.2 kg m⁻³ subduct in areas where MLDs are relatively shallow and Ekman pumping is relatively strong, in particular, along the coast (Fig. 11). While subduction of the lightest waters in the slope region is consistent with the model of Speer and Tziperman (1992), this is the region where the errors in the Argo data are largest. The presence of water below 22.2 kg m⁻³ may be an exaggeration of how light this water is. Last, and also consistent with the model of Speer and Tziperman (1992), we found that Ekman pumping makes a negligible contribution to ventilation where EDW subducts (Fig. 12).

c. EDW formation characteristics

Applying the density-based criterion $\Delta\rho_\theta > 0.125$ makes no qualitative difference in our volume estimates. The *t* statistics for the differences between the mean EDW thicknesses using the density- and temperature-based criteria over 2002–06 where the EDW thicknesses

are not zero do not show that there is a significant difference in the mean EDW thicknesses to 95% confidence. Because there is no EDW along the coasts and north of 45°N, this is consistent with where the MLD estimates are significantly different.

The density range between 25.2 and 27.8 kg m⁻³ accounts for most of the ventilated waters, including newly subducted EDW in the Sargasso Sea. Most of the ventilated EDW lies within the range of σ_θ between 26.0 and 26.8 kg m⁻³, consistent with the EDW densities of McCartney and Talley (1982). In 2005, the distribution of densities of ventilated EDW is wider than in previous years with a longer tail toward lower densities. This may be explained by the subduction and obduction of a wider spatial distribution of water parcels formed over a range of years.

EDW is found to form just south of the Gulf Stream. This subtropical mode water follows the North Atlantic recirculation gyre toward the south and subducts toward the edges of the recirculation gyre with a higher population of water between 17° and 19°C during springtime (Fig. 12). We find that there is weaker stratification at the top of the thermocline where there is EDW, consistent with the definition of EDW. There is virtually no EDW north of the Sargasso Sea because older EDW mixes and is modified by heat fluxes after entering the Gulf Stream. The rest of the water between 17° and 19°C is of subpolar origin.

We find EDW subduction rates closer to that of Kwon and Riser (2004) than Speer and Tziperman (1992). The Walin (1982) formation equates the diapycnal volume flux with the difference between the air–sea buoyancy and diffusive fluxes over outcrop areas. Since the air–sea buoyancy fluxes are spatially and temporally variable, there may be substantial error in the air–sea buoyancy flux term, propagating error to the annual formation rate estimate of EDW. Additionally, neglecting meso-scale variability that contributes to diapycnal volume flux may propagate error to the annual formation rate estimate of EDW. According to our estimates, as well as those of Kwon and Riser (2004), the Walin (1982) formulation appears to equate wintertime formation with annual formation.

More water between 17° and 19°C is formed than subducts (Table 4). The differences between the EDW formation rates of Kwon and Riser (2004) and Speer and Tziperman (1992) also appear to be explained by the differences between the mean annual volume of water between 17° and 19°C and the EDW volume. Our EDW subduction and obduction rates are also tabulated in Table 4. An increase in net EDW subduction is consistent with an increase in net subduction of all waters. However, it is apparent that a net positive subduction rate of EDW does not necessarily indicate an increase in

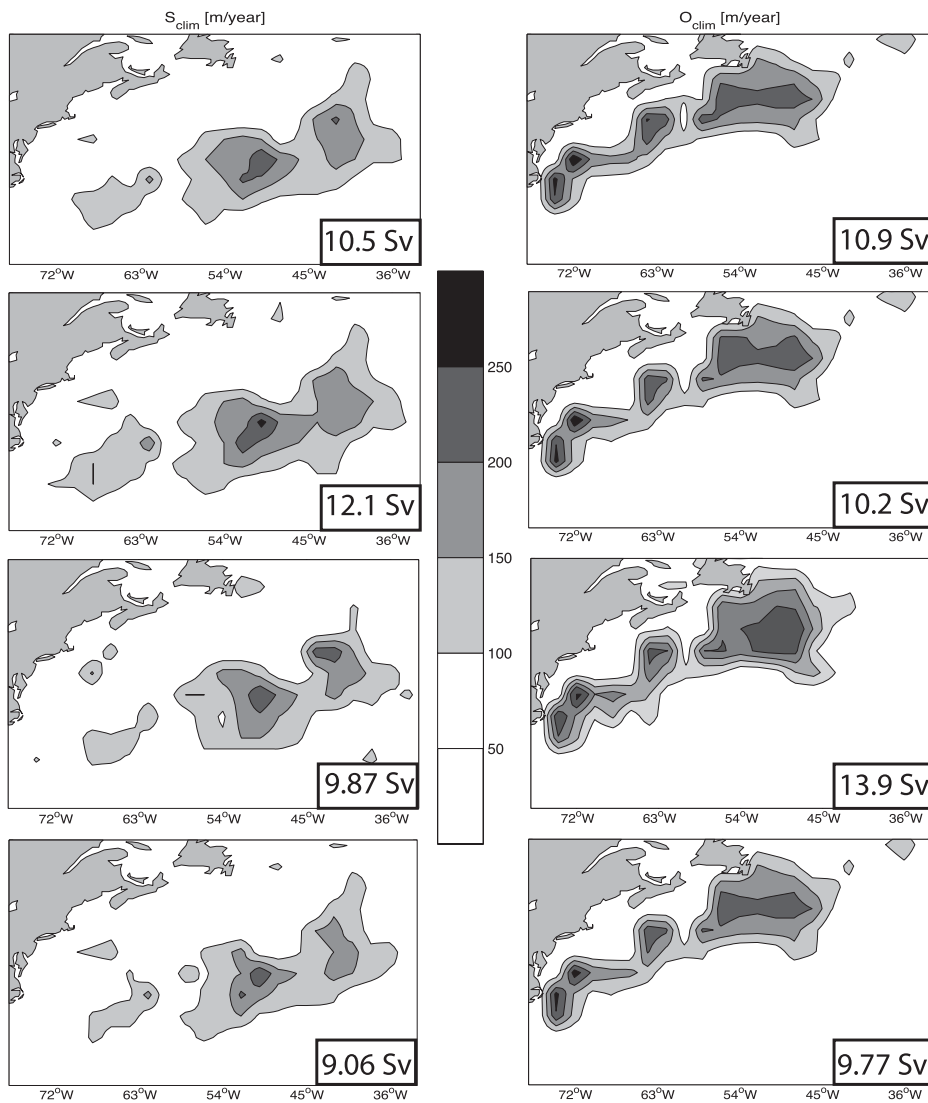


FIG. 8. The mean annual (left) subduction (m yr^{-1}) and (right) obduction (m yr^{-1}) rates using 1999–2006 average currents and a climatology of 2002–05 average monthly varying MLDs. (top) Using the $\Delta T > 0.3$ criterion with vertical current shear. (second from top) Using the $\Delta T > 0.3$ criterion with no vertical current shear. (second from bottom) Using the $\Delta \rho_\theta > 0.125$ criterion with vertical current shear. (bottom) Using the $\Delta T > 0.3$ criterion and vertical shear, but no Ekman currents ($w_e = 0$). The corresponding volume transport in and out of the permanent thermocline (Sv) is in the lower right-hand corner box of each plot.

EDW volume and, similarly, a net positive obduction rate of EDW does not dictate a decrease in EDW volume. The missing EDW from this picture is 1) EDW that is modified in the main thermocline, 2) newly formed EDW in the mixed layer that has not obducted, and 3) EDW that is not modified after obducting.

The spatial distribution of the errors in EDW volume is the same as that of the MLDs because the same objective mapping procedure was applied to the EDW thicknesses themselves. By estimating the thickness of water between 17° and 19°C via objectively mapped

Argo temperatures, we find similar volumes to the thickness objective mapping procedure until 2005 and 2006 when the thickness objective mapping procedure yields EDW volumes that are twice as large. The spatial distributions of EDW and seasonal variability of its volume between the order of $1 \times 10^{14} \text{ m}^3$ and about $10 \times 10^{14} \text{ m}^3$ are robust. Using a combination of objectively mapped temperatures from both Argo and WOA data yields EDW volumes with a depressed seasonal variation.

Subducted waters are about the same density on average as obducted waters. However, while obducted

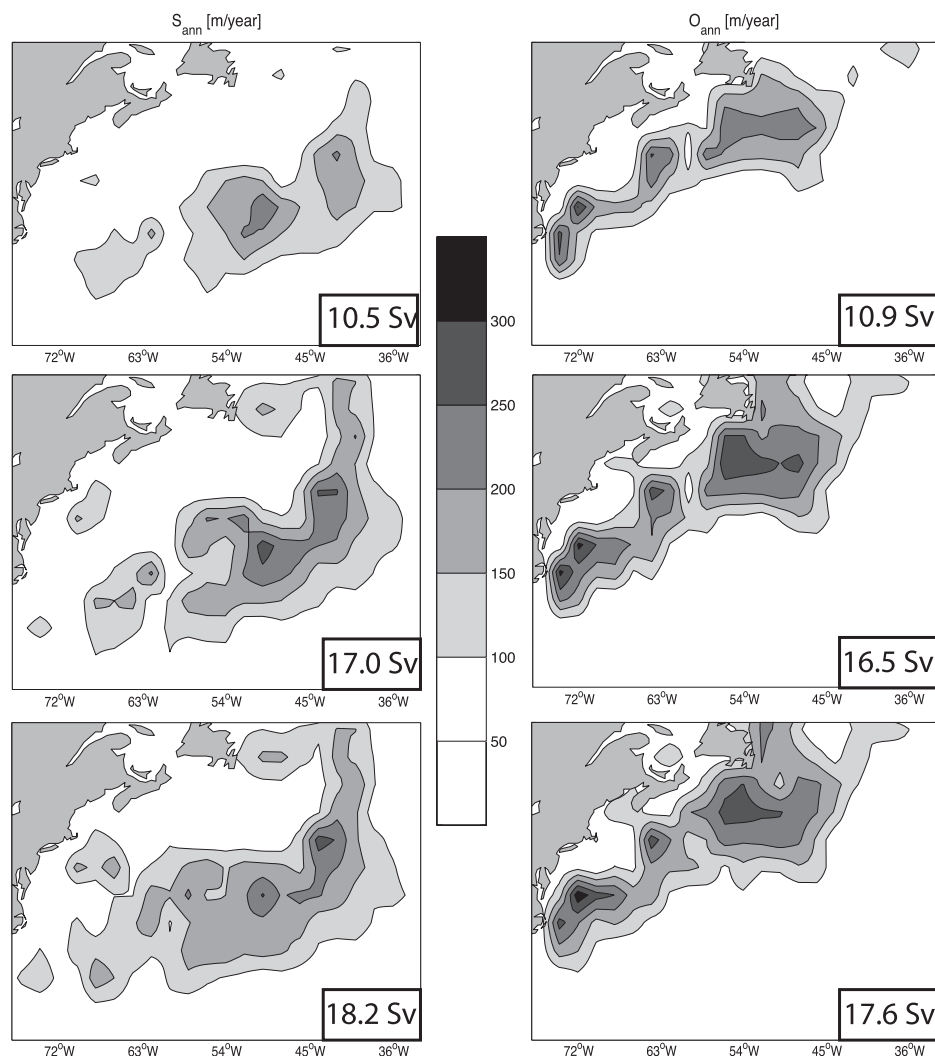


FIG. 9. The mean (left) subduction (m yr^{-1}) and (right) obduction (m yr^{-1}) rates over 2002–06 using (top) mean velocities and mean MLDs, (middle) mean velocities and estimated monthly MLDs, and (bottom) estimated weekly velocities and estimated monthly MLDs, all with shear. The corresponding volume transport in and out of the permanent thermocline (Sv) is in the lower right-hand corner box of each plot.

water may generally be colder than subducted water, obducted waters are generally warmer relative to their surroundings than subducted waters. Similarly, since EDW is cold relative to surrounding waters in the mixed layer, we reason that the mixed layer should be coldest in 2002, warm due to an increase in net subduction of EDW from 2002 to 2004, and cool due to a decrease in net subduction of EDW in 2005. Consistent with this, the winter (January through March average) mixed layer (potential) temperature means and standard deviations respectively for our domain are 16.3° and 4.76°C (4023 data points) for 2002, 16.7° and 4.82°C (4134 data points) for 2003, 16.8° and 5.02°C (4010 data points) for 2004,

17.6° and 4.77°C (3885 data points) for 2005, and 17.0° and 5.02°C (3987 data points) for 2006.

4. Conclusions

The wind stresses are significantly dependent upon the relative velocities between the winds and the geostrophic currents. This affects the Ekman divergences by about 10%–20% as well as the Sverdrup transports. Even so, vertical pumping is increasingly negligible relative to lateral induction farther away from the coast. Including a small vertical shear can make about a 10% difference in ventilation rates, albeit within our error.

TABLE 4. Ventilation and EDW rates and EDW volume from 2002 to 2006 (—indicates insufficient data).

Mean annual quantity	Year				
	2002	2003	2004	2005	2006
17°–19°C water volume (10^{14} m^3)	4.44	8.78	10.3	13.5	12.8
17°–19°C water subduction (Sv)	3.72	8.17	6.10	6.20	—
17°–19°C water obduction (Sv)	1.30	1.70	2.03	2.30	—
EDW volume (10^{14} m^3)	1.58	1.63	1.69	1.35	1.89
EDW subduction (Sv)	0.80	1.86	1.74	3.65	—
EDW obduction (Sv)	0.43	0.72	1.00	1.12	—
Subduction (Sv)	17.4	19.0	18.1	18.2	—
Obduction (Sv)	20.4	20.5	15.5	13.9	—

Because the ratio of Δh_m to τ is not constant, “analytical” estimates of ventilation rates may be inaccurate. The use of climatologies in our Lagrangian trajectory method may underestimate ventilation by similar reasoning. Nevertheless, the length of our time series may also explain the underestimates of ventilation. By how much averaging MLDs matters remains uncertain but appears to be marginally larger than our error: at most 50%. Winter mixed layer temperatures change in concert

with net (EDW) ventilation rates, which suggests that mixed layer temperature variability may be closely linked to ventilation rate variability. However, the criterion by which we calculate the MLDs may affect obduction estimates because of a difference in h_m where salinity profiles have significantly different gradients than elsewhere.

Subducted waters tend to come from south of the Gulf Stream, while obducted waters come from along the Gulf Stream. The lightest waters subduct at the coast where vertical velocities can dominate MLD variability. Water heavier than EDW is aided by the ageostrophic currents in subduction more than other waters. Both lateral induction and vertical pumping are nonnegligible in the Gulf Stream. MLD variability matters most for ventilation elsewhere.

WOA data have been used in related studies (e.g., Kwon 2003, see chapter 5; Joyce et al. 2000), but owing to the relatively unpronounced seasonal variability in WOA data the temporal resolution made possible by the Argo system allows us to investigate variability at higher frequencies. The importance of intra-annual variations in MLDs, for instance, must be emphasized in estimating EDW ventilation rates. These rates are greatest during the springtime when the mixed layer shoals. Some of the subducted EDW resides in the thermocline for a period

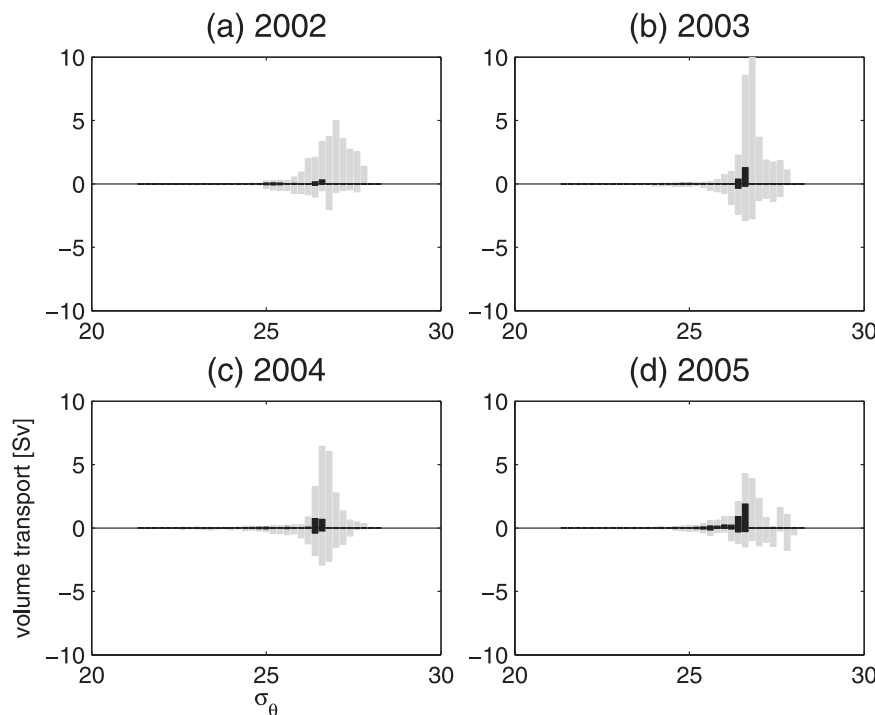


FIG. 10. Volume transports (Sv) categorized into subduction (gray positive bars), EDW subduction (black positive bars), obduction (gray negative bars), and EDW obduction (black negative bars), broken into intervals of 0.2 kg m^{-3} for (a) 2002, (b) 2003, (c) 2004, and (d) 2005.

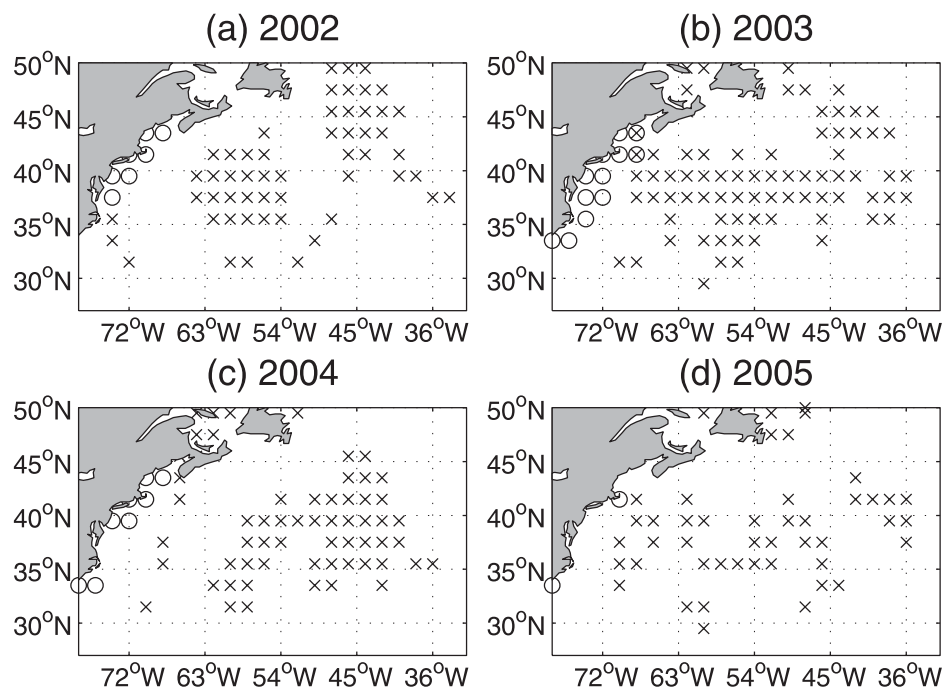


FIG. 11. Locations of where at anomalously large subduction rates of heavy water, 27.0 kg m^{-3} and above, (x) and light water, 22.0 kg m^{-3} and below, (o) occurs for (a) 2002, (b) 2003, (c) 2004, and (d) 2005.

longer than the length of our time series: at least 3.5 yr. This warrants future investigations because the EDW residence time may play a crucial role in the heat budget and a potential NAO–EDW coupled oscillation.

Acknowledgments. The authors would like to acknowledge Suzanne Dickinson for preparing the QuikSCAT gridded wind speed field and Li Ren for some of the Argo data.

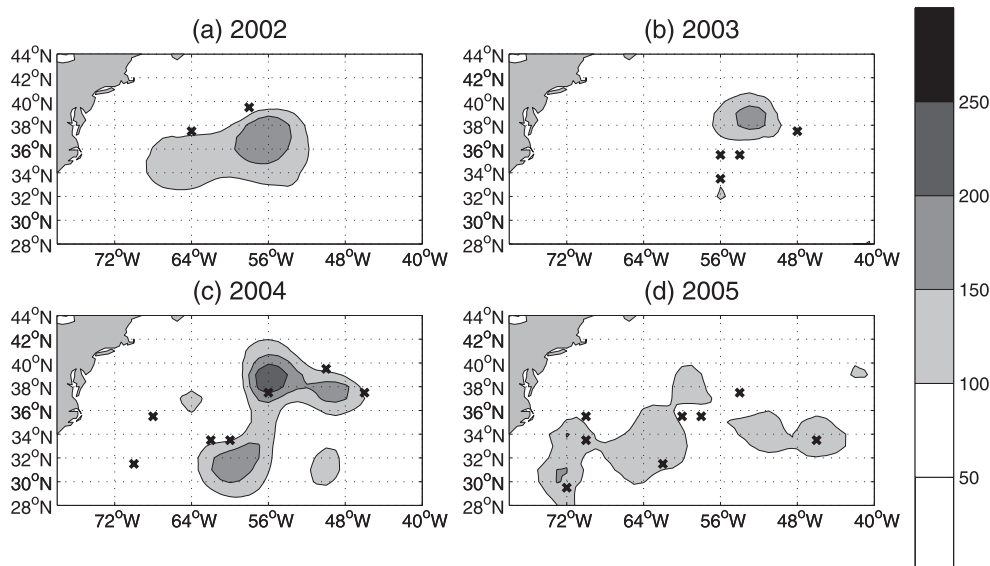


FIG. 12. Thickness of waters between 17° and 19°C (in m) in springtime (April–June) with locations where greater than average subduction rates of EDW occur in our domain (x) in (a) 2002, (b) 2003, (c) 2004, and (d) 2005.

APPENDIX

Derivation of Ekman Current Equations

Starting from the momentum-balance equations for an incompressible fluid and neglecting viscosity, we have the set of coupled partial differential equations (PDEs),

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} - f v = \frac{1}{\rho} \frac{\partial \tau^x}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial x}, \quad (\text{A1})$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + f u = \frac{1}{\rho} \frac{\partial \tau^y}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial y}, \quad (\text{A2})$$

where the zonal, meridional, and vertical currents are $u = u_e + u_g$, $v = v_e + v_g$, and $w = w_e$, respectively, the sums of their Ekman and geostrophic contributions. Ageostrophic currents, which include Ekman currents, are defined to be the difference between the total currents and the geostrophic currents. Here, non-Ekman ageostrophic currents and vertical geostrophic currents

are ignored. Furthermore, because we have defined the geostrophic current in terms of the gradient of the total pressure field, the pressure gradients cancel with the geostrophic currents without derivatives (Holton 2004).

Now we apply the following assumptions: (i) The geostrophic currents are vertically uniform in the Ekman layer. This is reasonable if we assume that the Ekman layer depth is finer than our vertical resolution. (ii) The geostrophic currents obey quasigeostrophic dynamics. That is, $\partial u_g / \partial t + u_g \partial u_g / \partial x + v_g \partial u_g / \partial y - f v_a = 0$ and $\partial v_g / \partial t + u_g \partial v_g / \partial x + v_g \partial v_g / \partial y + f u_a = 0$ to first order. Here, “a” denotes “ageostrophic” but non-Ekman. This essentially assumes that Rossby number, $\text{Ro} = U/fL$, is much less than unity. The ageostrophic current terms are of order Ro compared to the geostrophic velocities. Also, the vertical velocity w is of the order of Ro compared to aspect ratio scaling. Thus, by vertically integrating over the Ekman layer with depth δ_e we get the set of coupled PDEs:

$$\begin{aligned} & \frac{\partial (U_e + \delta_e u_g)}{\partial t} + \int_{-\delta_e}^0 u_e \frac{\partial u_e}{\partial x} dz + U_e \frac{\partial u_g}{\partial x} + u_g \frac{\partial U_e}{\partial x} + \delta_e u_g \frac{\partial u_g}{\partial x} + \int_{-\delta_e}^0 v_e \frac{\partial u_e}{\partial y} dz + V_e \frac{\partial u_g}{\partial y} + v_g \frac{\partial U_e}{\partial y} + \delta_e v_g \frac{\partial u_g}{\partial y} \\ & + \int_{-\delta_e}^0 w_e \frac{\partial u_e}{\partial z} dz - f V_e - f V_a = \frac{\tau^x}{\rho}; \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} & \frac{\partial (V_e + \delta_e v_g)}{\partial t} + \int_{-\delta_e}^0 u_e \frac{\partial v_e}{\partial x} dz + U_e \frac{\partial v_g}{\partial x} + u_g \frac{\partial V_e}{\partial x} + \delta_e u_g \frac{\partial v_g}{\partial x} + \int_{-\delta_e}^0 v_e \frac{\partial v_e}{\partial y} dz + V_e \frac{\partial v_g}{\partial y} + v_g \frac{\partial V_e}{\partial y} + \delta_e v_g \frac{\partial v_g}{\partial y} \\ & + \int_{-\delta_e}^0 w_e \frac{\partial v_e}{\partial z} dz - f U_e + f U_a = \frac{\tau^y}{\rho}. \end{aligned} \quad (\text{A4})$$

The integrated zonal Ekman currents are U_e and the integrated meridional Ekman currents are V_e . The Ekman currents are so smooth that, in fact, all first-order derivatives of them turn out to be negligible. This may also be seen by a scaling argument. Thus, with one further assumption, that (iii) terms to second order in u_e and v_e (e.g. $\int_{-\delta_e}^0 dz u_e \partial u_e / \partial x$) are negligible, we find

$$U_e \frac{\partial u_g}{\partial x} + u_g \frac{\partial U_e}{\partial x} + V_e \frac{\partial u_g}{\partial y} + v_g \frac{\partial U_e}{\partial y} - f V_e = \frac{\tau^x}{\rho}; \quad (\text{A5})$$

$$U_e \frac{\partial v_g}{\partial x} + u_g \frac{\partial V_e}{\partial x} + V_e \frac{\partial v_g}{\partial y} + v_g \frac{\partial V_e}{\partial y} + f U_e = \frac{\tau^y}{\rho}. \quad (\text{A6})$$

After introducing τ_{damp} to iteratively find a numerical solution, we get Eqs. (7) and (8).

We may, alternatively, solve for the Ekman transports, U_e and V_e , explicitly if we assume (i), (ii), (iii), and make further assumptions about the strengths of the Ekman and geostrophic currents and their derivatives in space with respect to one another: If (iv) $(\partial u_e / \partial x) / (\partial u_g / \partial x) \ll 1$, (v) $u_g / u_e \ll 1$ (similarly for derivatives in y and for the meridional currents), and (vi) products of derivatives in any currents are negligible, then we get

$$U_e \approx \frac{1}{\rho} \left\{ \frac{\tau^x \partial v_g / \partial y}{[f(f + \partial v_g / \partial x - \partial u_g / \partial y)]} + \frac{\tau^y}{(f + \partial v_g / \partial x)} \right\}; \quad (\text{A7})$$

$$V_e \approx \frac{1}{\rho} \left\{ \frac{\tau^y \partial u_g / \partial x}{[f(f + \partial v_g / \partial x - \partial u_g / \partial y)]} - \frac{\tau^x}{(f - \partial u_g / \partial y)} \right\}. \quad (\text{A8})$$

From these analytical results, we can find $w_e = \partial U_e / \partial x + \partial V_e / \partial y$ to get

$$w_e \approx \frac{1}{\rho} \frac{1}{\left(f + \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}\right)} \left\langle \frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y} - \frac{\beta}{f} \tau^x - \frac{\beta}{\left(f + \frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y}\right)} \left\{ (\tau^y - \tau^x) \left(\frac{\partial v_g}{\partial x} - \frac{\partial u_g}{\partial y} \right) + 2 \left[f(\tau^y - \tau^x) - \tau^x \left(\frac{\partial v_g}{\partial x} - \frac{\partial v_g}{\partial y} \right) + \tau^y \left(\frac{\partial u_g}{\partial x} - \frac{\partial u_g}{\partial y} \right) \right] \right\} \right\rangle. \quad (\text{A9})$$

It is assumption (v) that fails almost everywhere, but this is our first guess in numerically solving for the Ekman currents. Note that a negative value for w_e represents water transport from the Ekman layer down to the mixed layer because of how we have defined this quantity. A positive Ekman suction is a positive w_e and a positive Ekman pumping is a negative w_e .

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