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Quantitative estimate of the Milankovitch-forced contribution to observed Quaternary climate change

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Abstract

A number of records commonly described as showing control of climate change by Milankovitch insolation forcing are re-examined. The fraction of the record variance attributable to orbital changes never exceeds 20%. In no case, including a tuned core, do these forcing bands explain the overall behavior of the records. At zero order, all records are consistent with stochastic models of varying complexity with a small superimposed Milankovitch response, mainly in the obliquity band. Evidence cited to support the hypothesis that the 100 Ka glacial/interglacial cycles are controlled by the quasi-periodic insolation forcing is likely indistinguishable from chance, given the small sample size and near-integer ratios of 100 Ka to the precessional periods. At the least, the stochastic background "noise" is likely to be of importance.

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1. Introduction

The so-called Milankovitch hypothesis, that much of inferred past climate change is a response to near-periodic variations in the earth's position and orientation relative to the sun, has attracted a great deal of attention. Numerous textbooks (e.g., Bradley, 1999; Wilson et al., 2000; Ruddiman, 2001) of varying levels and sophistication all tell the reader that the insolation changes are a major element controlling climate on time scales beyond about 10,000 years. A recent paper begins "It is widely accepted that climate variability on time scales of 10^3 to 10^5 years is driven primarily by orbital, or so-called Milankovitch, forcing." (McDermott et al., 2001).

To a large extent, embrace of the Milankovitch hypothesis can be traced to the pioneering work of Hays et al. (1976), who showed, convincingly, that the expected astronomical periods were visible in deep-sea core records. Since that time, a torrent of papers has analyzed a huge variety of records seeking the Milankovitch frequencies, usually found them, and interpreted the records as implying control of climate by the corresponding insolation changes. In parallel with this description of the climate record, a theoretical and modelling literature has emerged (Saltzman, 2002) rationalizing the response of climate to the insolation driving.

In trying to quantify the degree of relationship between the orbital frequencies and climate change, two separate lines of evidence must be examined. One concerns climate variability in the broad band of frequencies lying above about one cycle/100 ka and below about one cycle/20 ka—where the major orbital perturbations lie. Is the variance there dominated by the astronomical forcing? The other line of evidence concerns the very clear energy excess at and about one cycle/100 ka, where the orbital forcing is extremely small, but where various hypotheses have been proposed whereby non-linear interactions in the climate system can rectify the higher frequency forcing into a very large lower frequency response. These two problems are related: if the 1/100–1/20 ka band is dominated by signals arising from the Milankovitch forcing, then there is a prima facie case for suspecting their non-linear interactions may play a major role. If, on the other hand, and as we will find, the $\frac{1}{100}\ \frac{1}{20}$ ka band is dominated by a continuum, the Milankovitch rectifier hypothesis is much less compelling. (The argument that there is a true, linear, resonant response to the small eccentricity forcing at 100 ka is set aside here, as being an extreme hypothesis.)

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A terminology for climate variability in the band of periods from about 10,000–100,000 years is needed. Following the constructs, "millennial", "decadal", etc., "myriennial" and "myriadic" are proposed, from the prefix "myria" meaning 10,000 (it is not an accepted SI prefix).

Over the years, beginning with some of the earliest papers on the subject (Mitchell, 1976; Imbrie and Imbrie, 1980; Winograd et al., 1992; Karner and Muller, 2000, and many others) doubts on various grounds have been expressed about the validity of the conclusions concerning climate control by Milankovitch driving. But these doubts and caveats have been brushed aside in much of the study of climate change. Indeed, the Devils Hole data (Winograd et al., 1992), suggesting a fundamental inconsistency between theory and observations, elicited a plaintive reaction (Broecker, 1992), and attempts to argue it away as being a purely local response (e.g., Herbert et al., 2001).

In the last 800 ka, the greatest inferred climate changes have been the glacial–interglacial shifts on roughly 100 ka time scales. These changes are so massive, and the Milankovitch forcing so slight (10% spatial redistributions of annual global insolation at periods much shorter than 100 ka), that a number of interesting hypotheses have been proposed to rationalize the observed shifts.

One can divide the problem into two parts, finding: (1) evidence that the orbitally controlled insolation changes drive the major climate shifts and, (2) the mechanisms by which that driving occurs. Problem (2) arises only if (1) exists. At least four identifiable "Milankovitch hypotheses", including Milankovitch's own, exist:

1. Northern hemisphere high latitude solar insolation controls climate change (a form of the original hypothesis).
2. Obliquity and precessional band energy is discernible in spectra of climate proxies.
3. Obliquity and precessional band energy *dominate* climate variability between about 18,000 yr and 42,000 yr periods.
4. Obliquity and/or precessional band energy, irrespective of (3) control, or "pace" the 100,000 yr interval characteristic of the glacial–interglacial shifts of the Pleistocene.

Item 4 is a specific version of item 1.

In a previous paper (Wunsch, 2003), it was shown that much of the observed variability was difficult to distinguish from comparatively simple random walk phenomena. Here, we extend those results by seeking a quantitative measure of the fraction of climate change for which orbital insolation changes clearly predominate. The measure used is a simple one: to find the fraction of the energy in the records ascribable to the direct, linear, response to orbital insolation forcing, and only then turn to the 100 ka-band energy. (The underlying mathematical structure is the Parseval/Rayleigh theorem expressing the record variance as the sum of its Fourier components; see Wunsch, 2000, Appendix.)

A major difficulty faced by anyone attempting to use cores to understand climate change is the need for an age model to convert from depth coordinates to time. If the underlying true record is dominated by the orbital frequency bands, errors in the age model can displace energy from those bands, thus reducing the apparent Milankovitch energies. Then, up to further problems of the unknown relative phase and of event identification, one can adjust the times of fluctuations to coincide with the astronomical forcing, and thus "tune" the core. Conversely however, tuning can take energy that properly belongs in the non-orbital bands, and improperly place it there (e.g., Neeman, 1993; Huybers and Wunsch, 2004). We will show however, that even in tuned records, the fraction of the variance in the myriadic band derived from the insolation forcing is so small that the inference it controls the overall record is not an obvious one. This point of view is an old one (e.g., Kominz and Pisias, 1979; Imbrie and Imbrie, 1980), but it has been not much heeded.

One is led to ask "what would be the nature of climate variability on a hypothetical earth with fixed obliquity in a circular orbit about a sun with unchanging output?" Given the chaotic/stochastic nature of weather and climate over the duration of the instrumental record—in a system driven by periodic forcing at diurnal and annual time scales—it is reasonable to propose that this hypothetical earth would similarly display a rich variability in climate, even in the absence of astronomical variations. Higher frequency phenomena, including weather, ENSO fluctuations, the Arctic Oscillation, etc., as well as much lower frequency, as yet undetected, atmospheric, oceanic, and cryospheric fluctuations would be expected to generate a much lower frequency variability even in the absence of orbital changes.

Milankovitch forcing and the general hypothesis of control are described in numerous papers and textbooks, including those already listed, and we will not further describe the details here. To limit the otherwise overly broad scope of this investigation, we focus primarily on the Quaternary, and within that period, on the Pleistocene.

2. Example records

2.1. Vostok deuterium and deuterium excess

Vostok core deuterium, $\delta D(t)$, data are a prime example of the sort of data used to depict the glacial/

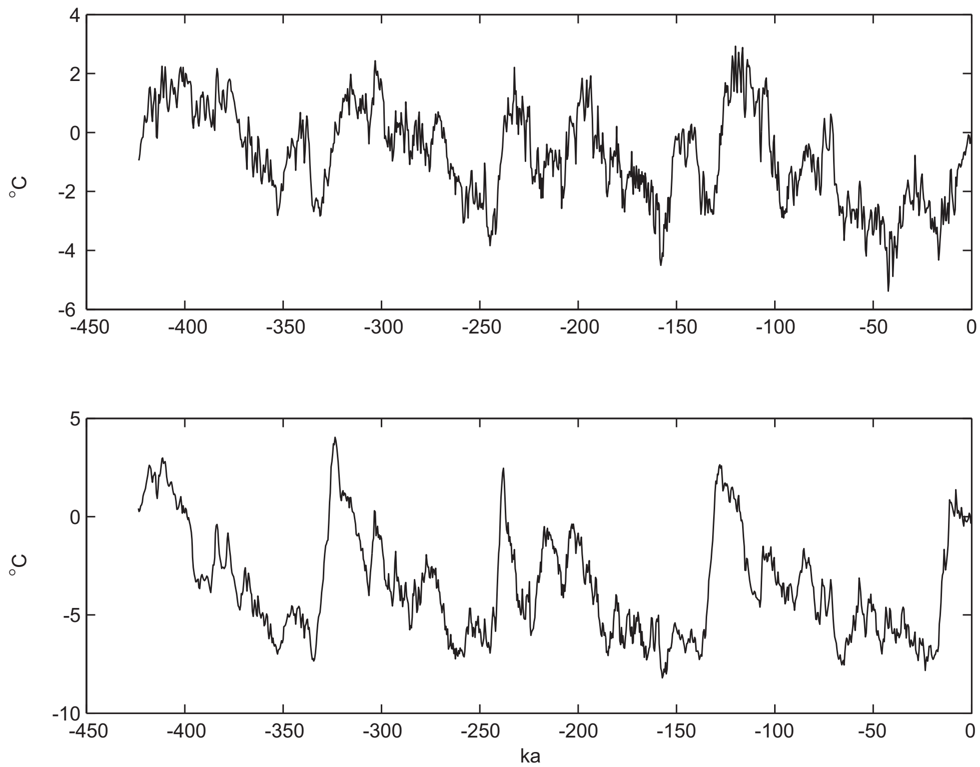

Fig. 1. $T_d(t)$ (upper) and $T_D(t)$ (lower) at the Vostok site as a function of time. Note time runs from left to right in this and all other figures. Interpolation and conversion from $\delta D(t)$ and $d(t)$ to temperatures was done by Vimeux et al. (2002).

interglacial behavior of climate on roughly 100 ka timescales. Fig. 1 shows the equivalent inferred local temperature, $T_D(t)$ as interpolated and computed by Vimeux et al. (2002). Vimeux et al. (2001) among others, claim that this record and the associated ‘‘excess deuterium’’, $d(t) = \delta D(t) - 8\ \delta^{18}O$, also shown in temperature terms, $T_d(t)$, in Fig. 1, support the inference that the 41 ka energy in the Milankovitch insolation forcing controls low-frequency climate change. (The terminology ‘‘site temperature’’, $T_D(t)$, and ‘‘source temperature’’, $T_d(t)$, will be used.) Analysis of the raw $\delta D(t)$, $d(t)$, makes only slight changes (Huybers, personal communication, 2003) in the inferences we draw. A power density spectral estimate of $T_D(t)$ is shown in Fig. 2, and one for $T_d(t)$ is in Fig. 3. The inference that obliquity dominates is derived from plotting figures like Fig. 2, in linear form, as shown in Fig. 4. The linear scale, common in paleoclimate studies, has the effect of exaggerating the importance of the peak at the obliquity period. The log–log form, to the contrary, suggests that the basic behavior might instead be a low-order autoregressive (AR, or equivalent) process, with perhaps a weak superimposed obliquity response. In Fig. 2 about 40% of the record variance lies in periods longer than about 92 ka, that is including the 100 ka energy, albeit there is no obvious ‘‘peak’’, about 3% lies between 40 and 45 ka periods, and about 4% between 18 and 22 ka. These obliquity and precession

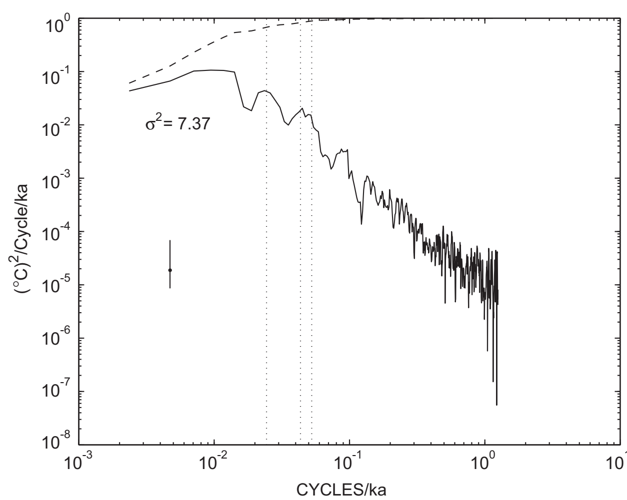

Fig. 2. Normalized (to unit variance) power density spectral estimate of $T_D(t)$ in the Vostok core. Obliquity and two precessional frequencies are marked by vertical lines. Dashed line is the cumulative power (asymptoting to 1) as a function of frequency. Here and in other spectral density plots, σ^2 is the record variance in dimensional units permitting conversion to an unnormalized power density should one wish that. All power density spectral estimates displayed here were computed using the multitaper method (e.g., Percival and Walden, 1993) and an approximate 95% confidence interval is shown.

band values include any contribution present from the background continuum as well as from direct insolation driving. Thus the Milankovitch bands carry less than

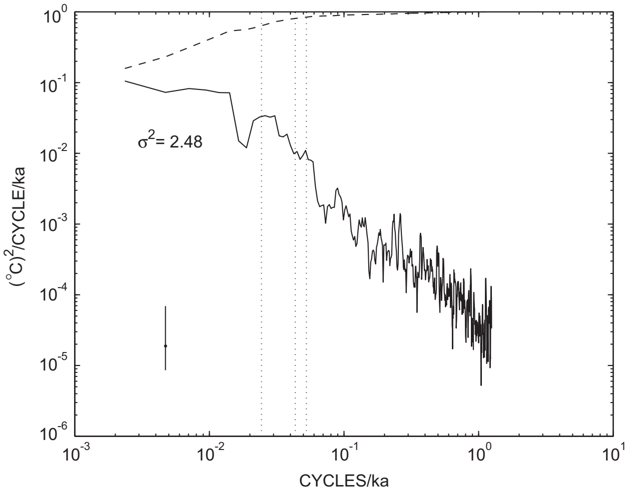

Fig. 3. As for Fig. 2, except for the source region, $T_d(t)$.

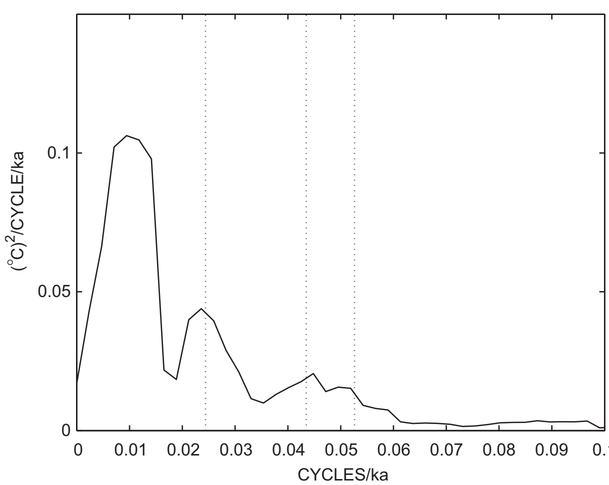

Fig. 4. Power density spectral estimate for $T_D(t)$ of Fig. 2 shown on a linear-linear scale. Vertical lines are again the 41 Kyr, and 21, 19 Kyr Milankovitch periods. Linear scales exaggerate the importance of the obliquity energy because estimates having energy levels less than 1% of the maximum energy plotted become invisible to the eye and are commonly suppressed. But there are a very large number of such estimates, commonly more than enough to outweigh the visible peak. On this linear scale, the confidence interval is proportional to the variance of the spectral element at each frequency and is not shown (it is identical to that in Fig. 2). Note that the frequency scale has been cut off at 0.1 cycles/ka because the curve becomes invisible beyond that frequency (compare Fig. 2 at high frequencies). The largest values here are in the 100 ka band.

about 7% of the record total variance, or about $0.07/0.6 \approx 12\%$ of the energy in the myriadic band; the remainder is indistinguishable from the background continuum. The equivalent numbers for $T_d(t)$ are about 35% of the variance in periods longer than about 92 ka, 5% in the obliquity band, and 5% in the precessional band, with the latter two accounting for about 15% of the myriadic energy. That the energy in the weak Milankovitch peaks actually controls the much more energetic 100 ka band is not the most immediately attractive hypothesis.

To explore the alternative hypothesis that the record is indistinguishable from stochastic, it is represented as an AR(N) process

$$T_D(t) = \sum_{n=1}^{N} a(n) T_D(t - n\Delta t) + \theta(t), \tag{1}$$

where the time step $\Delta t = 0.4$ ka. Here $\theta(t)$ is meant to be white noise, $\langle \theta(t) \rangle = 0$, $\langle \theta(t)\theta(t') \rangle = \sigma^2 \delta_{tt'}$, where the variance, σ^2, and the order N have to be determined from the data, and the hypothesis Eq. (1) tested after the fact. $\delta_{tt'}$ is the Kronecker delta, which vanishes for $t \neq t'$. Brackets $\langle \cdot \rangle$ denote ensemble averages.

Determination of $a(n)$, $\theta(t)$, is fundamentally a least-squares process (see Ljung, 1987), and determination of N is one of statistical inference. For $T_D(t)$, one finds, by the so-called Akaike's information theoretic criterion (AIC), that the best choice is $N = 2$ (although changes in N make little difference in skill). The curve fit explains about 97% of the total record variance. An AR(2) process is a minor extension of Hasselmann's (1976) model of climate change. Other representations are possible; one might prefer, for example, the equivalent moving average (MA) process

$$T_D(t) = \sum_{m=0}^{M} b(m)\theta(t - m\Delta t), \quad b(0) = 1$$

or a hybrid (ARMA). For more details, see Box et al. (1994).

Fig. 5 shows the comparison between the original record of $T_D(t)$, and the result obtained by using, $\tilde{a}(1) = 1.181$, $\tilde{a}(2) = -0.1984$ (the tildes are used to distinguish the estimates of parameters from the true values), and the estimate, $\tilde{\theta}(t)$, of $\theta(t)$ shown in Fig. 6. That the estimated $\theta(t)$ is indistinguishable from white noise is suggested by its autocovariance (not shown), which is itself indistinguishable from a delta function at $t = 0$. Equivalently, its power spectral density estimate is shown in Fig. 7, and is quite flat—apart from some decay at the very lowest frequencies. The very slight structure in the Milankovitch bands is all that remains of the hypothesis of dominance by obliquity. Some obliquity signal seems to be present, a result that is both interesting and useful for understanding the relationship between forcing and response. But its energy is so small as to make it of marginal significance in any description of the record as a whole. (The fitting procedure can be generalized to "color" the innovation so as to remove the remaining structure in its spectrum, as in the ARMA representation, but is not pursued here.) The histogram of $\tilde{\theta}(t)$ is displayed in Fig. 8, and deviations from normal are, visually, slight. It appears, to a good approximation, that $T_D(t)$ is well described as a simple stochastic

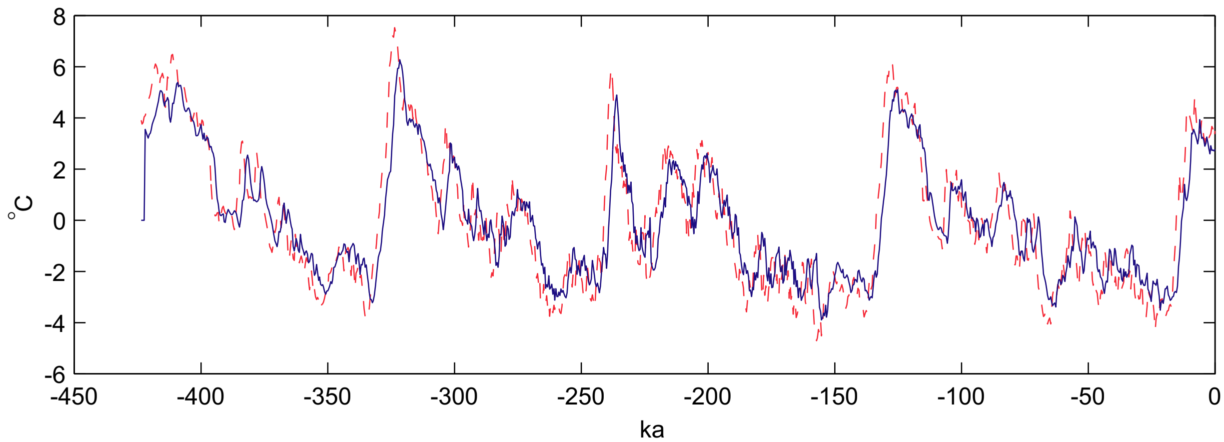

Fig. 5. Comparison of Vostok $T_D(t)$ (blue-solid) with the AR(2) autoregression fit (red-dashed), displayed as a 5-time-step prediction. Fit was only to the first half of the record shown. The curves are slightly displaced in time to render them more visible.

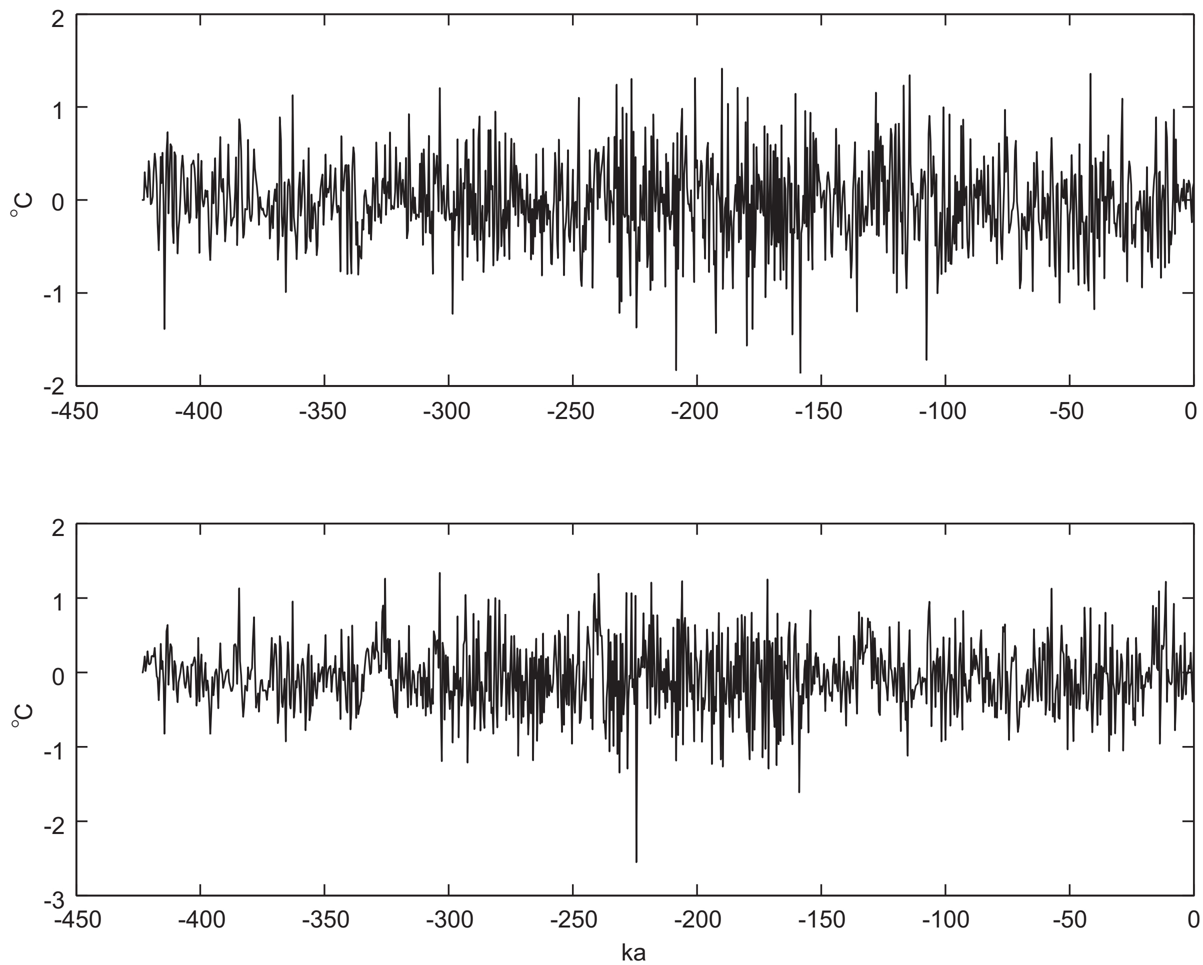

Fig. 6. Residuals, $\tilde{\theta}(t)$, of the AR fit to temperatures derived from $T_d(t)$ (upper), $T_D(t)$ (lower) at Vostok. A successful fit should produce a stationary white noise process.

process, with only a very slight orbital energy-band structure superimposed.

This is not however, quite the end of the story. $T_D(t)$ exhibits an asymmetry, with deglaciations being much more rapid than glaciations. Such behavior is not consistent with the simplest autoregressive process, and so something else is going on, too. If one examines the estimated innovation, $\tilde{\theta}(t)$, in the vicinity of the

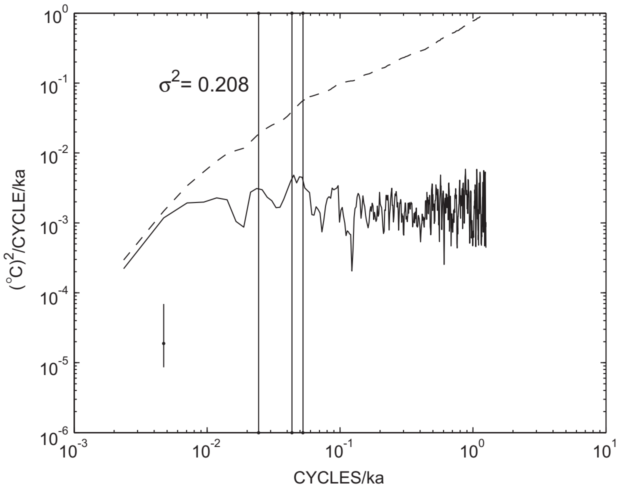

Fig. 7. Power density spectral estimate for $\theta(t)$ from AR fit to $\delta D(t)$. The result is nearly white, apart from a drop at the lowest frequencies, and a slight structure in the myriennial band.

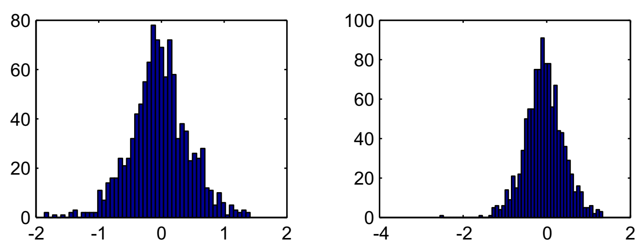

Fig. 8. Histograms of the white noise estimates for $T_d(t)$ (left panel) and $T_D(t)$ at Vostok. The results are roughly normal in character.

deglaciation, it is seen that they are all associated with a run of small positive values, but that are otherwise unremarkable (Fig. 9. A ''run'' consists of successive elements with the same sign, with a minimum length of one. Statistics of runs are described by Draper and Smith, 1981.)

The impulse responses of the AR(2) rules is shown in Fig. 10. $T_D(t)$ is associated with a very long decay time (O(40 ka)); an excess of positive values of $\tilde{\theta}(t)$ can accumulate over an extended period and thus produce a strong positive excursion.

That rapid negative excursions are not also seen can be traced back to the absence of similar negative runs in $\tilde{\theta}(t)$. Superficially at least, a tendency for positive runs of $\theta(t)$ ''explains'' the deglaciation. A physical interpretation might be that the small random disturbances driving the system have a greater probability of being positive when a deglaciation begins. Such behavior is not consistent with an ordinary Gaussian white noise process.

What are the innovations? It is first important to recognize that the simplicity of the AR(2) model does *not* imply a simple underlying physics. The model is merely a description that any complete theory needs to replicate; it does not itself constitute a theory. Intricate turbulent interactions produce simple power law physics (determining the $a(n)$), and phenomena such as the oceanic internal wave field, which also displays power law behavior, are sums of enormous numbers of stochastic propagating and strongly interacting waves. One can speculate that during a deglacial interval, these fluctuations, possibly deterministic in origin, might have preferred signs. Regulators (feedbacks) may exist to prevent the system from moving arbitrarily far to either an ice-covered earth or to very much warmer conditions. At least in these two Vostok records, the overall variance in the myriadic band is governed by a process indistinguishable from a stochastic one. A very small obliquity signal is superimposed. We have almost no information about the spatial structure of $T_D(t)$, and it is likely that the observed autoregressive behavior is a consequence of complicated space/time interactions.

2.2. ODP 677

Turning now to much longer records, consider ODP 677 from the Panama Basin extending to −3 ma, and described by Shackleton et al. (1990). It has not been tuned, was partially re-analyzed by Wunsch (2003), and so is not reproduced here. A slight obliquity peak appears, again superimposed upon a background red-noise continuum. Wunsch (2003) estimated it contains less than 11% of the record variance (and the slight structure in the precessional band contains even less). With a red spectrum, total record variance depends upon the record length, and so these fractions are not absolute statements. The tendency toward white noise beyond periods of order 100 ka however, renders the fractions quoted much more stable than if the spectrum were indefinitely red. About 15% of the record variance lies in the band of frequencies around 100 ka period and about 40% of the record variance is at periods longer than about 96 ka, so that the Milankovitch band energy is about 18% of the energy in the myriadic band—not negligible, but not dominant either. The timescale on Fig. 1 of Wunsch (2003) was inadvertently stretched to −300 Ka, as pointed out by L. Hinnov (pers. comm, 2004). The spectral density in Fig. 3 remains correct.

2.3. ODP 659

This core extending back to −5 ma, and thus beyond our focus on the Quaternary, was taken from the southeast North Atlantic, and was tuned by Tiedemann et al. (1994) to the orbital frequencies. The resulting spectrum (not shown here), clearly displays a sharp obliquity peak and weaker precessional peaks—as required by the tuning process. Despite the tuning, which drives energy into the Milankovitch frequencies,

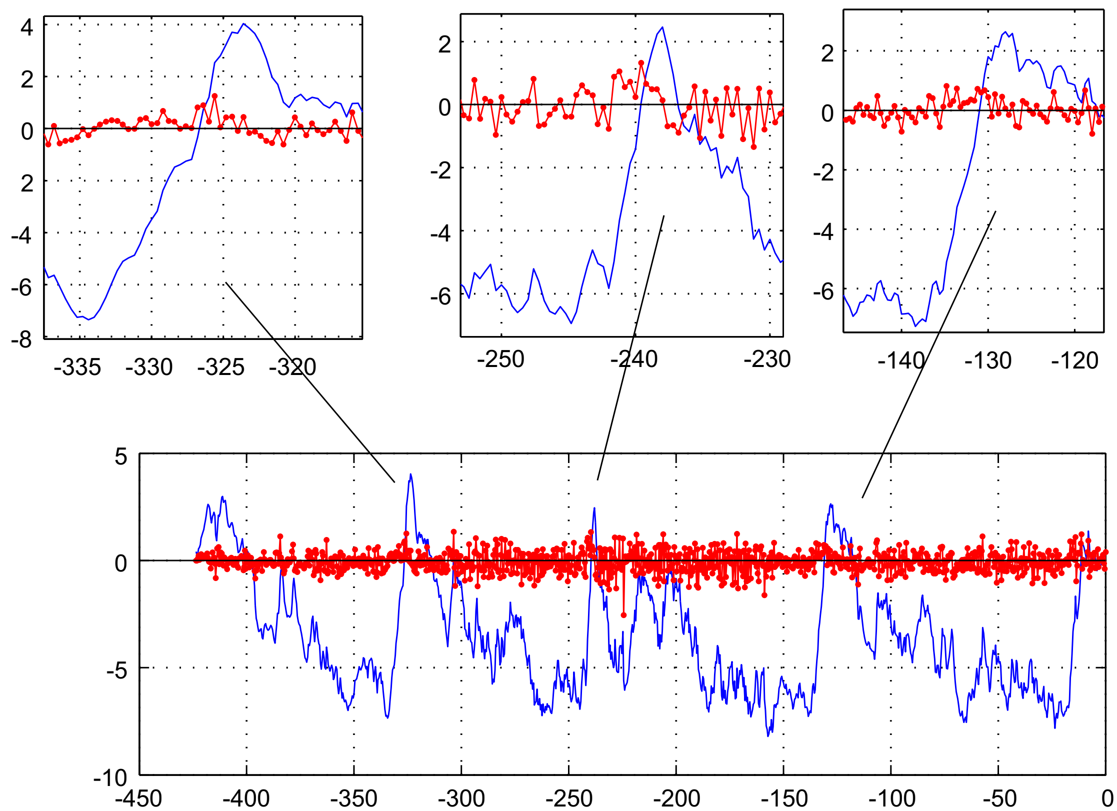

Fig. 9. $T_D(t)$ (solid, blue) and $\theta(t)$ (red, dots) and expanded plots in the vicinity of the deglaciations showing the positive runs in $\theta(t)$ when deglaciation occurs. Time is in kiloyears (ka).

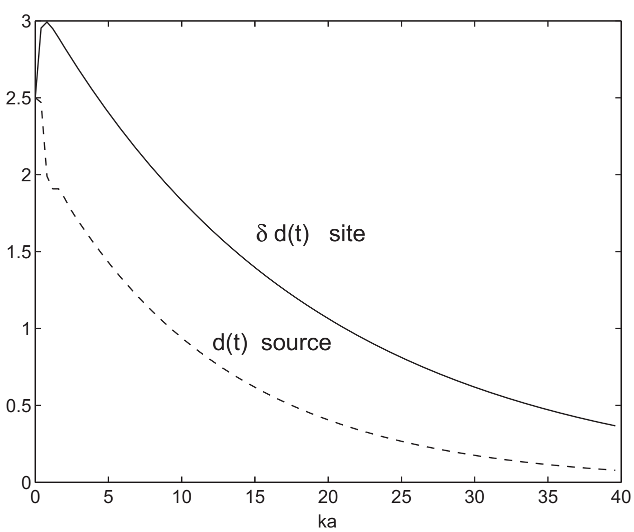

Fig. 10. Impulse responses from the estimated AR fit coefficients $\tilde{a}(n)$ for the two temperature time series. Time is in 10^3 yr.

Wunsch (2003) estimated that the obliquity and precessional peaks accounted for 8% and 3% of the record variance, respectively, as an upper bound. Thus approximately 89% of the record variance is unexplained.

2.4. DSDP 607

DSDP 607 from the subpolar North Atlantic was discussed recently by Raymo and Nisancioglu (2003). They describe the record (Fig. 11) before about −800 ka as being dominated by obliquity, and among other points, seek to rationalize the absence of precessional band energy. Fig. 12 displays the power density spectra before and after −800 ka. There is a quite remarkable obliquity band peak in the earlier part of the record (and this record was not tuned by Raymo and Nisancioglu, 2003). Note however, that the amount of energy at 40 ka period is hardly changed in the time after −800 ka, and notably, one sees the *addition* of energy at longer periods, without any significant reduction in obliquity-band energy.

It is important to note too, that the obliquity peak, generously, contains less than 10% of the record variance before −800 ka (and much less afterwards). Ninety percent of the energy in this record lies elsewhere, and an assertion that the pre −800 ka period is obviously dominated by obliquity forcing is surprising.

The simplest description of the myriadic band before −800 ka is again that of a rednoise process with a superimposed, conspicuous, obliquity peak. There is no

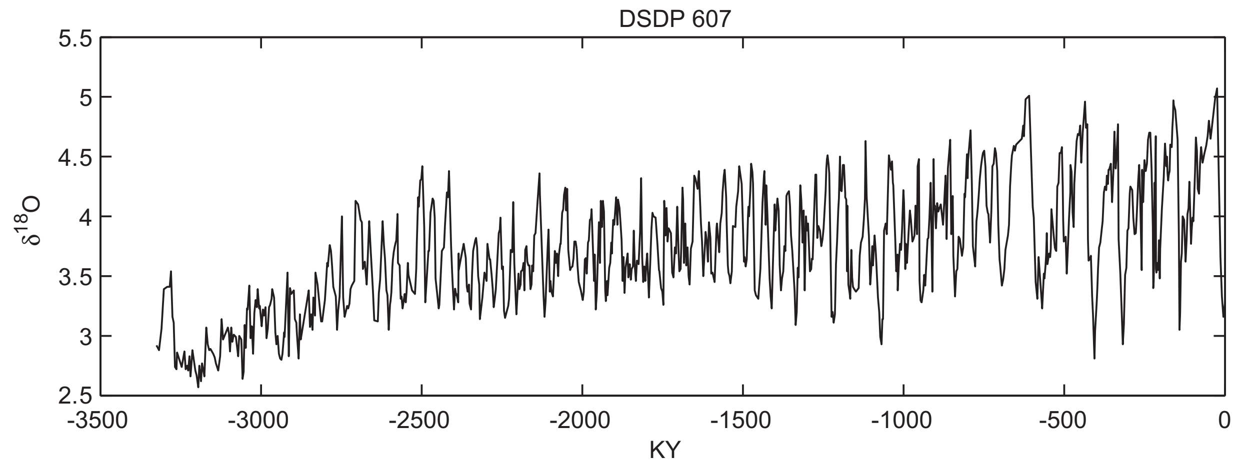

Fig. 11. δ^{18}O from DSDP607. Note the visual change in character occurring at about −800 Kyr.

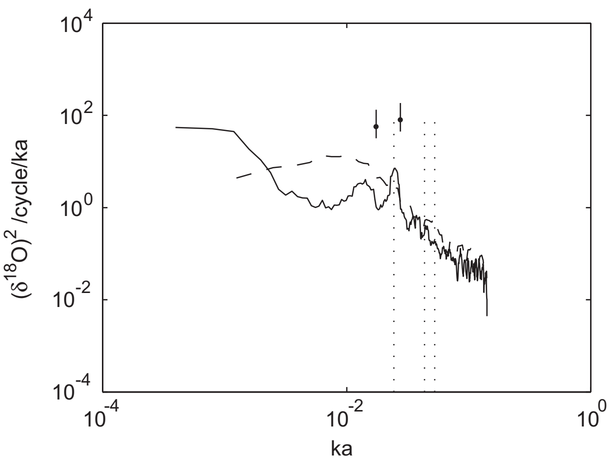

Fig. 12. Power density spectral estimates of the DSDP607 values of δ^{18}O before (solid line) and after −800 Kyr. The spectral density of the last 800 Kyr has much more energy at periods around 100 Kyr than one from before that time, but is, at higher frequencies, largely unchanged. The two approximate 95% confidence intervals are shown (rightmost one is for the recent interval).

evidence that obliquity controls the variability of the record apart from the less than 10% contribution.

3. The SPECMAP stack

The so-called SPECMAP stack (Imbrie et al., 1984) is an average of the δ^{18}O from five orbitally tuned deep-sea cores and is shown in Fig. 13, along with a depth-adjusted stack of 21 cores (Huybers and Wunsch, 2004). Averaging should enhance any deterministic components—which theoretically add constructively—relative to the incoherent stochastic components in the record. Assuming, in the absence of other information, that the background stochastic continuum is completely incoherent spatially, the power in the average background should be reduced by a factor of $\frac{1}{5}$ relative to the deterministic component as compared to its value in any individual core. Despite this anticipated background variance reduction, analysis of the SPECMAP stack (not shown; see Huybers and Wunsch, 2004) produced an upper bound of 11% of the stack variance in the obliquity band and 10% in the precessional one, leaving a minimum of 80% unaccounted for. (Failure to achieve a greater amplification of the deterministic components in the summation can arise from at least partial failure of any of the various assumptions, including fixed deterministic phases in all records, incoherence of the stochastic background, etc.) The presence of obliquity energy is clear—permitting orbital tuning with some skill—if phase relations are assumed; its lack of record dominance is equally clear.

4. Is the 100 ka energy deterministic?

Some of the several explanations proposed for the 100 ka-time-scale glacial/interglacial cycle rely upon its control by the weaker, higher frequency Milankovitch forcing. A representative example of the large literature on the subject is the paper of Ridgwell et al. (1999) who, using the SPECMAP stack, show the apparent coincidence of deglaciations with four *or* five precessional cycles measured at 65°N. Such an inference implies that the 100 ka band is deterministic and not stochastic, so that one way of obtaining insight into governing mechanisms is to examine that hypothesis.

First consider the particular inference. Ridgwell et al. (1999) employ the last seven glacial/interglacial events, providing six intervals, ΔT, whose approximate value is 100 ka. Taking the precessional period as nominally $T_p = 20$ ka, one has $\Delta T/T_p \approx 5$, so that as long as a dominant timescale of 100 ka is identifiable, finding an

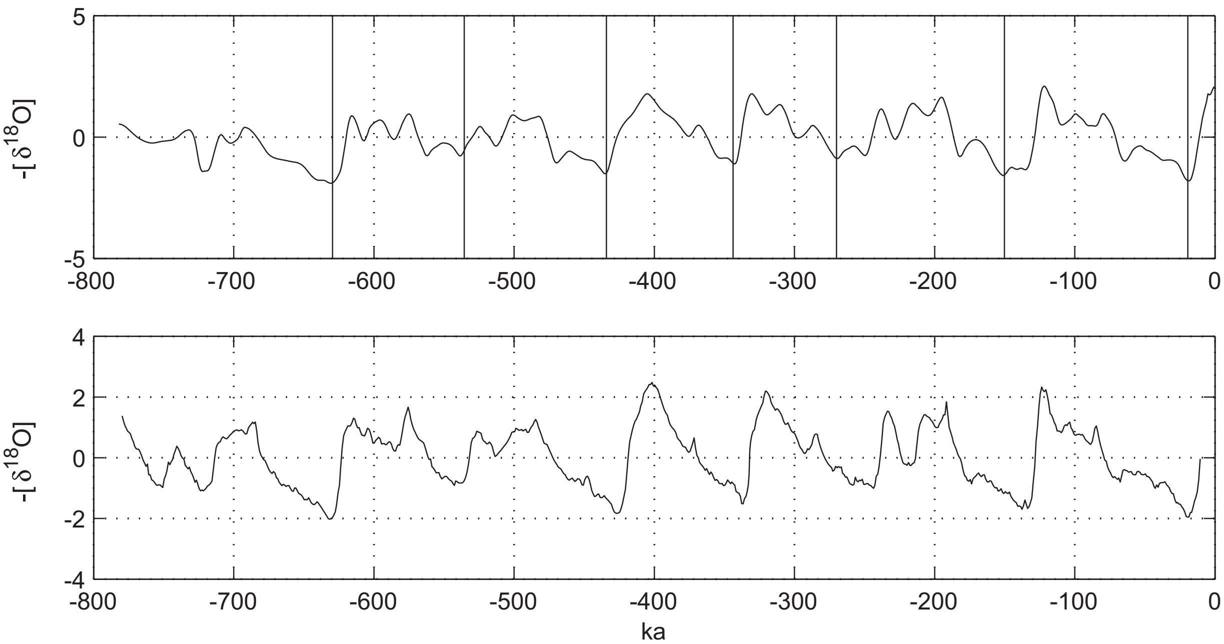

Fig. 13. (Upper panel) The SPECMAP stack, with origin time adjusted to the revised Brunhes-Matuyama transition. Vertical lines show the minimum (maximum glacial) times used as one of several ways to calculate intervals between cycles. (Lower panel) Same as the upper panel, except from the depth adjusted cores of Huybers and Wunsch (2004) in which no astronomical information was used. A gross similarity exists between the two curves, but they differ in important details.

average of five precession periods per glacial/interglacial is inevitable. To the extent that one then extends the recipe to include also as few as four precessional cycles as Ridgwell et al. (1999) do, the evidence for determinism begins to look (a) shaky, and (b) primarily a consequence of the two timescales. The SPECMAP stack has (optimistically) errors of ± 5 ka (see Huybers and Wunsch, 2004), which permits a further 25% adjustment in the precessional cycle relative to the 100 ka time-scale.

Formal tests exist for deterministic signals embedded in a continuum background. A particularly simple one is for signals which are sinusoidal: consider a sinusoid of amplitude a_m, which is a harmonic, m, of the record length T_1, so that its frequency is m/T_1. If a Fourier series coefficient (properly normalized) is computed from a record of length MT_1, where M is an integer, then one obtains a Fourier series amplitude from the new, longer record, which is nearly unchanged. On the other hand, if the energy at and near frequency m/T_1 is stochastic, its root-mean-square value should fall to $a_1/\sqrt{M}$. Thus by monitoring the behavior of the Fourier coefficient as a function of record length, one can test the hypothesis of a dominant sinusoid. Application of this idea (Huybers, personal communication, 2003) shows no evidence for periodicity near 100 ka period (although one cannot rule out the presence of a weak deterministic signal buried in the stochastic continuum).

Rather than employing another formal test (six cycles is a very small sample), let us use a more qualitative description. From Fig. 13, it is evident that deciding what is the duration of any particular glacial/interglacial cycle is not so easy. One might choose the intervals, ΔT, between the maxima (of the interglacial), of the minima (maxima of the glacial), or the downward-trend zero crossings (and other possibilities exist). Fig. 14 displays the intervals ΔT from each of these three criteria. None of them appears particularly stable or indicative of a periodic process. The estimated errors of ± 5 ka, if independent, do permit considerable adjustments in these values to make them more nearly uniform, but if those adjustments are made, one is then assuming periodicity rather than demonstrating it.

A simple conclusion is that describing the 100 ka energy band as stochastic is not in conflict with the SPECMAP stack (and similar results are found, e.g., from either record in Fig. 1). As in the conclusion of Roe and Allen (1999), the stochastic description cannot be rejected. The problem is that seven glacial cycles, producing six glacial intervals—is an extremely small sample for distinguishing competing hypotheses. (There is an analogue in the problem of explaining the so-called Titius–Bode law of planetary separation, and characterized e.g., by Efron (1971) as a problem which, "For a statistician, fitting a three-parameter curve of uncertain form to ten points with three exceptions certainly brings one to the far edge of the known world".)

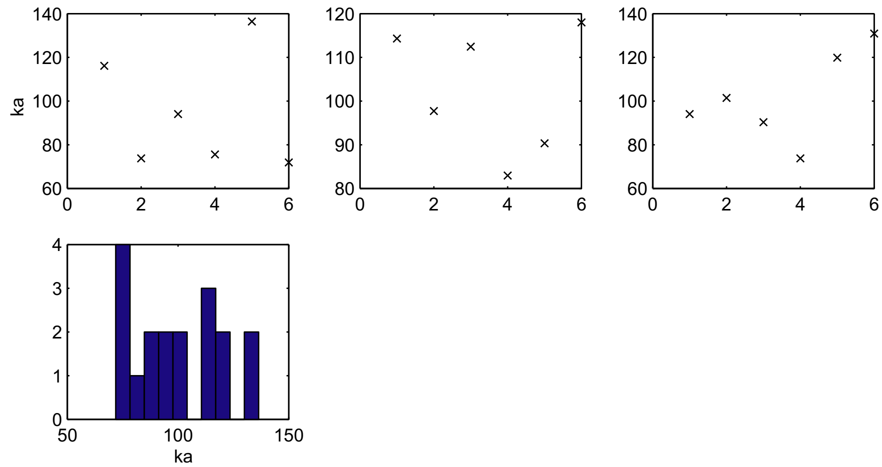

Fig. 14. Three time-differences defined for the SPECMAP stack shown in Fig. 13. In top panels, from left to right, the plotted time intervals are between the maxima, zero-crossings, and minima of the SPECMAP stack. Note the scale changes. The zero-crossing intervals are the most stable, although still showing a large scatter. One obtains similar results for the depth-tuned cores. Lower left panel is the histogram of all three sets of times.

5. Discussion

An almost unlimited number of records can be analyzed in this fashion, and it remains possible that some records will yet emerge in which the Milankovitch bands clearly dominate the climate variability. Although one can conceive of a climate system controlled by the small orbital perturbations, little concrete evidence exists that the real system has that character. As remarked above, one can hypothesize a system with no orbital forcing at all, and suggest that it would nonetheless demonstrate a great deal of climate variability. In practice, one expects response to *both* deterministic and stochastic forcing, and it is unrealistic to expect that the presence of one wholly precludes the other; a quantitative partition is required.

This study raises a number of issues. As in many paleoclimate studies, one must establish an age-depth model; orbital tuning is one approach. But even if a record is tuned to obliquity or precessional forcing, the resulting variance in those frequency bands is a fraction (commonly less than 10%) of the record energy. Depending upon the record, the Milankovitch bands contain a larger fraction of the myriadic band energy—again up to about 20%; exactly how to measure the importance of the insolation forcing, whether relative to the entire climate record, or relative only to the variance in the myriadic frequency band, becomes a matter of taste. But in either case, such results can hardly be said to ‘‘explain’’ the record.

Many records, with or without orbital tuning, show small energy excesses at the obliquity and precessional frequency. In no record we have seen, does one or both of these bands dominate by the simple measure of carrying most of the total record, or even just myriadic band, energy. Precessional band energy is also problematic, as discussed by Huybers and Wunsch (2003), as some unknown, and possibly very large, fraction can be an artifact of rectification in the recording mechanisms of the seasonal cycle. The presence of the strong obliquity band in DSDP607 prior to -800 ka supports some degree of astronomical tuning, albeit the phase relationships between forcing and response remain unknown.

One form of the Milankovitch hypothesis asserts that while the direct, linear, quaternary response in the Milankovitch bands is weak, that the much more energetic 100 ka glaciation time-scale is nonetheless controlled by interactions among the higher frequency precessional and obliquity forcing. Most of the records however, show that the 100 ka energy is indistinguishable from a broadband stochastic process. The most straightforward hypothesis is that it too, is a form of random walk, whether involving some type of large-scale instability of the climate system under full glacial conditions, as a number of authors have proposed, or merely one displaying the intermittent build up and asymmetric (in time) removal of ice, or some combination (Wunsch, 2003). Occam's razor suggests at least maintaining this hypothesis until it is proved untenable. The long-standing question of how the slight Milankovitch forcing could possibly force such an enormous glacial–interglacial change is then answered by concluding that it does not do so. (Huybers and Wunsch (2004), show that there is a conventional weak non-linear interaction of the 100 ka and obliquity frequency bands as well as self-interactions—producing still weaker sum and difference frequencies, with no indication of any stronger coupling among them.) The appeal of explaining the glacial/interglacial cycles by way of the Milankovitch forcing is clear: it is a deterministic story. But the rather modest variance directly explained by the orbital components supports the inference that the stochastic contribution cannot be ignored solely because it is not deterministic.

Many papers have made the point that there are problems with the Milankovitch hypothesis (e.g., Kominz and Pisias, 1979; Imbrie and Imbrie, 1980; Winograd et al., 1992; Karner and Muller, 2000). The simplest explanation of the results they describe is that they are not dominated by the Milankovitch forcing, but rather reflect the much more energetic background spectral continuum.

The shift from a 40 ka world to a 100 ka world requires some comment. As we have seen particularly in DSDP607 (and which has been remarked many times before), the 40 ka world is primarily distinguished by the absence of the 100 ka glacial–interglacial cycles, and there is a prominent obliquity peak. But the record is not actually dominated by the obliquity period energy. After about −800 ka, large glacial–interglacial changes begin to occur on an approximately 100 ka time-scale (but not periodically) superimposed upon the myriadic variability which continues largely unchanged. Why did 100 ka glacial–interglacials also became possible in addition to the myriadic ice volume variability? Lowering of global CO_2 below some critical threshold, or changes in continental configuration, or atmospheric circulation patterns, or all together, are among the conceivable possibilities (e.g., Raymo, 1998).

6. Summary

The main issue is the small sample size: seven Pleistocene ice ages with six intervals. The resulting record of myriadic climate variability in deep-sea and ice cores is dominated by processes indistinguishable from stochastic, apart from a very small amount (less than 20% and sometimes less than 1%) of the variance attributable to insolation forcing. Climate variability in this range of periods is difficult to distinguish from a form of random walk with small superimposed deterministic elements. Evidence that Milankovitch forcing "controls" the records, in particular the 100 ka glacial/interglacial, is very thin and somewhat implausible, given that most of the high frequency variability lies elsewhere. These results are not a proof of stochastic control of the Pleistocene glaciations, nor that deterministic elements are not in part a factor. But the stochastic behavior hypothesis should not be set aside arbitrarily—as it has at least as strong a foundation as does that of orbital control.

There is a common view in the paleoclimate community that describing a system as "stochastic" is equivalent to "unexplainable". Nothing could be further from the truth (e.g., Gardiner, 1985): stochastic processes have a rich physics and kinematics which can be described and understood, and even predicted.

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