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Constraints on the lake volume required for hydro-fracture through ice sheets

M. J. Krawczynski,[1] M. D. Behn,[2] S. B. Das,[2] and I. Joughin[3]

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[1] Water-filled cracks are an effective mechanism to drive hydro-fractures through thick ice sheets. Crack geometry is therefore critical in assessing whether a supraglacial lake contains a sufficient volume of water to keep a crack water-filled until it reaches the bed. In this study, we investigate fracture propagation using a linear elastic fracture mechanics model to calculate the dimensions of water-filled cracks beneath supraglacial lakes. We find that the cross-sectional area of water-filled cracks increases non-linearly with ice sheet thickness. Using these results, we place volumetric constraints on the amount of water necessary to drive cracks through ~1 km of sub-freezing ice. For ice sheet regions under little tension, lakes larger than 0.25–0.80 km in diameter contain sufficient water to rapidly drive hydro-fractures through 1–1.5 km of subfreezing ice. This represents ~98% of the meltwater volume held in supraglacial lakes in the central western margin of the Greenland Ice Sheet. **Citation:** Krawczynski, M. J., M. D. Behn, S. B. Das, and I. Joughin (2009), Constraints on the lake volume required for hydro-fracture through ice sheets, *Geophys. Res. Lett.*, *36*, L10501, doi:10.1029/2008GL036765.

1. Introduction

[2] During the summer much of the ablation zone of the Greenland Ice Sheet is covered with supraglacial melt water lakes. These lakes are controlled by bedrock topography and thus form in the same position from year to year, filling gradually during the melt season from June–August before in some cases draining rapidly in the late summer [*Box and Ski*, 2007]. This type of drainage has also been seen in smaller cold-temperate glaciers [*Boon and Sharp*, 2003]. *Das et al.* [2008] observed a 3-fold increase in the background flow rate of the Greenland Ice Sheet coinciding with the rapid (~2 hr) drainage of a large (~2.7 km average diameter) supraglacial lake near Jakobshavn Isbræ in the summer of 2006. This drainage event was characterized by rapid and coincident ice sheet uplift and elevated seismicity providing strong evidence that in this location surface melt water reached the base of the ice sheet by a hydro-fracture process. It remains unclear, however, whether hydro-fracture occurs beneath only the largest lakes and what fraction of the water contained in supraglacial lakes is transported to the base of the ice sheet. Thus, constraints on the dynamic advection of water from the surface to the bed and the subsequent impact on ice flow will be critical for climatologists to incorporate into models of the future behavior of ice sheets and accompanying changes in global sea level [*Intergovernmental Panel on Climate Change*, 2007].

[3] The goal of this study is to determine the extent to which supraglacial lakes can supply water to the base of thick, cold ice sheets via the propagation of water-filled cracks. The problem of crevasse propagation was originally outlined in a series of classic papers by *Weertman* [1971a, 1971b, 1973, 1996] in which he showed that due to the density contrast between ice and water, a water-filled crack will continue to propagate until it reaches the base of an ice sheet. If a crack remains water-filled during its evolution, which is likely the case for cracks formed directly beneath supraglacial lakes, the propagation depth is limited only by the volume of water available to fill the crack (Figure 1). Thus, determination of crack opening widths is of utmost importance in evaluating whether a supraglacial lake contains a sufficient volume of water to keep the crack water-filled until it reaches the bed. Here we adopt the Weertman dislocation-based fracture mechanics model for crack propagation in ice, and quantify the evolution of the crack shape. By doing so, we constrain the volume of water necessary to keep fractures water-filled and thus hydraulically drive them to depths corresponding to the thickness of the Greenland Ice Sheet under the ablation region. Because supraglacial lakes have finite volumes, calculating the crack opening geometry is a key improvement over previous studies that simply assumed an *a priori* crack width [e.g., *Alley et al.*, 2005; *van der Veen*, 2007].

2. Two-Dimensional Model for Water-Filled Fracture Geometry

[4] We model water-filled crack propagation in a 2-D ice sheet with an elastic half-space model geometry (Figure 1, inset). For an edge crack to propagate in a solid medium, the stress intensity at the crack tip must exceed the fracture toughness of the material. The maximum depth of propagation will depend on the deviatoric (longitudinal) stress, the hydrostatic pressure of the ice, the amount of water filling the crack, and the thickness of the ice sheet. Using the propagation depth determined above, we calculate the opening displacement from the integral of the dislocation density over the length of the crack, which is proportional to the depth of crack penetration [*Weertman*, 1971a, 1996]. See auxiliary material[1] for a complete description of the methods used to calculate crack geometry.

[1]MIT/WHOI Joint Program, Cambridge, Massachusetts, USA.
[2]Department of Geology and Geophysics, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA.
[3]Applied Physics Lab, University of Washington, Seattle, Washington, USA.

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[1]Auxiliary materials are available in the HTML. doi:10.1029/2008GL036765.

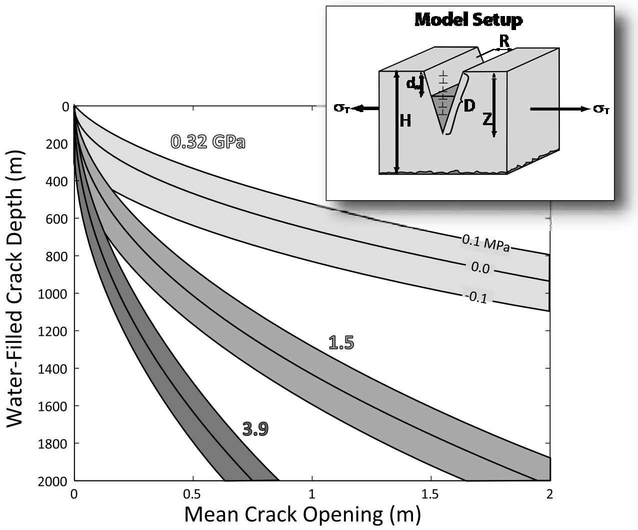

Figure 1. Mean opening width versus depth of a water-filled crack for varying far field stresses. Mean openings are shown for 3 different shear moduli of ice (0.32, 1.5, and 3.9 GPa illustrated in darkening shades of grey). Each field is centered on the curve for a neutral stress regime, and shows the range of differential stresses from −0.1 MPa (compression) to 0.1 MPa (tension). Calculations assume a 100% water filled crack. Figure inset shows the variables and geometry considered for our model.

[5] Using this approach we investigate the influences of longitudinal stress, ice sheet thickness, and the amount of water filling the crack on the predicted crack geometry. Because natural ice shows a variation in its elastic moduli due to variations in strain rate, grain size, impurities, and temperature, and because these conditions are poorly understood for ice in thick ice sheets, we use a range of values for the shear modulus of ice, from 0.32–3.9 GPa [*Vaughan*, 1995]. The cross-sectional area of a crack can then be determined by integrating the opening geometry over the depth of the crack, and extended in the third dimension to obtain a volume. We note that the cracks described here form on time scales (hours) short enough to ignore the viscous flow of ice, and are thus modeled as a purely elastic phenomenon.

[6] Field observations of crevasse opening widths at depth are difficult to make because most crevasses visible on the surface have been modified by melt-back, calving, plastic flow of ice, and/or 'healed' by water refreezing within the crack. In our model we only consider the simplest case of an isolated crack; however since a water-filled crevasse can propagate without longitudinal tension, reduction of longitudinal stress by the blocking effect of neighboring crevasses is not as important for water-filled cracks as it is for dry ones [*Nye*, 1955].

3. Model Results

[7] For a dry crack, the increase in overburden stress with depth will eventually cause the stress intensity at the crack tip to decrease below the fracture toughness and prevent further propagation of the crack. When considering far field stresses within the range observed in nature (0–0.5 MPa) dry cracks will only reach depths of order 10^1 m, consistent with field observations. However as has been shown by previous studies [e.g., *Weertman*, 1971a; *Alley et al.*, 2005; *van der Veen*, 2007], if a crack remains water-filled, the density of the water will offset the overburden stress and allow for continued propagation. Because the crack must remain nearly completely water-filled to reach the base of thick ice sheets (Figure S2), we use the crack volume to constrain the minimum water volume that must be contained in a supraglacial lake in order for it to be capable of driving a hydro-fracture to the base of the ice sheet.

[8] Our calculations show that typical opening widths vary between hundreds of microns for short cracks (<10 m; consistent with *Kenneally and Hughes* [2006]), up to 1–2 meters for either very deep cracks (>1500 m) or high longitudinal tensile stress gradients. In contrast to dry cracks which have 'V' shaped cross-sections, water-filled cracks are 'U' shaped with little variation in opening width for most of their depth. Our field observations support these predictions for both the magnitude of opening and a uniform opening distribution with depth (Figures S4 and S5). Intriguingly, we find that for any fixed volume of water, water-filled cracks are predicted to propagate deepest in regions with less tension (or even slight *compression*) because thinner cracks require less water to remain filled. Because supraglacial melt-water lakes often form in compressive stress regimes due to the concave topography of the lake basin, this surface geometry would facilitate the propagation of deep cracks that can fully drain a lake.

[9] Water flow through the crack systems is calculated to be turbulent, with the Reynolds number on the order of $\sim 10^6$. Using the average crack opening, we then estimate the flux of water through the ice sheet assuming turbulent flow for end member cases of channel and pipe flow [*White*, 1974]. We include the case for pipe flow because the initial planar crack is not infinite in its horizontal extent, and likely localizes to one or more discrete moulins with time during and following lake drainage [*Das et al.*, 2008], similar to the evolution in vent geometry observed for volcanic fissure eruptions [*Bruce and Huppert*, 1990].

[10] We calculate drainage time for a lake with a conduit established by hydro-fracture as a function of lake size and conduit geometry (Figure 2). For these calculations we generalize lake geometries to a conical pool of water with a diameter-to-depth aspect ratio of 100:1. This approximation is consistent with surface slopes and depth-to-diameter relationships observed in 2 lakes south of Jakobshavn Isbræ [*Das et al.*, 2008] and yields depths up to 10s of meters for lakes with diameters >1 km, consistent with lake depths reported by previous studies in Greenland [*Box and Ski*, 2007; *Sneed and Hamilton*, 2007; *McMillan et al.*, 2007]. Given this lake geometry and channel flow along a crack with a length equal to the lake diameter (as observed in the field) these calculations predict drainage times of 2–5 hours for lake diameters of 0.1–3 km, consistent with the drainage time observed by *Das et al.* [2008]. As discussed by *Alley et al.* [2005], the refreezing time of a 10-cm crevasse (the likely minimum width required to propagate through a 1-km thick ice sheet) in −20°C isothermal ice is 3–10 times

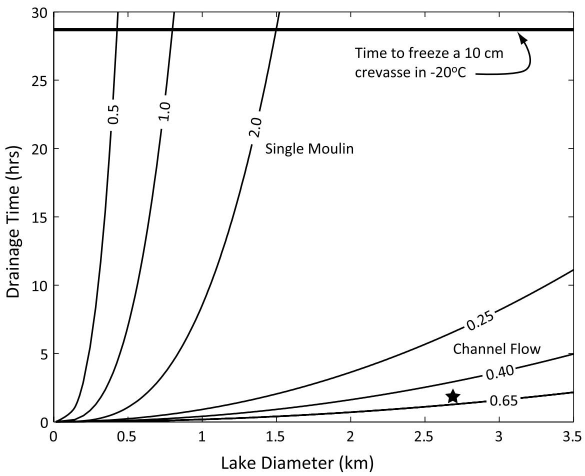

Figure 2. End member drainage times are calculated assuming pipe (moulin) and channel (crack) flow. Pipe flow curves are labeled with conduit diameter, and the channel flow curves are labeled by opening width (in meters). The time to refreeze a 10 cm crevasse is also shown (thick black line [*Alley et al.*, 2005]). All calculations assume zero differential stress. Star illustrates the drainage time of a 2.7 km diameter supraglacial lake observed by *Das et al.* [2008].

slower than the predicted drainage times (Figure 2), implying that it is unlikely a water-filled crack will heal before lake drainage is complete.

[11] Drainage times calculated for a single moulin assuming pipe flow are significantly longer than for channel flow along a crack. For example, a moulin with a 1-m diameter would take >100 hr to drain a 2-km diameter lake, while a ~0.5 m crack would drain the same lake in about 1.2 hrs. Moreover, to drain a 2-km lake through a moulin in 2 hours, similar to the time-scale observed by *Das et al.* [2008], would require a ~7.5 m diameter pipe. Our field observations suggest that the majority of rapid drainage events occur through long cracks, with localization of melt water flow into discrete moulins occurring only late in the drainage event after flow rates slow [*Das et al.*, 2008].

[12] It should also be noted that *Fountain et al.* [2005] have shown that in some small temperate alpine glaciers the hydrological system can be dominated by fractures that convey water at slow speed englacially, rather than through a direction surface-to-bed connection. While our model results can not entirely rule out this possibility, the ice sheet conditions encountered beneath most supraglacial lakes (subfreezing englacial temperatures and high overburden stress) are substantially different than those found in thin, temperate glaciers, and would argue against the formation and persistence of englacial conduits. Surface observations during a lake drainage event also suggest a direct surface-to-bed connection [*Das et al.*, 2008].

4. Implications for Supraglacial Lake Drainage

[13] As discussed above, the opening geometry of a crack is controlled by the depth of propagation, while the propagation depth is limited by the available volume of water (Figure 1). To apply our model results to natural settings, we calculated the minimum lake size required to drive water-filled cracks to the base of an ice sheet of given thickness (Figure 3). The total flux of water from supraglacial lakes to the ice sheet bed in a specific area can then be determined by summing the volume of all lakes larger than this minimum size. Because the most easily observable characteristic of supraglacial lakes is their surface area, we can use this area to calculate a volume assuming a conical shape and a diameter-depth ratio of 100:1 as discussed above. There are more sophisticated methods for measuring supraglacial lake volume, and there is a growing database available for seasonal lake volumes [*Sneed and Hamilton*, 2007; *Box and Ski*, 2007; *Das et al.*, 2008], which can be incorporated into future modeling studies.

[14] For this study we analyzed the maximum extent of supraglacial lakes that formed along a 500 km North-South stretch of the West Greenland Ice Sheet centered near Jakobshaven Isbræ between June–August 2006 (Figure 4a). The ice thickness in this region is ~1 km and we assume zero longitudinal stress. Under these conditions a water-filled crack will have a cross-sectional area of ~750 m^2, implying that lakes with surface diameter >250–800 m (depending on the shear modulus used) will deliver water to the bed. Lakes as small as 250 m can be hard to detect remotely due to the coarse resolution of the MODIS data used in this study (see auxiliary material), so to account for the volume of water held in small lakes we fit an exponential function of the form $y = y_o \exp(bx)$ to the size-frequency distribution of supraglacial lakes observed in this region in 2006 (Figure 4b). To determine the volume of melt water held in lakes with the potential to drain to the base, we integrated this function from the minimum lake size (diameter) calculated to deliver water to the bed (~500 m) to the maximum lake size observed (3.3 km). The total volume of melt water held in lakes large enough to drain is ~5.7 km^3, which represents ~98% of all the melt water held in supraglacial lakes in the study area. There is considerable uncertainty in this estimate because our model does not allow us to predict which lakes will drain in a given year. If a majority of lakes drain, however, the annual

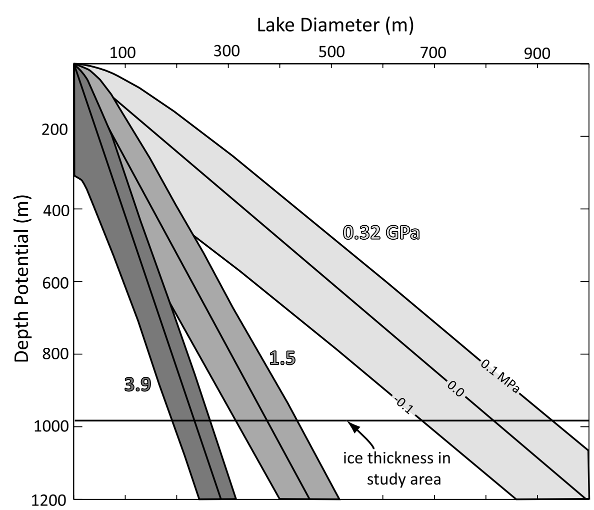

Figure 3. Maximum propagation depth of a water-filled crack versus lake-diameter. Stress and moduli contours are as in Figure 1. Thick black line shows ice thickness of 980 m in the western Greenland study area.

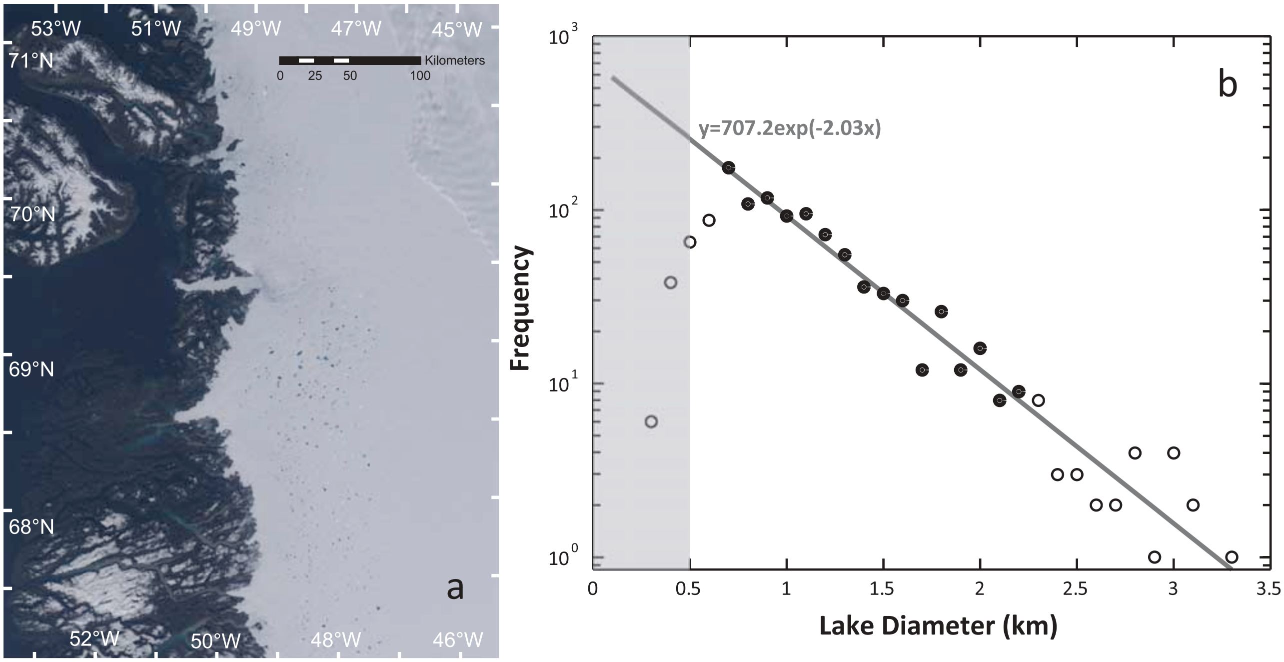

Figure 4. (a) Satellite view of the supraglacial lakes on the West Greenland Ice Sheet near Jakobshaven Isbræ during summer 2006. (b) Supraglacial lake size-frequency distribution binned in increments of 100 m diameter. Filled circles show data that were regressed, open circles show data for lakes that are too small to be reliably counted by the satellite resolution, or too large to be statistically meaningful. The shaded region shows the lakes that are too small to propagate a crack to the ice sheet bed. Based on this distribution, there are >1000 lakes that contain sufficient water to drive hydro-fractures to the ice bed.

flux of water to the ice sheet bed could even be larger than we calculate because once a lake has drained moulins typically remain open along the crack for the remainder of the melt season facilitating further drainage [*Das et al.*, 2008]. Our estimates are consistent with other independent estimates of melt water drained from the surface in this area of the Greenland Ice Sheet [*McMillan et al.*, 2007].

5. Conclusions

[15] Our calculations show that lakes that are only ~250–800 m across and 2–5 m deep contain a sufficient volume of water to drive a water-filled crack to the base of a 1 km-thick ice sheet. Lakes that are smaller may also be drained, however it requires fractures that are fed by multiple basins. This range in lake sizes represents the majority of supraglacial lakes in the ablation zone along the western margin of the Greenland Ice Sheet. Thus we propose that a large fraction of the melt water produced in the summer (on the order of several cubic kilometers) could rapidly reach the base of the ice sheet via this mechanism.

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M. D. Behn and S. B. Das, Department of Geology and Geophysics, Woods Hole Oceanographic Institution, 266 Woods Hole Road, Woods Hole, MA 02543, USA.
I. Joughin, Applied Physics Lab, University of Washington, 1013 NE 40th Street, Seattle, WA 98105-6698, USA.
M. J. Krawczynski, MIT/WHOI Joint Program, 77 Massachusetts Avenue, Cambridge, MA 02139, USA. (kraw@mit.edu)